Sequences in $\mathbb{R}^n$

Most of the properties of sequences in $\mathbb{R}$ are easy to generalize to sequences in $\mathbb{R}^n$. The main results is the following:

Theorem 1 *Consider a sequence* $\{x_k\} = \{(x_k^{(1)}, ..., x_k^{(n)})\}$ *in* $\mathbb{R}^n$. *It converges to* $x = (x^{(1)}, ..., x^{(n)})$ *iff* $x_k^{(i)} \longrightarrow x^{(i)}$ *for all* $i \in \{1, ..., n\}$.

Proof. To be added ■

Example 2 *1.Consider the sequence defined by* $x_k = (\frac{1}{k}, (-1)^k \frac{1}{k}) + e^{-k}$. *To analyze its convergence, we analyze each of its components.*

$$\begin{aligned} x_k^{(1)} &= \frac{1}{k} \\ &\longrightarrow 0 \\ x_k^{(2)} &= (-1)^k \frac{1}{k}) + e^{-k} \\ &\longrightarrow 1 \end{aligned}$$

We conclude that $x_k \longrightarrow (0, 1)$.
2. Consider now the sequence defined by $x_k = (\frac{1}{k}, \frac{2}{k}, (-1)^k)$. *Does it converge? Since its third component* $x_k^{(3)} = (-1)^k$ *does not converge, we can immediately conclude that it does not converge.*