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Lecture 2*

Optimal Fiscal and Monetary Policy

The study of fiscal and monetary policy will be done using the primal approach, in which the focus is on the choice of allocations that characterize equilibrium.

Structure of the economy:

- static, n consumption goods produced with labor
- Technology: constant return to scale

$$F(x_1, .., x_n, l) = 0$$

Firm's problem:

$$\max \sum_{i=1}^{n} p_i x_i - l$$

- Consumer's problem:

$$\max U(c_1, .., c_n, l)$$

$$s.t. \sum_{i=1}^{n} p_i(1+\tau_i)c_i \leq (1-\tau_l)l$$

- Government: proportional taxes on consumption goods and labor, $\tau_1, .., \tau_n, \tau_l$, and g_i is a given sequence of government spending of good i.The government budget constraint is:

$$\sum_{i=1}^{n} p_i \tau_i c_i = \sum_{i=1}^{n} p_i g_i.$$

A competitive equilibrium for this economy is allocations $c = (c_1, .., c_n), x = (x_1, .., x_n), l$, prices $p = (p_1, ..., p_n)$ and policy $\pi = (\tau_1, .., \tau_n, \tau_l)$ such that:

(i) c and l solve the consumer's maximization problem

(ii) x and l solve the firm's maximization problem

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(iii) the government budget constraint holds

(iv) resource constraint holds, $\forall\ i$

$$c_i + g_i = x_i$$

Recall that in equilibrium we have on redundant equation by Walras 'Law; the government budget constraint, the consumer's budget constraint; (iii) or (iv).

It's also straightforward to show that, if (c, x, l, p, π) is a competitive equilibrium with $\tau_l \neq 0$, then $(c, x, l, p, \widehat{\pi})$, with $\widehat{\pi} = (\widehat{\tau}_1, .., \widehat{\tau}_n, \widehat{\tau}_l)$ where $\widehat{\tau}_l = 0$ and $1 + \widehat{\tau}_i = \frac{1+\tau_i}{1+\widehat{\tau}_i}$ $\forall i$.This result says that one of the taxes is redundant. So, from now on, we will be considering models with only proportional taxes on the consumption good, unless otherwise stated.

Proposition 1:

(i) A C.E. allocation satisfies:

$$F(c_1 + g_1, ..., c_n + g_n, l) = 0 \tag{1}$$

and the implementability constraint:

$$\sum_{i=1}^{n} U_i c_i + U_l l = 0 \tag{2}$$

(ii) If an allocation satisfies (1) and (2), then there exits prices and policies which, together with the allocation, are a C.E.

Proof:

(i) We get (1) directly substituting the resource constraint in the equation that describes the technology. The implementability constraint comes is obtained substituting the prices and taxes in the consumer's budget constraint by the first order conditions.

(ii) Let $x_i = c_i + g_i$ and define:

$$p_i = -\frac{F_i}{F_l},$$

$$(1 + \tau_i) p_i = -\frac{U_i}{U_l}.$$

Ramsey Equilibrium

We now define equilibrium in a game in which the government is setting its policy to maximize the consumers' utility and to do so it forecasts the agents' behavior. Let Π be the set of policies for which a C.E. exits and let $(c(\pi), l(\pi), x(\pi))$ be an allocation rule if $(c(\pi), l(\pi), x(\pi), \pi)$ is part of a C.E. A *Ramsey Equilibrium* is a policy $\pi \in \Pi$, allocation rules $(c(\pi), l(\pi), x(\pi))$ and price functions such that

(i) π solves

$$\max_{\pi' \in \Pi} U(c(\pi'), l(\pi'))$$

(ii) $\forall\ \pi' \in \Pi$, $(c(\pi'), l(\pi'), x(\pi'), \pi', p(\pi'))$ is a C.E.

Given this definition of Ramsey equilibrium, the following proposition is derived as a corollary of proposition 1.

Proposition 2: If an allocation is part of a Ramsey equilibrium, it solves the Ramsey problem [1]:

$$\begin{aligned}
&\max U(c,l) \\
&s.t. \\
&F(c_1 + g_1, ..., c_n + g_n, l) = 0 \\
&\sum_{i=1}^{n} U_i c_i + U_l l = 0
\end{aligned}$$

Next, we will show the *Uniform Commodity Taxation* result.

Uniform Commodity Taxation

Proposition 3: If U is weakly separable between consumption and labor and homothetic in consumption, then it is optimal to set taxes $\tau_i = \tau_j$, $\forall i, j$.

