Probability Mass Functions and Probability Density Functions

The **probability mass function** or pmf, $f_X(x)$ of a discrete random variable X is given by $f_X(x) =$P$(X = x)$ for all x.

The **probability density function** or pdf, $f_X(x)$ of a continuous random variable X is the function that satisfies $F_X(x) = \int_{-\infty}^{x} f_X(t)\partial t$ for all x

A widely accepted convention which we will adopt, is to use an uppercase letter for the cdf and a lowercase letter for the pmf or pdf.

We must be a little more careful in our definition of a pdf in the continuous case. If we try to naively calculate P$(X = x)$ for a continuous random variable we get the following:

Since $\{X = x\} \subset \{x - \epsilon < X \leq x\}$ for any $\epsilon > 0$, we have from Theorem 2(3), P$\{X = x\} \leq$P$\{x - \epsilon < X \leq x\} = F_X(x) - F_X(x - \epsilon)$ for any $\epsilon > 0$.
Therefore, $0 \leq$ P$\{X = x\} \leq \underset{\epsilon \longrightarrow 0}{lim} [F_X(x) - F_X(x - \epsilon)] = 0$ by the continuity of F_X.

A note on notation: The expression "X has a distribution given by $F_X(x)$" is abbreviated symbolically by "$X \sim F_X(x)$," where we read they symbol "$\sim$" as is distributed as" or "follows".

Theorem 5: A function $f_X(x)$ is a pdf or pmf of a random variable X if and only if:
(1) $f_X(x) \geq 0$ for all x
(2) $\int_{-\infty}^{\infty} f_X(x)\partial x = 1$ (pdf) and $\sum_{\substack{X \\ x}} f(x) = 1$ (pmf)