Intermediate Microeconomics (Econ 3101)
Exam 1

Answers

INSTRUCTIONS: Write your name on the back of the exam. Answer all parts of all questions, and show work. Exactly 120 minutes will be provided to complete the exam. Notes, books, computers, cell phones, or communications devices may not be used. Calculators may be used. Communication with other students is not permitted. The test has 7 pages and 150 possible points.

1. [20 points] Consider a model with two periods, each with one composite consumption good. Prices are the stable and normalized to 1. Suppose a consumer is endowed with income 20 in the first period and 60 in the second period. The consumer receives a 10% interest rate on savings and is subject to a 20% interest rate on borrowing. Regulations prohibit the consumer from borrowing more than 20.
(a) Find this consumer's intertemporal budget constraint.
(b) Carefully graph the budget set. Label all intercepts.

(a) if saves in period 1

$$c_2 = m_2 + (1+r_s)(m_1 - c_1)$$

$$c_2 = 60 + 1.1(20 - c_1) \quad \text{if } c_1 \le 20$$

if borrows in period 1

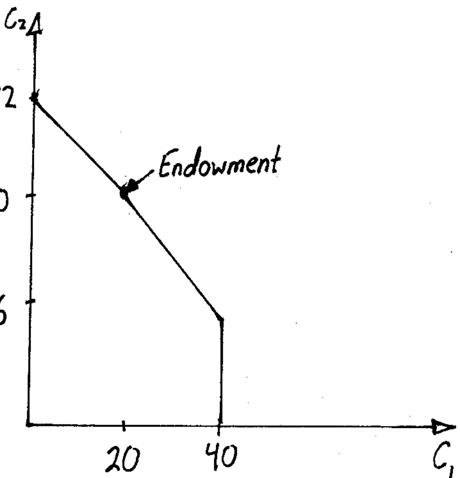
$$c_2 = m_2 - (1+r_b)(c_1 - m_1)$$

$$c_2 = 60 - 1.2(c_1 - 20) \quad \text{if } 20 < c_1 \le 40$$

together

$$c_2 = \begin{cases} 60 + 1.1(20 - c_1) & \text{if } c_1 \le 20 \\ 60 + 1.2(20 - c_1) & \text{if } 20 < c_1 \le 40 \end{cases}$$

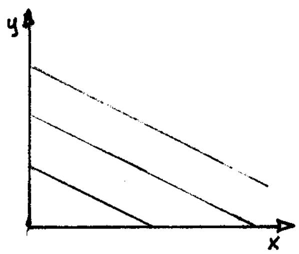
(b)



2. [25 points] Suppose a consumer's preferences are represented by the utility function $u(x, y) = x + 2y$. The price for good x is p_x, the price for good y is p_y, and the consumer has m to spend.

(a) Graph indifference curves given by these preferences.
(b) Derive the consumer's demand for good x as a function of p_x, p_y, and m.
(c) Derive the consumer's demand for good y as a function of p_x, p_y, and m.
(d) Calculate the consumer's optimal consumption bundle if $p_x = 8$, $p_y = 5$, and $m = 400$.

(a)



$u(x,y) = x + 2y$

$y = \frac{u}{2} - \frac{x}{2}$

(b,c) if $p_Y < 2p_x$, optimal to spend all on y

$\Rightarrow x = 0, \quad y = \frac{m}{p_Y}$

if $p_Y > 2p_x$, optimal to spend all on x

$\Rightarrow x = \frac{m}{p_x}, \quad y = 0$

Formally

$$\max_{x,y} \; x + 2y \quad \text{s.t.} \quad p_x x + p_Y y \leq m, \quad x \geq 0, \quad y \geq 0$$

or equivalently

$$\max_{x} \; x + 2\left(\frac{m}{p_Y} - \frac{p_x}{p_Y} x\right) \quad \text{s.t.} \quad 0 \leq x \leq \frac{m}{p_x}$$

First order cond. $\quad 1 - \frac{2p_x}{p_Y} = 0 \quad$ no critical points unless $p_Y = 2p_x$

