

Before , listing one of the most important theorems in Probability Theory, it is essential that we define the notion of conditional probability.

If A and B are events in Ω , and $P(B) > 0$, then the **conditional probability of A given B**, written $P(A|B)$, is:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}.$$

The intuition is that our original sample space in Ω , has been updated to B. All further occurrences are then calibrated with respect to their relation to B. In particular, note what happens to conditional probabilities of disjoint sets. Suppose A and B are disjoint so that $P(A \cap B) = 0$. It then follows that $P(A|B) = P(B|A) = 0$.

Baye's Theorem: Let A_1, A_2, \dots be a partition of the sample space, and let B be any given set. Then for each $i=1, 2, \dots$,

$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{\sum_{j=1}^{\infty} P(B|A_j)P(A_j)}$$

Example

Thirty percent of light aircraft that disappear while in flight in a certain country are subsequently discovered. Of the aircraft that are discovered, seventy percent have an emergency locator, whereas ninety percent of the aircrafts not discovered do not have such a locator. Suppose a light aircraft has disappeared.

(a) If it has an emergency locator, what is the probability that it will not be discovered?

(b) If it does not have an emergency locator, what is the probability that it will be discovered?

$$\begin{aligned} D &= \{\text{discovered flight}\}, D^C = \{\text{not discovered flight}\} \\ L &= \{\text{emergency locator exists}\}, L^C = \{\text{emergency locator does not exist}\} \\ P(D) &= 0.3, P(D^C) = 0.7 \\ P(L|D) &= 0.7, \implies P(L^C|D) = 0.3 \\ P(L^C|D^C) &= 0.9, \implies P(L|D^C) = 0.1 \end{aligned}$$

Use the Baye's Theorem:

(a)

$$P(D^C|L) = \frac{P(D^C)P(L^C|D^C)}{P(L^C|D^C)P(D^C)+P(L^C|D)P(D)} = \frac{(0.1)(0.7)}{(0.1)(0.7)+(0.7)(0.3)} = \frac{1}{4} = 0.25$$

(b)

$$P(D|L^C) = \frac{P(L^C|D)P(D)}{P(L^C|D)P(D)+P(L^C|D^C)P(D^C)} = \frac{(0.3)(0.3)}{(0.3)(0.3)+(0.9)(0.7)} = \frac{9}{72} = 0.125$$