Intermediate Microeconomics (Econ 3101)
Assignment 2

Answers

Due **Monday, 29 June 2009, 10:00 am**. Students are welcome to discuss homework in groups, but each student must prepare and submit a unique assignment and note the names of other group members. All assignments must be neat and professional. Answer all parts of all questions.

1. Suppose a consumer's preferences are represented by the utility function $u(x, y) = \min\{x, 2y\}$. The price for good x is p_x, the price for good y is p_y, and the consumer has m to spend.
(a) Derive the consumer's demand for good x as a function of p_x, p_y, and m.
(b) Calculate the consumer's optimal consumption bundle if $p_x = 3$, $p_y = 4$, and $m = 600$.
(c) Graph the Engel Curve for prices $p_x = 3$ and $p_y = 4$.

(a) Note that the consumer will always chose $x = 2y$.

$$x = 2y$$

$$p_x x + p_y y = m$$

$$p_x x + p_y\left(\frac{x}{2}\right) = m \qquad\qquad p_x(2y) + p_y y = m$$

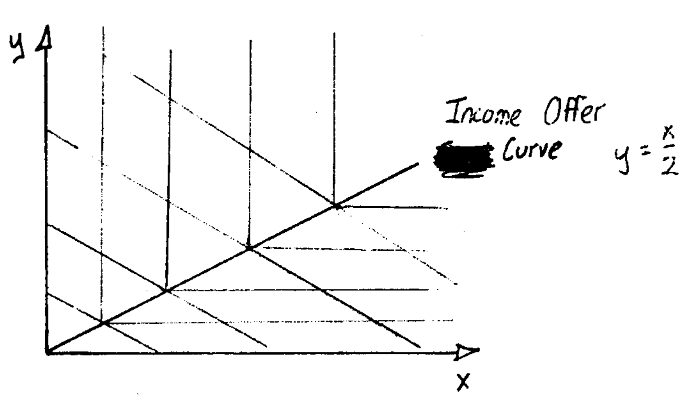
$$x = \frac{m}{p_x + \frac{1}{2}p_y} = \frac{2m}{2p_x + p_y} \qquad\qquad y = \frac{m}{2p_x + p_y}$$

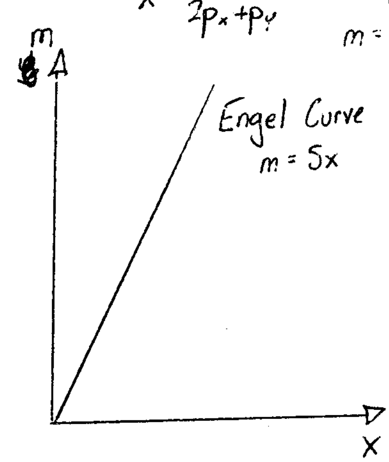
(b) $x = \dfrac{2(600)}{2\cdot 3 + 4} = 120 \qquad y = \dfrac{600}{2\cdot 3 + 4} = 60$

$$x = \frac{2m}{2p_x + p_y} \Rightarrow m = (p_x + \tfrac{1}{2}p_y)x$$

$$m = 5x$$

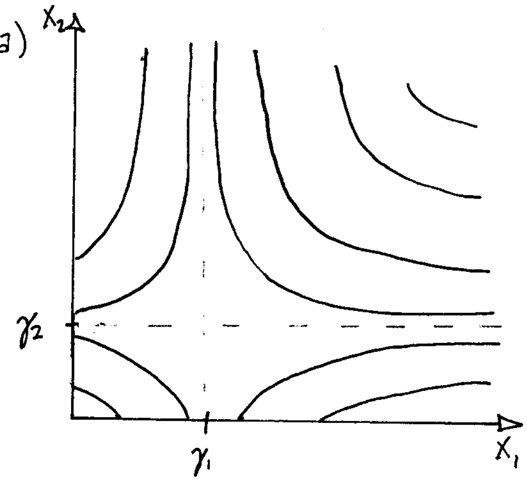
(c)





2. The Stone-Geary utility function, $u(x_1, x_2) = (x_1 - \gamma_1)^{\beta_1}(x_2 - \gamma_2)^{\beta_2}$, is an important generalization of the Cobb-Douglas Utility Function. Consider the special case where $\beta_1 = 1$, $\beta_2 = 1$, $\gamma_1 > 0$, and $\gamma_2 > 0$. The price for good x_1 is p_1, the price for good x_2 is p_2, and the consumer has m to spend.
(a) Graph the indifference curves associated with this preference.
(b) Derive the consumer's demand for good x_1 as a function of p_1, p_2, and m.
(c) Are goods x_1 and x_2 gross substitutes, gross complements, or unrelated goods?