We say U is weakly separable across the consumption goods if $\frac{U_i}{U_j}$ is independent of l.Recall that the assumptions about homothetic implies that:

$$U(c,l) = W(G(c), l)$$

with G homogeneous of degree one (and this implies that G_i is homogeneous of degree zero).

Proof:

The Ramsey problem is:

$$\begin{gathered}
\max U(c,l) \\
s.t. F(c_1 + g_1, ..., c_n + g_n, l) = 0 \\
\sum_{i=1}^{n} U_i c_i + U_l l = 0
\end{gathered}$$

The F.O.C.'s for this problem are, with γ and λ being the respective Lagrange multiplier associated with each constraint:

[1] The reverse is not necessarily true! However, it holds when given government policy, C.E. allocation is unique

$$
\begin{gathered}
U_j - \lambda[\sum_{i=1}^{n} U_{ij}c_i + U_{lj}l] = \gamma F_j, \\
\frac{U_i}{U_l} = \frac{F_i}{F_l}, \\
\frac{U_{lj}}{U_j}l = \frac{W_{21}G_j}{W_1G_j}.
\end{gathered}
$$

In a C.E. we have that:

$$
\begin{aligned}
(1+\tau_i)p_i &= -\frac{U_i}{U_l}, \\
p_i &= -\frac{F_i}{F_l} \\
\Rightarrow (1+\tau_i) &= \frac{U_i}{U_l}\frac{F_i}{F_l} \\
\Rightarrow \frac{(1+\tau_i)}{(1+\tau_j)} &= \frac{U_i}{U_j}\frac{F_i}{F_j} = 1
\end{aligned}
$$

Hence it's optimal to have $\tau_i = \tau_j, \forall i, j$.

The next result that will be showed, the *Intermediate Goods Theorem*, requires some changes in our original environment. We say a good is a primary good if it's in the consumption set or in the utility function or if it's an input or output in the aggregate technology set. If a good is not primary, we call it an intermediate good.

Now let's consider an economy with one final good ,x and an intermediate good, z. Both are assumed to be produced with a constant return to scale technology:

$$
\begin{aligned}
f(x, z, l_1) &= 0 \\
h(g, z, l_2) &= 0
\end{aligned}
$$

Note that we are also assuming 2 different types of labor.

Then we have the following firms' problem:

Firm x:

$$\max px - wl_1 - (1+\eta)qz$$

Firm z:

$$\max rg + qz - wl_2$$

We can write the Ramsey Problem for this economy as:

$$
\begin{gathered}
\max U(x, l) \\
s.t\ U_x x + U_l l = 0 \\
f(x, z, l_1) = 0 \\
h(g, z, l_2) = 0 \\
l_1 + l_2 = l
\end{gathered}
$$

The implementability constraint in this problem was obtained, as usual, substituting the consumers' first order condition on the budget constraint.

Having this new set up, the next result shows that it's optimal to have production efficiency, i.e., the intermediate goods should not be taxed.

Proposition 4:The solution to the Ramsey problem satisfies production efficiency; namely the marginal rates of transformation are equated across technologies. Equivalently, $\eta = 0$ is optimal.

Proof:

From the Ramsey's problem we get the F.O.C.'s:

$$\begin{gathered} \gamma f_z = -\mu h_z \\ U_l - \gamma f_l - \lambda(U_{lx}x + U_{ll}l + U_l) = 0 \\ U_l - \gamma h_l - \lambda(U_{lx}x + U_{ll}l + U_l) = 0 \\ \Rightarrow \ \frac{f_z}{f_l} = -\frac{h_z}{h_l} \end{gathered}$$

From the firms' F.O.C.'s, we have:

$$\begin{gathered} \frac{w}{q(1+\eta)} = \frac{f_l}{f_z} \\ \frac{w}{q} = -\frac{h_l}{h_z} \\ \Rightarrow \ \frac{f_z}{f_l} = -\frac{h_z}{h_l}(1+\eta) \Rightarrow \eta = 0 \end{gathered}$$

Another approach to intermediate goods taxation

Let there be a vector of N final goods $\{x_1, \ldots, x_N\}$, a vector of M types of inputs (labor) $\{l_1, \ldots, l_M\}$ and a vector of K intermediate goods $\{m_1, \ldots, m_K\}$. One can define technology as

$$Y = \left\{ (x_1, \ldots, x_N, -l_1, \ldots, -l_M) | \exists (m_1, \ldots, m_K) \ni \begin{array}{l} f^1(x_1, \ldots, x_N, -l_1, \ldots, -l_M, m_1, \ldots, m_K) = 0 \\ \vdots \\ f^p(x_1, \ldots, x_N, -l_1, \ldots, -l_M, m_1, \ldots, m_K) = 0 \end{array} \right\}$$

Production Efficiency implies

$$\frac{\frac{\partial f^i}{\partial m_l}}{\frac{\partial f^i}{\partial_k}} = \frac{\frac{\partial f^j}{\partial m_l}}{\frac{\partial f^j}{\partial_k}}$$

for all $i, j = 1, \ldots, p$ and all $l, k = 1, \ldots, K$.

