

The Rank of a Matrix

The **rank** of a matrix A , denoted by $\text{rank}(A)$, is the dimension of the column space of A . In other words, to determine the rank of a matrix, one needs to locate and count the pivot columns.

Example

Determine the rank of the matrix $A = \begin{bmatrix} 2 & 5 & -3 & -4 & 8 \\ 4 & 7 & -4 & -3 & 9 \\ 6 & 9 & -5 & 2 & 4 \\ 0 & -9 & 6 & 5 & -6 \end{bmatrix}$

Reduce A to echelon form (does not need to be in reduced echelon form since we are only interested for the number of pivot columns).

$$A = \begin{bmatrix} 2 & 5 & -3 & -4 & 8 \\ 4 & 7 & -4 & -3 & 9 \\ 6 & 9 & -5 & 2 & 4 \\ 0 & -9 & 6 & 5 & -6 \end{bmatrix} \sim \dots \sim \begin{bmatrix} 2 & 5 & -3 & -4 & 8 \\ 0 & -3 & 2 & 5 & -7 \\ 0 & 0 & 0 & 4 & -6 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Since there are three pivot columns then $\text{rank}(A) = 3$