

## Subspaces

This section focusses on important sets of vectors in  $\mathbb{R}^n$  called subspaces. Often subspaces arise in connection with some matrix  $A$  and they provide useful information about the equation  $A\mathbf{x} = \mathbf{b}$ .

A **subspace** of  $\mathbb{R}^n$  is any set  $\Phi$  in  $\mathbb{R}^n$  that has three properties:

- a. The zero vector is in  $\Phi$ .
- b. For each  $\mathbf{u}$  and  $\mathbf{v}$  in  $\Phi$ , the sum  $\mathbf{u} + \mathbf{v}$  is in  $\Phi$  (closed under vector addition).
- c. For each  $\mathbf{u}$  in  $\Phi$  and each scalar  $\alpha$ , the vector  $\alpha\mathbf{u}$  is in  $\Phi$  (closed under scalar multiplication).