

Basics of Probability Theory

For each event A in the sample space Ω we want to associate with A a number between zero and one that will be called the probability of A , denoted by $P(A)$. It would seem natural to define the domain of P (the set where the arguments of the function $P(\cdot)$ are defined) as all subsets of Ω ; that is, for each $A \subset \Omega$ we define $P(A)$ as the probability that A occurs. Unfortunately matters are not that simple. There are some technical difficulties to overcome. We will not dwell on these difficulties; although they are of importance. However, a firm understanding of statistics requires at least a passing familiarity with the following.

A collection of subsets of Ω is called a **sigma algebra** and is denoted by S , if it satisfies the following three properties:

- (1) $\emptyset \in S$ (the empty set is an element of the sigma algebra S),
- (2) if $A \in S$, then $A^C \in S$ (the sigma algebra S is closed under complementation),
- (3) if $A_1, A_2, \dots \in S$, then $\bigcup_{i=1}^{\infty} A_i \in S$ (the sigma algebra S is closed under countable unions)

Example of a sigma algebra

Power sets of sample spaces are always sigma algebras. Suppose that $\Omega = \{1, 2, 3\}$. A sigma algebra of Ω is the collection of sets S where

$S = \{\{\emptyset\}, \{1, 2, 3\}, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}\}$. Verify all three properties and convince yourself that S is a sigma algebra.

Suppose that there exists a new set $S' = \{\{\emptyset\}, \{1, 2, 3\}, \{1\}, \{2\}, \{1, 3\}, \{2, 3\}\}$. Is set S' a sigma-algebra?

If S' is not a sigma algebra, indicate the property that is violated.