Production Efficiency

An allocation of final goods,$x_1, \ldots, x_N, l_1, \ldots, l_M$, is said to be production efficient if there exists some vector of prics $p_1, \ldots, p_N, p_{N+1}, \ldots, p_{N+M}$ such that

$x_1, \ldots, x_N, l_1, \ldots, l_M$ solves

$$\begin{aligned}
&\max \sum_{i=1}^{N} p_i x_i - \sum_{j=1}^{N} p_{N+j} l_j \\
&s.t. \\
&(x, l) \in Y
\end{aligned}$$

Theorem: Any solution to Ramsey problem must satisfy production efficiency.(assume that Y is a convex cone)

Ramsey problem

$$\begin{aligned}
&U(c, l) \\
&s.t. \\
&\sum_{i=1}^{N} U_i c_i + \sum_{j=1}^{M} U_{l_j} l_j = 0 \\
&(c, l) \in Y
\end{aligned}$$

proof:
Using *Separating Hyper-plane Theorem*

Taxing Inelastically Supplied Inputs

Consider the following environment

- 1 type of consumption good,c
- 1 type of labor,l
- 1 type of "Land",T
- technology:$c = F(T, l)$
- r is return on land

Household problem is

$$\begin{aligned}
&\max U(c, l) \\
&s.t. \\
&p(1 - tau_c)c \leq (1 - \tau_l)l + rT
\end{aligned}$$

Firm's problem

$$\max pF(T, l) - l - rT$$

therefore, $r = pF_T(T, l)$.

FOC:

$$\frac{U_c}{U_l} = -\frac{p(1\tau_c)}{1 - tau_l}$$

which implies the following implementability constraint

$$U_c c + U_l l + \frac{UcF_T(T,l)}{1+\tau_c} T$$

Ramsey problem

$$\begin{aligned}
&\max U(c,l) \\
&s.t. \\
&U_c c + U_l l + \frac{UcF_T(T,l)}{1+\tau_c} T \\
&c = F(T,l)
\end{aligned}$$

Optimal taxation is to keep $\frac{1-\tau_l}{1+\tau_c} = 1$ and increase τ_c to collect all the revenue. Note, that this in fact means that optimal policy is Lump-Sum taxtion and not distorting household labor-lisure decision.

Optimal Fiscal Policy in One-Sector Growth Model

Consider the following environment. Government has expenditure g_t and levies tax on consumtion, τ_{ct}, labor income, τ_{lt} and return on capital, τ_{kt} and issues debt b_{t+1}.

A sequential market tax distorted competitive equilibrium is allocations $(c_t, k_{t+1}, b_{t+1}, l_t)$, prices (r_t, w_t, R_{bt}) and government policy $(\tau_{ct}, \tau_{lt}, \tau_{kt}, g_t)$, such that,

i) given government policy and prices, allocations solve household problem

$$\begin{aligned}
&\max \sum_{t=0}^{\infty} \beta^t U(c_t, l_t) \\
&s.t. \\
&(1+\tau_{ct})c_t + k_{t+1} + b_{t+1} \leq (1 + (r_t - \delta)(1-\tau_{kt}))k_t + w_t(1-\tau_{lt})l_t + R_{bt}b_t \\
&k_{t+1} \geq 0 \\
&b_{t+1} \geq -\underline{\mathrm{B}}
\end{aligned}$$

ii) Firm's maximization

$$\begin{aligned}
r_t &= F_k(k_t, l_t) \\
w_t &= F_l(k_t, l_t)
\end{aligned}$$

iii) Allocations are feasible

$$c_t + g_t + k_{t+1} = F(k_t, l_t) + (1-\delta)k_t$$

iv) Government budget constraint holds

$$g_t + R_{bt}b_t = b_{t+1} + \tau_{ct}c_t + \tau_{kt}(r_t - \delta)k_t + w_t\tau_{lt}l_t$$

Assuming interior solution we can characterize equilibrium allocations using first order conditions of the household, which implies the following

$$
\begin{aligned}
\frac{U_l(t)}{U_c(t)} &= -\frac{w_t(1-\tau_{lt})}{1+\tau_{ct}} \\
\beta^t U_c(t) &= \lambda_t(1+\tau_{ct}) \\
\lambda_t &= \lambda_{t+1}(1+(r_{t+1}-\delta)(1-\tau_{kt+1})) \\
R_{bt+1} &= (1+(r_{t+1}-\delta)(1-\tau_{kt+1}))
\end{aligned}
$$

