Matrix Equations

An equation in the form $A\mathbf{x} = \mathbf{b}$ is called a **matrix equation**.

If **A** is an m×n matrix, with columns $\mathbf{a}_1, \mathbf{a}_2, ..., \mathbf{a}_n$, and if **x** is in $\Re^n$, then the product of **A** and **x**, denoted by **Ax**, is the linear combination of the columns of **A** using the corresponding entries in **x** as weights, that is:

$$A\mathbf{x} = \begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \cdots & \mathbf{a}_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ . \\ x_n \end{bmatrix} = x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + ... + x_n\mathbf{a}_n \ .$$

Note that **Ax** is defined only if the number of columns of **A** equals the number of entries in **x**.

Example

Compute $A\mathbf{x}$, where $A= \begin{bmatrix} 2 & 3 & 4 \\ -1 & 5 & -3 \\ 6 & -2 & 8 \end{bmatrix}$ and $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$.

$$A\mathbf{x} = \begin{bmatrix} 2 & 3 & 4 \\ -1 & 5 & -3 \\ 6 & -2 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2x_1 + 3x_2 + 4x_3 \\ -x_1 + 5x_2 - 3x_3 \\ 6x_1 - 2x_2 + 8x_3 \end{bmatrix}$$

Rule for computing $A\mathbf{x}$

If the product $A\mathbf{x}$ is defined, then the ith entry in $A\mathbf{x}$, is the sum of the products of corresponding entries from row i of A and from the vector **x**.

Properties of $A\mathbf{x}$

If **A** is an m×n, **u** and **v** are vectors in $\Re^n$, and α is a scalar, then:
a. $\mathbf{A}(\mathbf{u} + \mathbf{v}) = \mathbf{Au} + \mathbf{Av}$
b. $\mathbf{A}(\alpha u) = \alpha(\mathbf{Au})$