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Econ. 8108 **Spring 2004**

Lecture 4*

Monetary Economics

We now introduce monetary models. Our first model consist of a cash-credit economy, where one of the two goods present in the economy is constrained by a Clower constraint. In this economy household can be interpreted as consisting of a worker-shopper pair, at any period in time the household chooses the amount of cash to allocate to the shopper, then the pair divides, the shopper chooses the amount of consumption goods and the worker supplies labor earning cash for next period. Cash good will be denoted by c_1 and credit good by c_2, feasibility then requires

$$c_{1t} + c_{2t} = l_t, \quad \forall t.$$

Preferences of the household are given by

$$\sum_{t=0}^{\infty} \beta^t u\left(c_{1t}, c_{2t}, l_t\right)$$

timing is crucial in determining the constraint faced by the household: every period t begins with the securities market where the household receives invoices from the credit good bought in the previous period, together with labor income and returns on outstanding loans; immediately after the labor and good market open the worker supplies work in exchange for cash in the next period, the shopper buys cash goods given current cash and credit goods given the period budget constraint. Then next period begins and the cycle repeats. Our budget constraint at each period will be given by

$$A_{t+1} \leq (M_t - p_t c_{1t}) + w_t l_t - p_t c_{2t} + R_{t+1} B_t + T_t, \quad \forall t,$$

where the term in parenthesis is the unspent cash of the household, R_{t+1} is the gross nominal interest rate and A_{t+1} chosen in security market at the beginning of the period is given by $A_{t+1} = M_{t+1} + B_{t+1}$; purchase of cash goods is constrained by the following cash-in advance constraint

$$p_t c_{1t} \leq M_t, \quad \forall t,$$

note how, introducing this constraint, we are implicitelly assigning to money a special role, since it allows us to purchase cash goods. In order to avoid Ponzi schemes we introduce a lower bound on credit, that is

$$B_t \geq \bar{B}, \quad \forall t.$$

The HH problem is then given by

$$\max \sum_{t=0}^{\infty} \beta^t u\left(c_{1t}, c_{2t}, l_t\right)$$

$$\begin{aligned} s.t. \quad M_{t+1} + B_{t+1} &\leq (M_t - P_t c_t) + w_t l_t - p_t c_{2t} + R_{t+1} B_t + T_t, \quad \forall t \\ p_t c_{1t} &\leq M_t, \quad \forall t \\ B_t &\geq \bar{B}, \quad \forall t \end{aligned}$$

*These notes are prepared by Laurence Ales, Roozbeh Hosseini , Pricila Maziero and Miguel Ricaurte. They are preliminary and possibly contain errors. Comments and feedbacks are welcome.

(for now on we set transfers T_t equal to zero) a profit maximizing firm solves the following problem

$$\begin{aligned} &\max p_t (c_{1t} + c_{2t}) - w_t l_t, \\ &s.t. \quad c_{1t} + c_{2t} \leq l_t \end{aligned}$$

Since the household are identical in equilibrium we must have $B_t = 0$. A competitive equilibrium is a sequence of allocation $\{c_{1t}, c_{2t}, l_t\}_{t=0}^{\infty}$ prices $\{p_t, w_t, R_t\}_{t=0}^{\infty}$ a monetary policy $\{M_t\}_{t=0}^{\infty}$ and a bond $\{B_t\}_{t=0}^{\infty}$ such that household maximize, firm maximize and market clears

$$\begin{aligned} c_{1t} + c_{2t} &= l_t, \\ B_t &= 0, \\ M_t &= \bar{M} \end{aligned}$$

(we also must have as an initial condition $M_0 + B_0 \leq \bar{M}$). Also note how we can not consider the Ponzi scheme constraint at the moment since in equilibrium we must have $B_t = 0$. The first order condition of the household problem are given by

$$\beta^t u_{c1}(t) = (\lambda_t + \mu_t) p_t \quad (1)$$

$$\beta^t u_{c2}(t) = \lambda_t p_t \quad (2)$$

$$-\beta^t u_l(t) = \lambda_t w_t \quad (3)$$

$$\lambda_t + \mu_t = \lambda_{t-1} \quad (4)$$

$$\lambda_t = R_t \lambda_{t+1} \quad (5)$$

where with λ_t we denote the lagrange multiplier for the budget constraint and μ_t for the budget constraint. From the firm problem we have $p_t = w_t$.

