

Basic definitions

That problem, known as finding a **global maximum** of f , is in general difficult, so we will introduce a simpler one: find a **local maximum** of f .

Definition 1 We say that x^* is a **global maximum** of f iff $f(x^*) \geq f(x)$ for all $x \in \mathbb{R}^n$.

We say that x^* is a **local maximum** of f iff there exists $\epsilon > 0$ such that $f(x^*) \geq f(x)$ for all $x \in B(x^*, \epsilon)$.

Example 2 To illustrate the previous distinction, consider the function $f :$

$$\mathbb{R} \longrightarrow \mathbb{R} \text{ given by } f(x) = \begin{cases} x & \text{if } x \leq 0 \\ -x & \text{if } x \in (0, 1] \\ -2 + x & \text{if } x \in (1, 3] \\ 4 - x & \text{if } x > 3 \end{cases}$$

Then (see figure) 0 is a local maximum (with $f(0) = 0$) and 3 is a global maximum (with $f(3) = 2$).