

The random variables X and Y are **identically distributed** if for every set $A \in \mathcal{S}$, $P(X \in A) = P(Y \in A)$

Note that two random variables that are identically distributed are not necessarily equal. That is, the above definition does not say that $X=Y$.

Consider the experiment of tossing a fair coin three times. Define the random variables $X :=$ number of heads observed and $Y :=$ number of tails observed.

The distribution of X is:

ω	<u>HHH</u>	<u>HHT</u>	<u>HTH</u>	<u>THH</u>	<u>TTH</u>	<u>THT</u>	<u>HTT</u>	<u>TTT</u>
$X(\omega)$	3	2	2	2	1	1	1	0

The distribution of Y is:

ω	<u>TTT</u>	<u>TTH</u>	<u>THT</u>	<u>HTT</u>	<u>HHT</u>	<u>HTH</u>	<u>THH</u>	<u>HHH</u>
$Y(\omega)$	3	2	2	2	1	1	1	0

That is for each $A = \{0, 1, 2, 3\}$, we have $P(X \in A) = P(Y \in A)$. So X and Y are identically distributed but for no point ω we have that $X(\omega) = Y(\omega)$.

Theorem 4: The following two statements are equivalent.

- (1) The random variable X and Y are identically distributed,
- (2) $F_X(x) = F_Y(x)$ for every x .