

Linear Combinations in a Crusoe Economy

Suppose that, as the book by Daniel Defoe envisions, a man (Robinson Crusoe) is stranded in a series of islands. Assume that the commodities available on island A are coconuts, pineapples and mangos while the commodities available on island B are strawberries and blueberries. Thus the commodity space lives in \mathbb{R}^5 (since there are 5 commodities available to Crusoe). To survive Crusoe has two options:

- (1) to consume daily 2 coconuts, 2 pineapples and 3 mangos, or
- (2) to consume daily 2 coconuts, 1 pineapple, 10 strawberries and 10 blueberries.

His survival consumption bundles can be entered in a vectorial form where

$$\mathbf{a} = \begin{bmatrix} \mathbf{a}_1 := \text{coconuts} \\ \mathbf{a}_2 := \text{pineapples} \\ \mathbf{a}_3 := \text{mangos} \\ \mathbf{a}_4 := \text{strawberries} \\ \mathbf{a}_5 := \text{blueberries} \end{bmatrix} \subset \mathbb{R}^5$$

His survival option 1 is $\mathbf{a}_1 = \begin{bmatrix} 2 \\ 2 \\ 3 \\ 0 \\ 0 \end{bmatrix}$ and his survival option 2 is $\mathbf{a}_2 = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 10 \\ 10 \end{bmatrix}$

- (a) What economic interpretation can be given to a vector $10\mathbf{a}_1 + 5\mathbf{a}_2$?

$$10\mathbf{a}_1 + 5\mathbf{a}_2 = 10 \begin{bmatrix} 2 \\ 2 \\ 3 \\ 0 \\ 0 \end{bmatrix} + 5 \begin{bmatrix} 2 \\ 1 \\ 0 \\ 10 \\ 10 \end{bmatrix} = \begin{bmatrix} 20 \\ 20 \\ 30 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 10 \\ 5 \\ 0 \\ 50 \\ 50 \end{bmatrix} = \begin{bmatrix} 30 \\ 25 \\ 30 \\ 50 \\ 50 \end{bmatrix}$$

This vector indicates the survival consumption bundle of Crusoe for a 15-day period. You can interpret this, as the bundle he has to accumulate if for 15 days, he wants to lie in the sun doing nothing, just consuming his daily survival portions, or maybe, the bundle he needs to accumulate to safeguard against a potential sickness that can keep him in his hut for a 15-day period.

- (b) What if Crusoe wants to know how many days he needs to consume the survival bundle 1, and how many days he needs to consume the survival bundle 2, if he is planning on consuming during a given time period the bundle

$$\begin{bmatrix} 22 \\ 20 \\ 27 \\ 20 \\ 20 \end{bmatrix} ?$$

Recall the vector equation $x_1\mathbf{a}_1 + x_2\mathbf{a}_2 = \mathbf{b}$

$$x_1 \begin{bmatrix} 2 \\ 2 \\ 3 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 1 \\ 0 \\ 10 \\ 10 \end{bmatrix} = \begin{bmatrix} 22 \\ 20 \\ 27 \\ 20 \\ 20 \end{bmatrix}$$

$$\text{Therefore } \begin{bmatrix} 2 & 2 & 22 \\ 2 & 1 & 20 \\ 3 & 0 & 27 \\ 0 & 10 & 20 \\ 0 & 10 & 20 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 11 \\ 0 & -1 & -2 \\ 0 & -3 & -6 \\ 0 & 10 & 20 \\ 0 & 10 & 20 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 9 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The reduced echelon form indicates that option 1 needs to be consumed for 9 days and option 2 needs to be consumed for 2 days.