In this section, we consider only square matrices.

Invertible Matrices

If A is an $n \times n$ matrix, it often happens that there exists another $n \times n$ matrix A^{-1} such that $AA^{-1} = A^{-1}A = I$ where I is an $n \times n$ identity matrix. In this case, we say that A is an **invertible** (or **nonsingular) matrix** and we call A^{-1} *the* **inverse** of A.

Notice that, we call the matrix A^{-1}, "the" inverse of A and not, "an" inverse of A, because A^{-1} is *unique*.

The proof is fairly simple. Suppose not; then, there exist *at least* 2 matrices that when multiplied with A, produce the $n \times n$ identity matrix I. Without loss of generality, assume that there exist 2 such inverse matrices. Call the first inverse matrix A^{-1}, and the second inverse matrix Λ^{-1}. Recall that, by the definition of an inverse matrix we have, $AA^{-1} = I$ and $A\Lambda^{-1} = I$.

Thus, $A^{-1} = A^{-1}I = A^{-1}(\ A\Lambda^{-1}) \overset{associativity}{=} (A^{-1}A)\Lambda^{-1}) = I\Lambda^{-1} = \Lambda^{-1}$.

Therefore, when A is invertible then, its inverse is unique.