

## Homogeneous Linear Systems

A system of linear equations is said to be **homogeneous** if it can be written in the form  $A\mathbf{x} = \mathbf{0}$ , where  $A$  is an  $m \times n$  matrix and  $\mathbf{0}$  is the zero vector in  $\mathfrak{R}^n$ . Such a system has always at least one solution, namely  $\mathbf{x} = \mathbf{0}$ . This zero solution is called the **trivial solution**. For a given equation  $A\mathbf{x} = \mathbf{0}$ , the important question is whether there exists a **nontrivial solution**, that is, a nonzero vector  $\mathbf{x}$  that satisfies  $A\mathbf{x} = \mathbf{0}$ . The homogeneous equation  $A\mathbf{x} = \mathbf{0}$  has a nontrivial solution if and only if the equation has at least one free variable.