We can multiply household budget constraint at each date by the lagrange multiplier,λ_t and sum over time. Using transversality conditions

$$
\begin{aligned}
\lim_{t\to\infty} \lambda_{t+1}k_{k+1} &= 0 \\
\lim_{t\to\infty} \lambda_{t+1}b_{k+1} &= 0
\end{aligned}
$$

and first order conditions to eliminate prices we can derive the following implementability condition

$$
\sum_{t=0}^{\infty} \beta^t(U_c(t)c_t + U_l(t)l_t) = \frac{U_c(0)}{1+\tau_{c0}}((1+(r_0-\delta)(1-\tau_{k0}))k_0 + R_{b0}b_0)
$$

Ramsy Problem

$$
\begin{aligned}
&\max_{c_t,l_t,k_{t+1},\tau_{c0},\tau_{k0},R_{b0}} \sum t = 0^{\infty}\beta^t U(c_t,l_t) \\
&s.t. \\
&\sum_{t=0}^{\infty} \beta^t(U_c(t)c_t + U_l(t)l_t) = \frac{U_c(0)}{1+\tau_{c0}}((1+(r_0-\delta)(1-\tau_{k0}))k_0 + R_{b0}b_0) \\
&c_t + g_t + k_{t+1} = F(k_t,l_t) + (1-\delta)k_t
\end{aligned}
$$

With no restrictions on problem, optimal taxation is lump-sum taxation (taxing capital at date zero). In order to make the problem more interesting we assume τ_{c0},τ_{k0} and R_{b0} are given.
Let λ be lagrange multiplier on implementability constraint and μ_t be lagrange multiplier on feasibility constraint. Define

$$
\begin{aligned}
W(c,l;\lambda) &= U(c,l) + \lambda(U_c c + U_l l) \\
W_0(c,l;\lambda) &= U(c,l) - \lambda\frac{U_c(0)}{1+\tau_{c0}}((1+(r_0-\delta)(1-\tau_{k0}))k_0 + R_{b0}b_0)
\end{aligned}
$$

Then Ramsy problem is the same as the following

$$
\begin{aligned}
&\max_{c_t,l_t,k_{t+1}} \sum t = 1^{\infty}\beta^t W(c_t,l_t;\lambda) + W_0(c_0,l_0;\lambda) \\
&s.t. \\
&c_t + g_t + k_{t+1} = F(k_t,l_t) + (1-\delta)k_t
\end{aligned}
$$

For $t \geq 1$

$$
W_{ct} = \beta W_{ct+1}(F_{kt+1} + 1 - \delta)
$$

And this implies that in steady state it must be true that

$$1 = \beta(F_{kt+1} + 1 - \delta)$$

This is known as *Chamley-Judd* result. Note that, household FOC implies

$$\frac{U_c(t)}{1+\tau_{ct}} = \beta\frac{U_c(t)}{1+\tau_{ct}}(1 + (r_{t+1} - \delta)(1 - \tau_{kt+1}))$$

And this implies that in steady state it is not optimal for government to distort inter-temporal decisions of household.

Optimal Taxation in Stochastic Environment

Let $S = \{s^1, \ldots, s^N\}$ and the technology be of the form

$$c(s^t) + g(s^t) = z(s^t)l(s^t)$$

Government levies state contingent taxes $\tau(s^t)$ on labor income and issues state contingent debt $b(s^t)$ with return $R_b(s^t)$. Household problem is

$$\begin{aligned}
&\max \sum_t \sum_{s^t} \beta^t \mu(s^t) U(c(s^t), l(s^t)) \\
&s.t. \\
&c(s^t) + b(s^t) \leq (1 - \tau(s^t))w(s^t)l(s^t) + R_b(s^t)b(s^{t-1}) \\
&b(s^t) \geq -\underline{B} \qquad (\text{some } \underline{B} \geq 0) \\
&b(s^{-1}) \quad \text{given}
\end{aligned}$$

Firm's problem

$$\begin{aligned}
&\max c(s^t) + g(s^t) - w(s^t)l(s^t) \\
&s.t. \\
&c(s^t) + g(s^t) \leq z(s^t)l(s^t)
\end{aligned}$$

Competitive equilibrium is defined as usual (straight-forward). One can use household FOC to derive the following implementability condition

$$\sum_t \sum_{s^t} \beta^t \mu(s^t)(c(s^t)U_c(s^t) + l(s^t)U_l(s^t)) \leq U_c(s_0)R(s_0)b(s^{-1})$$

Characterization

Suppose allocations $(c(s^t), l(s^t))$ and government expenditure $g(s^t)$ satisfy the above implementability together with the resource constraint. We want to find the prices and policies taht support this allocation in a competitive equilibrium.

