The Length of a Vector

The **length** or **norm** of $\mathbf{v}$ is the nonnegative scalar $\|\mathbf{v}\|$ defined by:

$$\|\mathbf{v}\| = \sqrt{<\mathbf{v},\mathbf{v}>} = \sqrt{\mathbf{v}_1^2 + \mathbf{v}_2^2 + \cdots + \mathbf{v}_n^2}, \quad \text{and } \|\mathbf{v}\|^2 = <\mathbf{v},\mathbf{v}> .$$

Suppose that $\mathbf{v}$ is in $\Re^2$, say $\mathbf{v} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$.

If we identify $\mathbf{v}$ with a geometric point in the plane, then $\|\mathbf{v}\|$, coincides with the usual notion of the length of the line segment from the origin to $\mathbf{v}$.

Hence $\left\| \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right\| = \sqrt{< \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix} >} = \sqrt{3^2 + 4_2^2} = \sqrt{25} = 5.$