(d) $x(8,5,400) = 0$

$y(8,5,400) = \frac{400}{5} = 80$

Comparing end points

x	$U(x)$
0	$2m/p_Y$
m/p_x	m/p_x

$u(0) > u(m/p_x) \Leftrightarrow \frac{2m}{p_Y} > \frac{m}{p_x}$

so $x(p_x, p_Y, m) = \begin{cases} 0 & \text{if } p_Y < 2p_x \\ \frac{m}{p_x} & \text{if } p_Y > 2p_x \end{cases}$

$y(p_x, p_Y, m) = \begin{cases} m/p_Y & \text{if } p_Y < 2p_x \\ 0 & p_Y > 2p_x \end{cases}$

3. [25 points] Suppose a consumer's preferences are represented by the utility function $u(x, y) = -\frac{1}{x} - \frac{1}{y}$. The price for good x is p_x, the price for good y is p_y, and the consumer has m to spend.

(a) Derive the consumer's demand for good x as a function of p_x, p_y, and m. (Note: The solution is interior.)

(b) Calculate the price elasticity of demand for good x.

(c) Calculate the income elasticity for good x.

(d) Is the good x a normal good or an inferior good?

(a) $\max_{x,y} -\frac{1}{x} - \frac{1}{y}$ s.t. $p_x x + p_y y = m$

$$\frac{\partial u}{\partial x} = \frac{1}{x^2} \qquad \frac{\partial u}{\partial y} = \frac{1}{y^2}$$

tangency condition

$$\frac{-1/x^2}{1/y^2} = \frac{-p_x}{p_y}$$

$$\frac{y^2}{x^2} = \frac{p_x}{p_y}$$

$$y = \sqrt{\frac{p_x}{p_y}}\, x$$

budget equation

$$p_x x + p_y y = m$$

$$p_x x + p_y \sqrt{\frac{p_x}{p_y}}\, x = m$$

$$x(p_x, p_y, m) = \frac{m}{p_x + \sqrt{p_x p_y}}$$

(b) $$x(p_x, p_y, m) = \frac{m}{p_x + \sqrt{p_x p_y}}$$

$$\frac{\partial x}{\partial p_x} = \frac{-m}{(p_x + \sqrt{p_x p_y})^2} \cdot \left(1 + \frac{p_y}{2\sqrt{p_x p_y}}\right)$$

$$\frac{p_x}{x}\frac{\partial x}{\partial p_x} = \frac{-m p_x}{(p_x + \sqrt{p_x p_y})^2}\left(1 + \frac{p_y}{2\sqrt{p_x p_y}}\right)\frac{p_x + \sqrt{p_x p_y}}{m}$$

$$\frac{p_x}{x}\frac{\partial x}{\partial p_x} = \frac{-p_x}{p_x + \sqrt{p_x p_y}}\left(1 + \frac{p_y}{2\sqrt{p_x p_y}}\right)$$