We now introduce a stationary equilibrium, defined as a monetary equilibria with all real quantities constant, that is $c_{1t} = c_1$, $c_{2t} = c_2$, $l_t = l$ for all t and also

$$\frac{M_t}{p_t} = \frac{\bar{M}}{p}, \quad \forall t. \quad (6)$$

We have that the marginal rate of transformation between l and the credit good is equal to one

$$-\frac{U_l}{U_{c2}} = 1,$$

also in a stationary equilibria we have that (substituting (2) and (4) into (1))

$$\frac{\beta^t u_{c1}(t)}{p_t} = \frac{\beta^{t-1} u_{c2}(t-1)}{p_{t-1}}, \quad (7)$$

then for $t \geq 1$ we have that (assuming constant price levels) $\beta u_{c1} = u_{c2}$, so that

$$-\frac{u_l}{\beta u_{c1}} = 1,$$

note that $u_1 > u_2$ since buying cash good the household is loosing the opportunity cost of interest, the households will then hold an inefficiently low amount of cash goods.

Summarizing the stationary equilibria is characterized by

$$-\frac{u_l}{u_2} = 1, \quad (8)$$

$$-\frac{u_l}{\beta u_1} = 1, \quad (9)$$

$$c_1 + c_2 = l, \quad (10)$$

also the price level is determined by $c_1 = \frac{\bar{M}}{p}$. We say that this type of economy exhibits neutrality, that is scaling the nominal variables by λ leaves real allocation unaffected.

Neutrality of money is deeply connected with homogeneity of degree zero in allocation for prices (we are really only changing the unit of measure).

An allocation $\{c_{1t}, c_{2t}, l_t\}_{t=0}^{\infty}$ is efficient if solves the following maximization problem

$$\max \sum_{t=0}^{\infty} \beta^t u\left(c_{1t}, c_{2t}, l_t\right)$$
$$s.t. \quad c_{1t} + c_{2t} \leq l.$$

Proposition A stationary monetary equilibria is inefficient. This follows immediately from first order conditions.

We now introduce monetary policy in our model , consider money growing according to $M_t = gM_{t-1}$ with $g \geq 0$ (g is the gross rate of money supply). If we look at the conditions imposed for a stationary monetary equilibrium, we observe how, by looking at equation (6) price must also grow at rate g. In particular from (7) we have that

$$\beta \frac{u_1}{u_2} = \frac{p_t}{p_{t-1}},$$

so that

$$\frac{u_1}{u_2} = \frac{g}{\beta},$$

if the allocation is efficient we would have $\frac{u_1}{u_2} = 1$, so if $g = \beta$ (money supply contracting at rate β) from the above we have that the allocation is efficient; also note how a high value of g gives a more inefficient allocation. We now look at the gross nominal interest rate, from (5) and (2) we have

$$\frac{\beta^t u_2\left(t\right)}{p_t} = \frac{\beta^{t+1} u_2\left(t+1\right)}{p_{t+1}} R_{t+1}$$

so that $R = \frac{g}{\beta}$. So high inflation economies are economies with high interest rate. Recall that from Fisher rule we have R =real interest rate + expected inflation rate. If $g < \beta$ we can solve the f.o.c but we will have no equilibrium since nominal interest rate would be negative. So in theory I could arbitrage issuing debt, getting interest and paying back less money.

