Definiteness of Matrices

Matrix A of dimension $n \times n$ is **negative semidefinite** if,
$\mathbf{x}^T A\mathbf{x} \leq 0$ for all $\mathbf{x} \in \Re^n$.

If the inequality is *strict* for all $\mathbf{x} \neq 0$, then the matrix A is **negative definite.**

On the contrary, matrix A of dimension $n \times n$ is **positive semidefinite** if,
$\mathbf{x}^T A\mathbf{x} \geq 0$ for all $\mathbf{x} \in \Re^n$.

If the inequality is *strict* for all $\mathbf{x} \neq 0$, then the matrix A is **positive definite.**

Example

The simplest $n \times n$ matrices are the diagonal matrices. They also correspond to quadratic form since

$$\begin{bmatrix} x_1 & x_2 & \cdots & x_n \end{bmatrix} \begin{bmatrix} a_1 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & a_2 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot & a_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix} = a_1x_1^2 + a_2x_2^2 + \cdots + a_nx_n^2$$

Clearly, this quadratic form will be positive definite if and only if all of the diagonals in A are positive with at least one strictly positive and vice versa.