(c) $$\frac{\partial x}{\partial m} = \frac{1}{p_x + \sqrt{p_x p_y}}$$

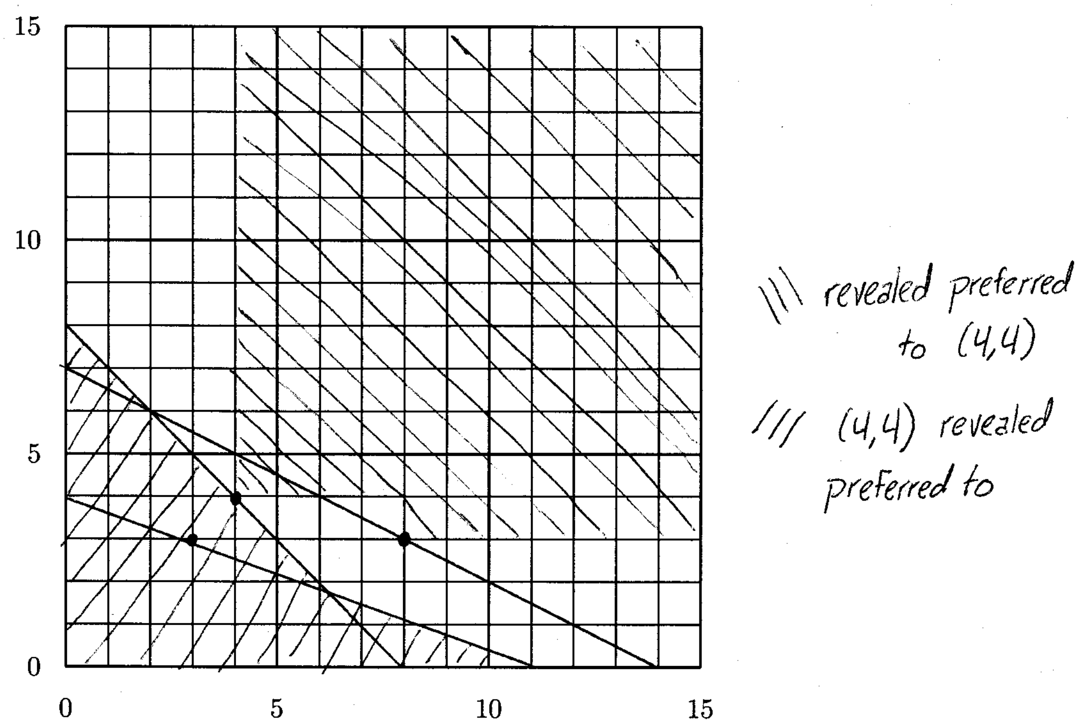
$$\frac{m}{x}\frac{\partial x}{\partial m} = \frac{m(p_x + \sqrt{p_x p_y})}{m}\,\frac{1}{p_x + \sqrt{p_x p_y}} = 1$$

(d) $\frac{\partial x}{\partial m} > 0 \Rightarrow x$ is a normal good

4. [20 points] Consider a consumer with preferences over goods x and y that satisfy the completeness, reflexivity, continuity, transitivity, and strict monotonicity properties. Let p_x denote the price of good x, and p_y the price of good y. The consumer's income fluctuates. Suppose the consumer is observed making the following consumption decisions:

- when $p_x = 2$ and $p_y = 2$ the consumer chose $x = 4, y = 4$
- when $p_x = 1$ and $p_y = 3$ the consumer chose $x = 3, y = 3$
- when $p_x = 1$ and $p_y = 2$ the consumer chose $x = 8, y = 3$

Graph the budget lines and consumption points corresponding to each decision. Indicate the area revealed to be preferred to (4,4) and the area to which (4,4) is revealed to be preferred.



5. [20 points] Suppose a consumer's preferences are represented by the utility function $u(x, y) = xy$. The price for good x is p_x, the price for good y is p_y. The consumer is endowed with ω_x of good x and ω_y of good y.

(a) Derive the consumer's gross demand for good x as a function of p_x, p_y, ω_x, and ω_y.

(b) Derive the consumer's net demand or net supply for good x as a function of p_x, p_y, ω_x, and ω_y.

(a) $\max \; xy \quad \text{s.t.} \quad p_x x + p_y y = p_x \omega_x + p_y \omega_y$

interior solution, since utility function is Cobb-Douglas

$$\frac{\partial U / \partial x}{\partial U / \partial y} = \frac{y}{x} = \frac{p_x}{p_y}$$

$$p_x x = p_y y$$

budget equation $\quad p_x x + p_y y = p_x \omega_x + p_y \omega_y$

$$2 p_x x = p_x \omega_x + p_y \omega_y$$

$$x(p_x, p_y, \omega_x, \omega_y) = \frac{p_x \omega_x + p_y \omega_y}{2 p_x} \qquad \text{gross demand}$$

