Diagonalization

In a previous section, we examined the LU Decomposition of matrix A. In this section, we examine another important factorization of A. **Diagonalization** is a display of A in the form $A = PDP^{-1}$. Diagonalization enables us to compute A^k quickly for large values of k, a fundamental idea in several applications of linear algebra. The D in the factorization stands for the diagonal matrix. Powers of such a D are trivial to compute.

Example

If $D = \begin{bmatrix} 11 & 0 \\ 0 & 9 \end{bmatrix}$, calculate D^2 and D^3.

$$D^2 = \begin{bmatrix} 11 & 0 \\ 0 & 9 \end{bmatrix} \begin{bmatrix} 11 & 0 \\ 0 & 9 \end{bmatrix} = \begin{bmatrix} 121 & 0 \\ 0 & 81 \end{bmatrix} = \begin{bmatrix} 11^2 & 0 \\ 0 & 9^2 \end{bmatrix}$$

$$D^3 = \begin{bmatrix} 11 & 0 \\ 0 & 9 \end{bmatrix} \begin{bmatrix} 11 & 0 \\ 0 & 9 \end{bmatrix} \begin{bmatrix} 11 & 0 \\ 0 & 9 \end{bmatrix} = \begin{bmatrix} 11 & 0 \\ 0 & 9 \end{bmatrix} \begin{bmatrix} 121 & 0 \\ 0 & 81 \end{bmatrix}$$

$$= \begin{bmatrix} 1331 & 0 \\ 0 & 729 \end{bmatrix} = \begin{bmatrix} 11^3 & 0 \\ 0 & 9^3 \end{bmatrix}$$

In general, $D^p = \begin{bmatrix} d^p & 0 \\ 0 & d^p \end{bmatrix}$ for $p \geq 1$