Macroeconomic Theory **V.V.Chari**
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Lecture 3[*]

Policy without commitment[1]

Static Model: Suppose household makes decision on its consumption, saving and labor supply each period in two phases: In each period household endowed with w unit of consumption good which it can consume, c_1, or save, k. Return of saving is R (given) and household claims it in the afternoon. There is a government that has expenditure g and finances that expenditure using tax on household saving, δ,and labor income, τ. Household decides about its labor supply, l, and second portion of its consumption, c_2,in the afternoon. So, household's problem is[2]

$$\begin{aligned}
&\max U(c_1+c_2,l)\\
&s.t.\\
&c_1+k\leq w\\
&c_2\leq R(1-\delta)k+(1-\tau)l\\
&c_1\geq 0\\
&c_2\geq 0\\
&k\geq 0
\end{aligned}$$

Government budget constraint is

$$g=\delta Rk+\tau l$$

Competitive equilibrium is allocations $\{c_1,c_2,k,l\}$ and government policy $\{\tau,\delta,g\}$ such taht: 1)given government policy, allocations solve household problem. 2) Government budget constraint holds.

Commitment

Consider the case when government has commitment. The timing is as the following:

1. Government chooses policy, $\pi=(\tau,\delta)$
2. Household chooses (c_1,k)
3. Household chooses (c_2,l)

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[*] These notes are prepared by Laurence Ales, Roozbeh Hosseini , Pricila Maziero and Miguel Ricaurte. They are preliminary and possibly contain errors. Comments and feedbacks are welcome.

[1] See Chari and Kehoe's "Sustainable Plan", *JPE*,1990.

[2] Wage is normalized to one

Household's reaction to government policy in a competitive environment is as the following

$$\begin{aligned} R(1-\delta) > 1 &\quad\Rightarrow\quad k = w, c_1 = 0 \\ R(1-\delta) < 1 &\quad\Rightarrow\quad k + 0, c_1 = w \\ R(1-\delta) = 1 &\quad\Rightarrow\quad \text{indifferent} \end{aligned}$$

And in interior solution, $1 - \tau = -\frac{U_l}{U_c}$.

Define δ^* such that, $(1-\delta^*)R = 1$. Now consider the following two cases

1. $g \leq \delta^* Rw$, the government can finance its expenditure entirely by levying (lump-sum) tax on k. The optimal allocations and taxes are

   $$\begin{array}{ll} c_1 = 0 & k = w \\ U_l = U_c & \tau = 0, \delta = \frac{g}{Rw} \end{array}$$

   and $c_2 = R(1-\delta)w + l$

2. $g \geq \delta^* Rw$, This is the interesting case since government has to raise revenue through distortionary taxation. It can be shown[3] that in this case the *unique Ramsey outcome* has $\delta = delta^*$, $c_1 = 0$, $k = w$, and τ, l and c_2 are solution to the following problem

   $$\begin{aligned} &\max U(c_2, l) \\ &s.t. \\ &c_2 \leq w + (1-\tau)l \\ &-\frac{U_l}{U_c} = (1-\tau) \\ &g \leq \delta * w + \tau l \end{aligned}$$

No commitment

Consider now the following timing

1. Household chooses (c_1, k)
2. Government chooses policy, $\pi = (\tau, \delta)$
3. Household chooses (c_2, l)

Claim: The unique equilibrium outcome has $c_1 = w$, $k = 0$, $\delta > \delta^*$ and τ, c_2 and l are chosen to solve

$$\begin{aligned} &\max U(c_2 + w, l) \\ &s.t. \\ &c_2 \leq (1-\tau)l \\ &-\frac{U_l}{U_c} = (1-\tau) \\ &g \leq \tau l \end{aligned}$$

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[3] Proposition 1 in Chari-Kehoe, *JPE*,1990

Deep sources of time inconsistency problem [To be added later]

Repeated economy and Sustainable Plans

Now suppose the economy repeats every period. Household' problems is

$$
\begin{aligned}
&\max \sum_{t=0}^{\infty} \beta^t U(c_{1t} + c_{2t}, l_t) \\
&s.t. \\
&c_{1t} + k_t \leq w \\
&c_{2t} \leq R(1-\delta_t)k_t + (1-\tau_t)l_t \\
&c_{1t} \geq 0 \\
&c_{2t} \geq 0 \\
&k_t \geq 0
\end{aligned}
$$

And government budget is the same. Competitive equilibrium is defined as usual.

