Set Theory Definitions

In an experiment, a statistician needs to identify all possible outcomes. The set Ω of all possible outcomes of the experiment, is called the **sample space**.

For example, suppose that the experiment is to cast a fair die. The sample space is $\Omega = \{1, 2, 3, 4, 5, 6\}$.

On the other hand, an **event** is a collection of possible outcomes of an experiment.

Suppose that event A:= {even numbers when you cast a die}.
Then A= $\{2, 4, 6\}$. Notice that an event is a subset (does not have to be proper) of a sample space. Therefore, we say that an event A occurred if and only if the realized outcome of the experiment is in the set A.

Now given any two events A and B, we have the following elementary operations:

Union: The union of A and B, denoted by $A \cup B$, is the set of elements that belong to either A or B or both. Thus, $A \cup B = \{x: x \in A \text{ or } x \in B\}$.

Intersection: The intersection of A and B, denoted by $A \cap B$, is the set of elements that is common to both A and B. Thus, $A \cap B = \{x: x \in A \text{ and } x \in B\}$

Complementation: The complementation of A, denoted by A^C, is the set of elements in Ω that are not common to A. Thus, $A^C = \{x: x \in \Omega \text{ and } x \notin A\} = \{x: x \in \Omega \backslash A\}$

The elementary set operations can be combined, somewhat akin to the way addition and multiplication can be combined. As long as we are careful, we can treat sets as of they were numbers. We can now state the following useful properties of set operations.

For any three events, A, B, C, defined on a sample space Ω, we have the following relationships:

(1) $A \cup B = B \cup A$ and $A \cap B = B \cap A$ (by *commutativity*),
(2) $A \cup (B \cup C) = (A \cup B) \cup C$ and $A \cap (B \cap C) = (A \cap B) \cap C$ (by *associativity*),
(3) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ and $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (by the *Distributive Laws*),
(4) $(A \cup B)^C = A^C \cap B^C$ and $(A \cap B)^C = A^C \cup B^C$ (by the *Demorgan's Laws*).

Two events A and B are **disjoint** (**mutually exclusive**) if $A \cap B = \varnothing$. Furthermore, the events $A_1, A_2, \cdots$ are **pairwise disjoint** if $A_i \cap A_j = \emptyset$ for all $i \neq j$.

If $A_1, A_2, \cdots \in \Omega$, are pairwise disjoint and $\bigcup_{i=1}^{\infty} A_i = \Omega$, then the collection $A_1, A_2, \cdots$ forms a **partition** of Ω.