Equivalence Between Vector Equations and Matrix Equations

If $\mathbf{A}$ is an m×n matrix, with columns $\mathbf{a}_1, \mathbf{a}_2, ..., \mathbf{a}_n$, and if $\mathbf{b}$ is in $\Re^m$, the matrix equation $A\mathbf{x} = \mathbf{b}$ has the same solution set as the vector equation $x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + ... + x_n\mathbf{a}_n = \mathbf{b}$ which in turn, has the same solution set as the system of linear equations whose augmented matrix is $\begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \cdots & \mathbf{a}_n & \mathbf{b} \end{bmatrix}$.

The theorem provides a powerful tool for gaining insight into problems in linear algebra because we can now view a system of linear equations in three different but equivalent ways: as a system of linear equations, as a matrix equation, or as a vector equation.