The Rank Theorem

If a matrix A has n columns, then $rank(A) + dim(Nul(A)) = n$.

Example

a. If A is a 7×9 matrix with a two-dimensional null space, what is the rank of A?

b. Could a 6×9 matrix (call it B) have a two-dimensional null space?

a. By the Rank theorem, the rank of A must be 7 since $rank(A)+dim(Nul(A)) = n = rank\ A + 2 = 9$.

b. No since by the Rank Theorem the rank of B must be 7. But the columns of B are vectors in $\Re^6$, and so the dimension of $Col\ B$ cannot exceed 6; that is the rank of B can not exceed 6.