Vectors

A matrix with only one column (m×1) is called a **column vector** (or simply a vector). A vector is a list of numbers. For example,

$\mathbf{v} = \begin{bmatrix} -3 \\ -2 \end{bmatrix}$ is a vector in $\Re^2$ and $\mathbf{u} = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$ is a vector in $\Re^3$.

The usual algebraic properties of reals apply also to vectors. Given two vectors $\mathbf{u}$ and $\mathbf{v}$ in $\Re^2$, their sum is the vector $\mathbf{u} + \mathbf{v}$ obtained by adding the corresponding entries of $\mathbf{u}$ and $\mathbf{v}$. For example,

$$\begin{bmatrix} 3 \\ -4 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3+2 \\ -4+3 \end{bmatrix} = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$$

Given a vector $\mathbf{u}$ and a scalar α (scalars are reals in $\Re^1$), the scalar multiple of $\mathbf{u}$ is the vector $\alpha\mathbf{u}$ obtained by multiplying each entry in $\mathbf{u}$ by α. For example,

if $\alpha = 3$ and $u = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ then $\alpha\mathbf{u} = 3 \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} (3)(2) \\ (3)(-1) \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \end{bmatrix}$

Algebraic Properties of $\Re^n$

For all vectors $\mathbf{u}$, $\mathbf{v}$, $\mathbf{z}$ in $\Re^n$ and all scalars α and β:

(i) $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$

(ii) $(\mathbf{u} + \mathbf{v}) + \mathbf{z} = \mathbf{u} + (\mathbf{v} + \mathbf{z})$

(iii) $\mathbf{u} + 0 = 0 + \mathbf{u} = \mathbf{u}$

(iv) $\mathbf{u} + (-\mathbf{u}) = -\mathbf{u} + \mathbf{u} = 0$ where $-\mathbf{u}$ denotes $(-1)\mathbf{u}$

(v) $\alpha(\mathbf{u} + \mathbf{v}) = \alpha\mathbf{u} + \alpha\mathbf{v}$

(vi) $(\alpha + \beta)\mathbf{u} = \alpha\mathbf{u} + \beta\mathbf{u}$

(vii) $\alpha(\beta\mathbf{u}) = (\alpha\beta)\mathbf{u}$

(viii) $1\mathbf{u} = \mathbf{u}$