

What is a Linear Equation

A **linear equation** is one that can be written in the generic form of

$$\alpha_1 x_1 + \alpha_2 x_2 + \dots + \alpha_n x_n = \beta \quad \Leftrightarrow \quad \sum_{i=1}^n \alpha_i x_i = \beta \quad (1)$$

where β and the coefficients $\alpha_1, \dots, \alpha_n$ can be any real or complex numbers. The subscript n is a positive finite integer.

In a typical real life problem, the coefficients $\alpha_1, \dots, \alpha_n$ are known whereas the x_1, \dots, x_n vary from observation to observation hence they are appropriately called **variables**.

Note that a linear equation can take many different forms as long as it can be expressed in the generic form outlined in (1). Hence an equation of the form

$$\frac{\alpha_1 x_1 - \alpha_2 x_2 + \beta - \alpha_4 x_4}{x_3} = \alpha_3$$

is by all means linear since it is equivalent to

$$\alpha_1 x_1 - \alpha_2 x_2 - \alpha_3 x_3 - \alpha_4 x_4 = -\beta.$$

Yet, equations such as

$$\alpha_1 x_1 x_2 + \beta - \alpha_4 x_4 = \alpha_3 x_3$$

or

$$x_1^3 - x_2 + \beta - \alpha_4 x_4 = \alpha_3 x_3$$

are not linear.

A **system of linear equations** comprises, as you might have guessed, of a number of linear equations involving the same variables. For example

$$\begin{aligned} 2x_1 - 2x_2 + 8x_3 &= 10 \\ 4x_1 - 6x_2 + 2x_3 &= 4 \\ 2x_2 + x_3 &= 6 \end{aligned}$$

is a system of three linear equations. Note that the coefficient α_1 in the third linear equation is taken to be 0 thus $\alpha_1 x_1$ is omitted, for convenience.