(a)



(b) if income is sufficiently high then the solution will be interior

$$-\frac{\partial u/\partial x_1}{\partial u/\partial x_2} = -\frac{p_1}{p_2}$$

$$\frac{\partial u}{\partial x_1} = x_2 - \gamma_2 \qquad \frac{\partial u}{\partial x_2} = x_1 - \gamma_1$$

$$\frac{x_2 - \gamma_2}{x_1 - \gamma_1} = \frac{p_1}{p_2} \Rightarrow p_2 x_2 - \gamma_2 p_2 = p_1 x_1 - \gamma_1 p_1$$

$$p_1 x_1 + p_2 x_2 = m$$

$$p_1 x_1 + (p_1 x_1 - \gamma_1 p_1 + \gamma_2 p_2) = m$$

$$x_1 = (m + \gamma_1 p_1 - \gamma_2 p_2)/2p_1$$

(c) $$\frac{\partial x_1}{\partial p_2} = -\frac{\gamma_2}{2p_1} < 0$$

$\Rightarrow$ good x_1 is a gross complement to good x_2

3. Is the consumer behavior described below consistent with a model of maximizing behavior? Explain why or why not.

Bundle	Observed Consumption	Prices	Cost under P_A	P_B	P_C
A	(10, 27, 6)	(2, 5, 3)	173	150	145
B	(17, 5, 11)	(3, 4, 2)	92	92	127
C	(22, 15, 14)	(4, 3, 4)	161	154	189

When the consumer chose A, bundles B and C were also afforable, so

$A \succ B$ and $A \succ C$.

When the consumer chose B, B was the only affordable bundle.
When the consumer chose C, A and B were affordable, so

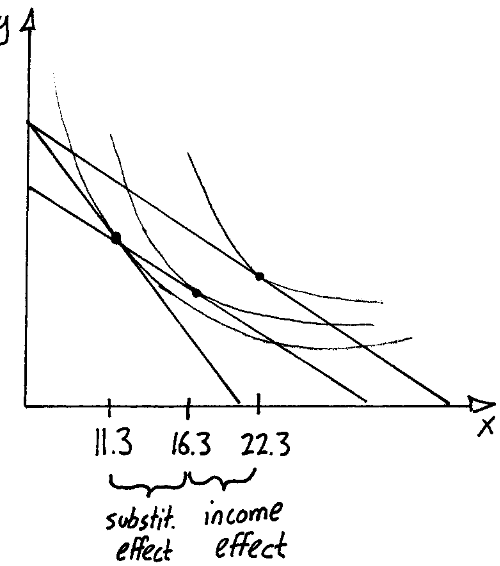
$C \succ A$ and $C \succ B$.

The consumer chose C over A after A was revealed preferred to C and so is not maximizing.

4. Suppose a consumer has a demand function of the form $x(p_x, p_y, p_z, m) = 1 + m(\frac{1}{4p_x} + \frac{1}{20p_z} + \frac{m}{400p_x^2})$. Suppose that while the price of good x decreases from 4 to 2, income and other prices remain constant at $m = 100$, $p_y = 3$, and $p_z = 2$. Find the change in quantity demanded. Find the magnitudes of the Slutsky substitution effect and income effect.

$$x(4,3,2,100) = 1 + 100\left(\frac{1}{16} + \frac{1}{40} + \frac{100}{400 \cdot 16}\right) = \frac{724}{64} \approx 11.3$$

$$x(2,3,2,100) = 1 + 100\left(\frac{1}{8} + \frac{1}{40} + \frac{100}{400 \cdot 4}\right) = \frac{89}{4} \approx 22.3$$



Finding the income associated with the pivoted budget line: $\Delta m = x \cdot \Delta p_x = (11.3)(2) = 22.6$

$$m' = 100 - 22.6 = 77.4$$

$$x(2,3,2,77.4) = 1 + 77.4\left(\frac{1}{8} + \frac{1}{40} + \frac{77.4}{400 \cdot 4}\right) \approx 16.3$$

Total change $\frac{175}{16} \approx 11$

substitution effect $16.3 - 11.3 = 5.0$

income effect $22.3 - 16.3 = 6.0$

5. Suppose an expected utility maximizer has preferences over monetary outcomes represented by $u(x) = \sqrt{x}$. Further suppose the agent has an investment that provides the entire monetary payoff. With probability $\frac{3}{4}$, no disaster strikes and the agent receives a payoff of 16. Disasters strike with probability $\frac{1}{4}$ and reduce the payoff to 0.

(a) Is the agent risk averse, risk neutral, or risk loving?

(b) What is the maximum amount the agent would pay in advance to fully insure against disasters.

(c) If the agent must pay $\frac{c}{2}$ to buy coverage that awards c during a disaster, what is the optimal coverage for the agent?

(a) $\sqrt{x}$ is concave $\Rightarrow$ agent is risk averse

(b) $$\sqrt{16-x} = \frac{3}{4}\sqrt{16} + \frac{1}{4}\sqrt{0}$$

$$\sqrt{16-x} = 3$$

$$16 - x = 9$$

$$x = 7$$

(c) $$\max_c \frac{3}{4}\sqrt{16 - c/2} + \frac{1}{4}\sqrt{c - c/2}$$

$$\max_c \frac{3}{4}\left(16 - \frac{c}{2}\right)^{1/2} + \frac{1}{4}\left(\frac{c}{2}\right)^{1/2}$$

FOC $$\frac{3}{8}\left(16 - \frac{c}{2}\right)^{-1/2}\left(-\frac{1}{2}\right) + \frac{1}{8}\left(\frac{c}{2}\right)^{-1/2} \cdot \left(+\frac{1}{2}\right) = 0$$

$$\frac{3}{\sqrt{16 - c/2}} = +\frac{1}{\sqrt{c/2}}$$

$$\frac{9c}{2} = 16 - \frac{c}{2}$$

$$c = \frac{16}{5}$$

SOC $$-\frac{3}{64}(16 - c/2)^{-3/2} - \frac{1}{64}\left(\frac{c}{2}\right)^{-3/2}\Big|_{c=\frac{16}{5}} < 0$$