Kolmogorov's Axioms

What follows are the so-called Kolmogorov's Axioms (after A. Kolmogorov, one of the fathers of probability theory) or the Axioms of Probability.

Given a sample space Ω and an associated sigma algebra S, a **probability function** is a function P with domain S, that satisfies:
(1) P(A)$\geq$ 0 for all A$\in$ S,
(2) P(Ω) = 1,
(3) if A$_1$,A$_2$,$\cdots \in S$ are pairwise disjoint, then P($\overset{\infty}{\underset{i=1}{\cup}}A_i$) = $\overset{\infty}{\underset{i=1}{\sum}}$ P(A$_i$)

Theorem 1: If P is a probability function and A is any set in S, then:
(1) P($\varnothing$) = 0 where $\varnothing$ is the empty set,
(2) P(A) $\leq$ 1,
(3) P(A^C) = 1$-$P(A).

Theorem 2: If P is a probability function and A and B are any given sets in S, then:
(1) P(B$\cap$A^C) =P(B)$-$P(A$\cap$B),
(2) P(A$\cup$B) =P(A)+P(B)-P(A$\cap$B)
(3) if A$\subset$B, then P(A)$\leq$P(B)