Proposition: Equilibrium under commitment has the same outcome as infinitely repeated sequence of static environment with commitment.(proof is easy)

No commitment

Define history of government policy as $h_{1t} = \{\pi_s | s = 0, \ldots, t-1\}$, where $\pi_t = (\delta_t, \tau_t)$. And $h_{2t} = \{h_{1t}, \pi_t\}$. Also, define household's allocation rule $f_t = \{f_{1t}(h_{1t}), f_{2t}(h_{2t})\}_{t=0}^{\infty}$ as $f_{1t}(h_{1t}) = (c_{1t}(h_{1t}, k_t(h_{1t}))$ and $f_{2t}(h_{2t}) = (c_{2t}(h_{2t}, l_t(h_{2t}))$. Let $\sigma_t(h_{1t})$ be government's policy rule. After any history h_{1t}, a policy rule σ induces future histories according to

$$
\begin{aligned}
h_{2t} &= (h_{1t}, \sigma_t(h_{1t})) \\
h_{1t+1} &= h_{2t} = (h_{1t}, \sigma_t(h_{1t})) \\
h_{2t+1} &= (h_{1t}, \sigma_t(h_{1t}), \sigma_{t+1}(h_{1t}, \sigma_t(h_{1t})))
\end{aligned}
$$

and so on.

Sustainable plan

And allocation and policy rule (f, σ) is a sustainable plan if

1. i Given a policy rule σ continuation of allocation rule f is optimal after any history h_{1t},i.e. $\{f_{1s}(h_{1s}), f_{2s}(h_{2s})\}_{s=t}^{\infty}$ solves

$$
\begin{aligned}
&\max \sum_{s=t}^{\infty} \beta^{s-t} U(c_{1s} + c_{2s}, l_s) \\
&s.t. \\
&c_{1s} + k_s \leq w \\
&c_{2s} \leq R(1-\delta_s(h_{1s}))k_s + (1-\tau_t(h_{1s}))l_s
\end{aligned}
$$

for all $s \geq t$

ii Given a policy rule σ continuation of allocation rule f is optimal after any history h_{2t},i.e. $f_{2t}(h_{2t})$ together with $\{f_{1s}(h_{1s}), f_{2s}(h_{2s})\}_{s=t+1}^{\infty}$ solves

$$\begin{aligned}
&\max U(c_{1t}(h_{1t}) + c_{2t}, l_t) + \beta \sum_{s=t+1}^{\infty} \beta^{s-t} U(c_{1s} + c_{2s}, l_s) \\
&s.t. \\
&c_{2t} \leq R(1-\delta_t)k_t(h_{1t}) + (1-\tau_t)l_t \\
&c_{1s} + k_s \leq w \\
&c_{2s} \leq R(1-\delta_s(h_{1s}))k_s + (1-\tau_t(h_{1s}))l_s
\end{aligned}$$

for all $s \geq t+1$

2. Given allocation rule f, continuation of σ solves the following for any history h_{1t}

$$\begin{aligned}
&\max \sum_{s=t}^{\infty} \beta^{s-t} U(c_{1s} + c_{2s}, l_s) \\
&s.t. \\
&g \leq \delta_s R k_s(h_{1s}) + \tau_s l_s(h_{2s})
\end{aligned}$$

where h_{2s} is induced by σ.

Define a static utility U^s as the following

$$\begin{aligned}
&U^s = \max_{\tau,l} U(w + (1-\tau)l, l) \\
&s.t. \\
&-\frac{U_l}{U_c} = (1-\tau) \\
&g \leq \tau l
\end{aligned}$$

And let σ^s be a policy plan that sets $\delta_t = 1$ and τ_t according to the solution to static problem above, for all t. For each level of saving in the first stage, k, define

$$\begin{aligned}
&U^d(k) = \max_{\tau,l,\delta,c_2} U(w - k + c_2, l) \\
&s.t. \\
&c_2 \leq (1-\tau)l \\
&-\frac{U_l}{U_c} = (1-\tau) \\
&g \leq \tau l + \delta R k
\end{aligned}$$

Note that $U^d(0) = U^s$.

Characterization theorem

An arbitrary sequences policy π and allocation x is a sustainable outcome if and only if (i) (π, x) is a competitive equilibrium for all t. (ii) the following inequality holds for all t

$$\sum_{s=t}^{\infty} U(c_{1s} + c_{2s}, ls) \geq U^d(k_t) + \frac{\beta}{1-\beta} U^s$$