(b) $$x - \omega_x = \frac{p_x \omega_x + p_y \omega_y}{2 p_x} - \frac{2 p_x \omega_x}{2 p_x}$$

$$= \frac{p_y \omega_y - p_x \omega_x}{2 p_x} \qquad \text{net demand}$$

6. [20 points] Suppose a farmer is growing a crop where the only risk is an early frost. Early frosts are known to occur with probability $\frac{1}{10}$. If a early frost occurs, the farmer's harvest is worth 0. Otherwise, the farmer will have harvest worth 25. The farmer may insure the harvest against an early frost at a percent insurance premium $\gamma = 0.2$, so if the farmer selects coverage that will pay C in an early frost, the farmer pays $0.2C$ in both cases. The farmer has expected utility representation with an expected utility function of $u(x) = \sqrt{x}$. Find the optimal insurance coverage C.

with coverage C

$25 - 0.2C$ with prob. $9/10$

$C - 0.2C$ with prob. $1/10$

$$\max_{C} \frac{9}{10} u(25-0.2C) + \frac{1}{10} u(0.8C)$$

$$\max_{C} \frac{9}{10}\sqrt{25-0.2C} + \frac{1}{10}\sqrt{0.8C}$$

First deriv. condition $$\frac{9}{20}\frac{-0.2}{\sqrt{25-0.2C}} + \frac{1}{20}\frac{0.8}{\sqrt{0.8C}} = 0$$

$$\frac{1.8}{\sqrt{25-0.2C}} = \frac{0.8}{\sqrt{0.8C}}$$

$$9\sqrt{0.8C} = 4\sqrt{25-0.2C}$$

$$81(0.8C) = 16(25-0.2C)$$

$$64.8C = 400 - 3.2C$$

$$C = \frac{100}{17} \approx 5.88$$

7. [20 points] For each of the following utility functions, graph the indifference curves passing through (3,3) and (6,6) and find the marginal rate of substitution.

(a) $u(x, y) = (x + y)^{\frac{3}{2}}$

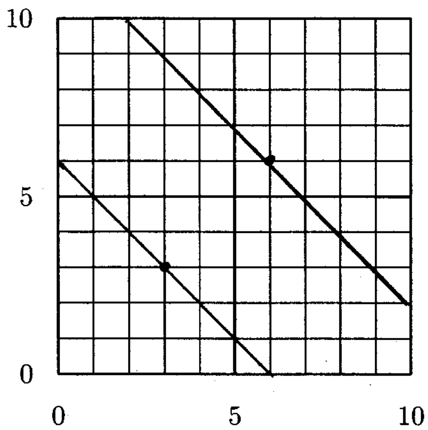
$u(3,3) = 6^{3/2}$

$u(6,6) = 12^{3/2}$

$u(x,y) = (x+y)^{3/2}$

$u^{2/3} = x + y$

$y = u^{2/3} - x$



$MRS = -\dfrac{\partial U/\partial x}{\partial U/\partial y}$

$\dfrac{\partial U}{\partial x} = \dfrac{3}{2}(x+y)^{1/2}$

$\dfrac{\partial U}{\partial y} = \dfrac{3}{2}(x+y)^{1/2}$

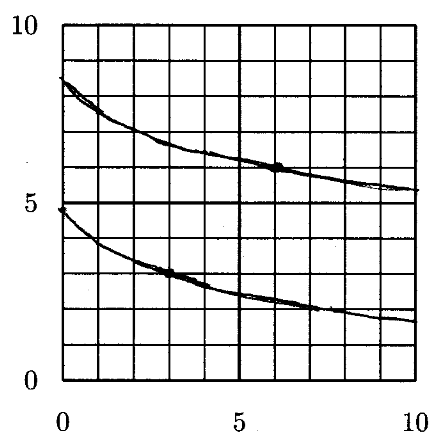
$MRS = -\dfrac{3/2\,(x+y)^{1/2}}{3/2\,(x+y)^{1/2}} = 1$

(b) $u(x, y) = \sqrt{x} + y$

$u(3,3) = 3 + \sqrt{3}$

$u(6,6) = 6 + \sqrt{6}$

$y = u - \sqrt{x}$



$\dfrac{\partial u}{\partial x} = \dfrac{1}{2\sqrt{x}}$

$\dfrac{\partial u}{\partial y} = 1$

$MRS = -\dfrac{1/2\sqrt{x}}{1} = -\dfrac{1}{2\sqrt{x}}$