Some observations

1. Consider $u\left(c_1, c_2, l\right) = u\left(c_1, c_2\right) + V\left(l\right)$, recall that

$$\frac{U_1\left(c_1, c_2, l\right)}{U_2\left(c_1, c_2, l\right)} = R$$

   that is setting $c = c_1 + c_2$

$$\frac{U_1\left(\frac{M}{p}, c - \frac{M}{p}\right)}{U_2\left(\frac{M}{p}, c - \frac{M}{p}\right)} = R$$

   then, we have the following functional relation $f\left(M, c\right) = R$ or $\frac{M}{p} = L\left(R, c\right)$; given this last relation by Taylor expansion we have

$$\log \frac{M}{p} = L_0 + L_1 \log c + L_2 \log R + \varepsilon;$$

   thus we can estimate parameters L_0, L_1, L_2 using a regression.

2. We now compare two economies; in the first one monetary policy is determined via open market operations, in the second using lump sum transfer. For the first economy we have the following budget constraint for the household

$$M_t B_t = (M_{t-1} - p_{t-1}c_{1t-1}) - p_{t-1}c_{2t-1} + w_{t-1}l_{t-1} + R_t B_{t-1},$$

the cash in advance constrain is given by

$$p_t c_{1t} \leq M_t,$$

and the government budget constraint

$$\bar{M}_t - \bar{M}_{t-1} + \bar{B}_t = R_t \bar{B}_{t-1}.$$

In the second economy household are constrained by

$$\begin{aligned} p_t c_{1t} + p_t c_{2t} + M_{t+1} &\leq w_t l_t + M_t + T_t, \\ p_t c_{1t} &\leq M_t + T_t \end{aligned}$$

and the government budget constraint is given by

$$\bar{M}_{t+1} = \bar{M}_t + T_t.$$

It can be shown that these two economies are equivalent (refer to homework 3).

We now consider a stochastic version of our economy. Consider a random variable taking a finite number of values $s = (s_0, \ldots, s_N)$. We denote with s^t the partial history up to time t. We consider two types of shocks in the economy: in the production function $F(l_t, s^t) = z(s^t) l(s^t)$ and in the government expenditure, that is $g = g(s^t)$. The resource constraint for this economy is given by

$$c_1\left(s^t\right) + c_2\left(s^t\right) \leq z\left(s^t\right) l\left(s^t\right), \tag{11}$$

and money growth is given by

$$M\left(s^t\right) = g\left(s^t\right) M\left(s^{t-1}\right). \tag{12}$$

Household maximize the following future flow of discounted utilities

$$\sum_t \sum_{s^t} \beta^t \pi\left(s^t\right) u\left(c_1\left(s^t\right), c_2\left(s^t\right), l\left(s^t\right)\right),$$

the constraints for the securities market are given by

$$\begin{aligned} M\left(s^t\right) + B\left(s^t\right) &= \left(M\left(s^{t-1}\right) - p\left(s^{t-1}\right) c_1\left(s^{t-1}\right)\right) + \qquad (13) \\ &\quad + w\left(s^t\right) l\left(s^{t-1}\right) - p\left(s^{t-1}\right) c_2\left(s^{t-1}\right) + R\left(s^{t-1}\right) B\left(s^{t-1}\right) + T\left(s^t\right), \\ p\left(s^t\right) c_1\left(s^t\right) &\leq M\left(s^t\right). \qquad (14) \end{aligned}$$

Firms solve the following problem

$$\begin{aligned} &\max p\left(s^t\right)\left(c_1\left(s^t\right) + c_2\left(s^t\right)\right) - w\left(s^t\right) l\left(s^t\right) \\ &s.t.\ \ c_1\left(s^t\right) + c_2\left(s^t\right) \leq z\left(s^t\right) l\left(s^t\right) \end{aligned}$$

