Independence

In some cases it may happen that the occurrence of a particular event B, has no effect on the probability of another event, say A. Symbolically, we are saying that P(A|B) =P(A).

Two events, A and B, are statistically **independent** if P(A∩B) =P(A)P(B).

Many gambling games provide models of independent events. The spins of a roulette wheel and the tosses of a pair of dice are both series of independent events.

Notice, that the conditional probability of A given B becomes
$\mathrm{P(A|B)} = \frac{\mathrm{P(A\cap B)}}{\mathrm{P(B)}} \overset{independence}{=} \frac{\mathrm{P(A)P(B)}}{\mathrm{P(B)}} = \mathrm{P(A)}$.

Similarly, the conditional probability of B given A becomes
$\mathrm{P(B|A)} = \frac{\mathrm{P(B\cap A)}}{\mathrm{P(A)}} \overset{independence}{=} \frac{\mathrm{P(B)P(A)}}{\mathrm{P(A)}} = \mathrm{P(B)}$.

Example
Suppose A, B and C are independent events, with P(A) = 0.8, P(B) = 0.7 and P(C) = 0.2. What is the probability of the event A∪B∪C?

Since A, B and C are independent, we have P(A∩B)=P(A)P(B), P(A∩C)= P(A)P(C), P(B∩C) = P(B)P(C) and P(A∩ B∩C) = P(A)P(B)P(C). Now use the formula:

Recall from Theorem 2 (2) that P(A∪B) =P(A)+P(B)-P(A∩B)

Therefore P(A∪B∪C)= P(A) + P(B) +P(C) − P(A∩B)−P(A∩C)− P(B∩C)+ P(A∩B∩C) = $(0.8)+(0.7)+(0.2)-(0.8)(0.7)-(0.8)(0.2)-(0.7)(0.2)+(0.8)(0.7)(0.2) = 0.8 + 0.7 + 0.2 - 0.56 - 0.16 - 0.14 + 0.112 = 0.952$