

Distances

Remember that our objective, is to be able to use the concept of distance to arrive at a solution vector $\hat{\mathbf{x}}$, which is the closest possible solution to the linear system $A\mathbf{x} = \mathbf{b}$. We are ready now, to describe proximity between two vectors. Recall that if α and β are real numbers, the distance on the number line between the two numbers is $|\alpha - \beta|$. For \mathbf{v} and \mathbf{w} in \mathbb{R}^n , the **distance** between \mathbf{v} and \mathbf{w} , written as $dist(\mathbf{v}, \mathbf{w})$, is the length of the vector $\mathbf{v} - \mathbf{w}$. That is, $dist(\mathbf{v}, \mathbf{w}) = \|\mathbf{v} - \mathbf{w}\|$.

Example

Compute the distance between the vectors $\mathbf{v} = (3, 4)$ and $\mathbf{w} = (3, 2)$.

Step 1:

Calculate

$$\mathbf{v} - \mathbf{w} = \begin{bmatrix} 3 \\ 4 \end{bmatrix} - \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

Step 2:

Calculate

$$\|\mathbf{v} - \mathbf{w}\| = \sqrt{0^2 + (2)^2} = \sqrt{4} = 2$$