From the household first order condition we have

$$-\frac{u_l\left(s^t\right)}{u_2\left(s^t\right)} = z\left(s^t\right), \tag{15}$$

$$\frac{u_1\left(s^t\right)}{u_2\left(s^t\right)} \leq R\left(s^t\right), \tag{16}$$

with equality if the cash in advance constraint is binding. We can now write the household infinite horizon budget constraint as

$$\sum_t \sum_{s^t} \beta^t \pi\left(s^t\right)\left(c_1 u_1 + c_2 u_2 + l u_l\right) = \frac{u_2\left(s^0\right)}{p\left(s^0\right)} A\left(s^{-1}\right),$$

where all of the variables in the double sum are dependent on s^t; with $A\left(s^{-1}\right)$ we denote the value of the initial assets deflated by the price level. In a deterministic economy a stationary equilibria exists if $R > 1$, in fact the household first order condition are given by (let $\lambda\left(s^t\right)$ be the lagrange multiplier on budget constraint and $\mu\left(s^t\right)$ on the cash in advance constraint)

$$\begin{aligned}
\beta^t \pi\left(s^t\right) u_{c1}\left(s^t\right) &= \sum_{s^{t+1}} \left(\lambda\left(s^{t+1}\right) + \mu\left(s^{t+1}\right)\right) p\left(s^t\right), & (17)\\
\beta^t \pi\left(s^t\right) u_{c2}\left(s^t\right) &= \sum_{s^{t+1}} \lambda\left(s^{t+1}\right) p\left(s^t\right), & (18)\\
\lambda\left(s^t\right) - \mu\left(s^t\right) &= \sum_{s^{t+!}} \lambda\left(s^{t+1}\right), & (19)\\
\lambda\left(s^t\right) &= R\left(s^t\right) \sum_{s^{t+1}} \lambda\left(s^{t+1}\right), & (20)
\end{aligned}$$

then

$$R\left(s^t\right) \sum_{s^{t+1}} \lambda\left(s^{t+1}\right) - \mu\left(s^t\right) = \sum_{s^{t+1}} \lambda\left(s^{t+1}\right),$$

so that the cash in advance constraint is not binding if the interest rate is 1.

We now show how the above formulation is equivalent to an economy where the return on debt is state contingent. For this second economy the household budget constraint for this economy is given by

$$\begin{aligned}
M\left(s^t\right) + \hat{B}\left(s^t\right) &= \left(M\left(s^{t-1}\right) - p\left(s^{t-1}\right) c_1\left(s^{t-1}\right)\right) + & (21)\\
&\quad + w\left(s^t\right) l\left(s^{t-1}\right) - p\left(s^{t-1}\right) c_2\left(s^{t-1}\right) + \hat{R}\left(s^t\right) B\left(s^{t-1}\right) + T\left(s^t\right),
\end{aligned}$$

if we look at the first order condition for debt, we have

$$\lambda\left(s^t\right) = \sum_{s^{t+1}} \hat{R}\left(s^{t+1}\right) \lambda\left(s^{t+1}\right) \tag{22}$$

compared with the previous first order condition given by equation (20); to show partial equivalence, given an equilibrium allocation in economy define $R\left(s^t\right)$ so that

$$R\left(s^t\right) \sum_{s^{t+1}} \lambda\left(s^{t+1}\right) = \sum_{s^{t+1}} \hat{R}\left(s^{t+1}\right) \lambda\left(s^{t+1}\right),$$

and we also set

$$R\left(s^{t-1}\right) B\left(s^{t-1}\right) = \hat{B}\left(s^t\right),$$

and finally

$$R\left(s^{t-1}\right) B\left(s^{t-1}\right) + T\left(s^t\right) = R\left(s^t\right) \hat{B}\left(s^{t-1}\right) + \hat{T}\left(s^t\right). \tag{23}$$

So the equilibrium allocation given for economy two is also an equilibrium allocation for economy 1 once we redefine the lump sum transfers according to (23)

