The Characteristic Equation

Before describing the characteristic function, let's recall some of the important properties of determinants:

Let A and B be $n \times n$ matrices.
(1) A is invertible if and only if $\det A \neq 0$.
(2) $det\ AB = (\det A)(\det B)$
(3) $det\ A^T = \det A$
(4) If A is triangular, then $\det A$ is the product of the entries on the main diagonal of A.

Statement (1) implies that we can use a determinant test to determine when a matrix $A - \lambda I$ is *not* invertible. The scalar equation $det(A - \lambda I) = 0$ is called the **characteristic equation** of A. A scalar λ is an eigenvalue of an $n \times n$ matrix A if and only if λ satisfies the characteristic equation $det(A - \lambda I) = 0$.

Example

Find the characteristic equation of $A = \begin{bmatrix} 0.6 & 0.3 \\ 0.4 & 0.7 \end{bmatrix}$

The characteristic equation is given by $det(A - \lambda I) = 0$.

Therefore

$$\det \left\{ \begin{bmatrix} 0.6 & 0.3 \\ 0.4 & 0.7 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\} = \det \left\{ \begin{bmatrix} 0.6 & 0.3 \\ 0.4 & 0.7 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right\}$$

$$= \det \begin{bmatrix} 0.6 - \lambda & 0.3 \\ 0.4 & 0.7 - \lambda \end{bmatrix} = (0.6 - \lambda)(0.7 - \lambda) - (0.4)(0.3)$$

$$= (0.6 - \lambda)(0.7 - \lambda) - 0.12$$

$$= 0.42 - 1.3\lambda + \lambda^2 - 0.12 = \lambda^2 - 1.3\lambda + 0.30 = (\lambda - 1)(\lambda - 0.3) = 0$$

Notice that λ can *only* be an eigenvalue if it is either 1 or 0.3.