First order conditions from firm and household's problem imply

$$\begin{aligned} w(s^t) &= z(s^t) \\ -\frac{U_l(s^t)}{U_c(s^t)} &= w(s^t)(1-\tau(s^t)) \end{aligned}$$

Which pins down $w(s^t)$ and $\tau(s^t)$.

Next, define $H(s^t) = c(s^t)U_c(s^t) + l(s^t)U_l(s^t)$, then

$$\begin{aligned} H(s^t) &= U_c(s^t)[c(s^t) + l(s^t)\frac{U_l(s^t)}{U_c(s^t)}] \\ &= U_c(s^t)[c(s^t) + z(s^t)l(s^t)(1-\tau(s^t))] \\ &= U_c(s^t)[-g(s^t) + z(s^t)l(s^t) - z(s^t)l(s^t) + z(s^t)l(s^t)\tau(s^t)] \\ &= U_c(s^t)[z(s^t)l(s^t)\tau(s^t) - g(s^)] \end{aligned}$$

therefore, $H(s^t)$ is the value of government surplus in terms of current history consumption good [2].

For any history s^r define $b(s^r)$ as

$$b(s^r) = \sum_{t=r+1}^{\infty} \sum_{s^t|_{s^r}} \beta^{t-r}\mu(s^t|s^r)\frac{H(s^t)}{H(s^r)}$$

Therefore we only need to define $R(s^t)$. There in no unique $R(s^t)$ to support the allocation described above. Any $R(s^t)$ that satisfies the following will do[3]

$$U_c(s^t) = \sum_{s^{t+1}|_{s^t}} \beta\mu(s^{t+1}|s^t)U_c(s^{t+1})R(s^{t+1})$$

Ramsey Problem

$$\begin{aligned} &\max \sum_t \sum_{s^t} \beta^t \mu(s^t) U(c(s^t), l(s^t)) \\ &s.t. \\ &\sum_t \sum_{s^t} \beta^t \mu(s^t)(c(s^t)U_c(s^t) + l(s^t)U_l(s^t)) \leq U_c(s_0)R(s_0)b(s^{-1}) \\ &c(s^t) + g(s^t) = z(s^t)l(s^t) \end{aligned}$$

Let λ be the lagrange multiplier on implementability constraint, define

$$\begin{aligned} W(c,l;\lambda) &= U(c,l) + \lambda[cU_c + lU_l] \\ W_0(c(s_0), l(s_0); \lambda) &= U(c(s_0), l(s_0)) + \lambda[c(s_0)U_c(s_0) + l(s_0)U_l(s_0) - U_c(s_0)R(s_0)b(s^{-1})] \end{aligned}$$

[2]Notice that $\beta^t \frac{H(s^t)}{U_c(s_0)}$ is exactly the value of government surplus at each history in terms if date zero consumption. That means that the implementability constraint, is actually a constraint on expected value of government surplus for entire future.

[3]See Chari and Kehoe's "Optimal Fiscal and Monetary Policy" pages 44-47 for more discussion.

Now we can rewrite the Ramsey problem

$$
\begin{aligned}
&\max \mu(s_0)W_(c(s_0), l(s_0); \lambda) + \sum_{t=1}^{\infty}\sum_{s^t} \beta^t \mu(s^t) W(c(s^t), l(s^t); \lambda) \\
&s.t. \\
&c(s^t) + g(s^t) = z(s^t) l(s^t) \qquad ; \alpha(s^t)
\end{aligned}
$$

Then first order conditions for $t \geq 1$[4]

$$
\begin{aligned}
W_c(s^t) &= \alpha(s^t) \\
-W_l(s^t) &= \alpha(s^t) z(s^t)
\end{aligned}
$$

Which implies $-\frac{W_l(s^t)}{W_c(s^t)} = z(s^t)$. Therefore, if $g(s^t) = g(s^r)$ and $z(s^t) = z(s^r)$, then we have[5]

$$
c(s^t) = c(s^r) \quad , \quad l(s^t) = l(s^r)
$$

Examples

[Will be added later]

[4] the sam eis true for $t = 0$ if $b(s^{-1}) = 0$

[5] For more details read Chari and Kehoe's "Optimal Fiscal and Monetary Policy" pages 44-47