Optimal monetary policy

Consider now a central government having as available instruments, proportional taxes on labor income and money printing. Considering a stochastic environment we have the following feasibility constraint

$$c_1\left(s^t\right)+c_2\left(s^t\right)+g\left(s^t\right)=z\left(s^t\right)l\left(s^t\right), \tag{24}$$

household will maximize utility, constrained by the following securities market constraint and cash in advance constraint, both given by

$$M\left(s^t\right)+B\left(s^t\right)=\left(M\left(s^{t-1}\right)-p\left(s^{t-1}\right)c_1\left(s^{t-1}\right)\right)+ \tag{25}$$
$$+w\left(s^t\right)l\left(s^{t-1}\right)\left(1-\tau\left(s^{t-1}\right)\right)-p\left(s^{t-1}\right)c_2\left(s^{t-1}\right)+R\left(s^{t-1}\right)B\left(s^{t-1}\right)+T\left(s^t\right),$$
$$p\left(s^t\right)c_1\left(s^t\right)\leq M\left(s^t\right). \tag{26}$$

the government budget constrained is given by

$$\begin{aligned} M\left(s^t\right)-M\left(s^{t-1}\right)+B\left(s^t\right) &= R\left(s^{t-1}\right)B\left(s^{t-1}\right)+p\left(s^{t-1}\right)g\left(s^{t-1}\right)+ \\ &\quad -w\left(s^{t-1}\right)\tau\left(s^{t-1}\right)l\left(s^{t-1}\right). \end{aligned}$$

firms solve the following problem

$$\begin{aligned} &\max p\left(s^t\right)\left(c_1\left(s^t\right)+c_2\left(s^t\right)+g\left(s^t\right)\right)-w\left(s^t\right)l\left(s^t\right) \\ &s.t.\ \ c_1\left(s^t\right)+c_2\left(s^t\right)+g\left(s^t\right)\leq z\left(s^t\right)l\left(s^t\right) \end{aligned}$$

a competitive equilibrium is allocations$\{c_1\left(s^t\right),c_2\left(s^t\right),l\left(s^t\right),g\left(s^t\right),M\left(s^t\right),B\left(s^t\right)\}$ a price system $\{p\left(s^t\right),w\left(s^t\right),R\left(s^t\right)\}$ and a policy $\{\tau\left(s^t\right)\}$ such that: household, given prices and policies, are maximizing, firm are maximizing, the resource constraint hold, and $M\left(s^t\right)$ is given by (12). We now state a characterization theorem, so that given the initial price level $p\left(s_0\right)$, allocations are an equilibrium if the following hold

$$\sum_{t,s^t}\sum_{s^t}\beta^t\pi\left(s^t\right)\left(c_1u_1+c_2u_2+lu_l\right) = \frac{u_2\left(s^0\right)}{p\left(s^0\right)}A\left(s_0\right), \tag{27}$$

$$\frac{u_1}{u_2} = 1, \tag{28}$$

$$c_1\left(s^t\right)+c_2\left(s^t\right)+g\left(s^t\right) = z\left(s^t\right)l\left(s^t\right). \tag{29}$$

Then the Ramsey problem would be to maximize

$$\sum_t\sum_{s^t}\beta^t\pi\left(s^t\right)u\left(c_1\left(s^t\right),c_2\left(s^t\right),l\left(s^t\right)\right)$$

subject to (27), (28) and (29). Note that if A_0 in (27) is positive then the government would set p_0 as high as possible in order to make the right hand side of the constraint as small as possible; so in order to have a competitive equilibria we must set $A_0=0$. If we now assume a utility function homotetic in consumption and separable in leisure, by the intermediate good theorem we must have the same tax on c_1 and c_2, then we must have

$$\frac{u_1\left(s^t\right)}{u_2\left(s^t\right)}=1$$

this translates in a optimal monetary policy that prescribes a nominal interest rate equal to zero, so that the Friedman rule holds, again refer to Hw 3.