Supplement to “Endogenous Comparative Advantage”

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Contents

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This documents contains a discussion about empirical evidence supporting some implications of the model, additional calculations, and proofs that for the sake of brevity have been omitted from the paper "Endogenous Comparative Advantage". To make the derivations self-contained some equations have been duplicated from the main paper.

B Empirical evidence on wage differences

B.1 The relationship between wage differentials and GDP

We have investigated the relationship between the skill premium and wage differences in the data. Given the large differences in standard of living, it is hardly surprising that the wage differential tends to be larger in richer countries. However, we find it striking that the relationship is almost one to one. In Figure 1 we plot GDP against the education wage differential and, for comparison, also the wage ratio (the labels are Alpha-3 ISO country codes). The underlying data, from Hendricks [1], is a collection of Mincerian regressions by several authors (see references in Hendricks [1]) using micro data from a number of different countries.[1] We have used the coefficients of these regressions to compute the predicted wage of an average male worker with 8 and 12 years of education. All observations are converted to 1989 PPP-adjusted dollars, and we have only included the 26 studies using wage data between 1987 and 1991. The correlation between the wage differential and GDP is 0.98, while the correlation between GDP and the wage ratio is -0.4.

Obviously, there may be other reasons for the wage differential to rise with GDP. Differences in total factor productivity (henceforth TFP) would generate a scatterplot like the one in Figure 1, but such TFP differences is part of what a theory of development should explain, and there seems to be little consensus about what could account for differences in measured TFP.[2] This paper provides one simple theory for how measures of TFP can differ, in spite of all countries having access to the same technology.

Note also that because workers look at differences in expected utility, incentives are improved if all wages are multiplied by the same constant. This is a robust feature of the model, which is true irrespective of the number of signals and degree of risk aversion. The effects explored in this paper would therefore tend to strengthen the effect if TFP would rise for some other reason.

To sum up, the existing empirical literature measures the skill premium as a ratio for a good reason: the neoclassical growth model has implications about ratios and is silent about differences. Our model, on the other hand, has implications about differences and is silent on ratios. This is not a technical detail, but comes directly from the economics of the model. An individual compares her expected utility from investing with the expected utility if not investing, which means that the difference rather than the ratio of expected

[1] We are grateful to Lutz Hendricks who generously shared his dataset.

[2] In fact, Prescott [2] argues that the crucial difference between rich and poor countries is total factor productivity.

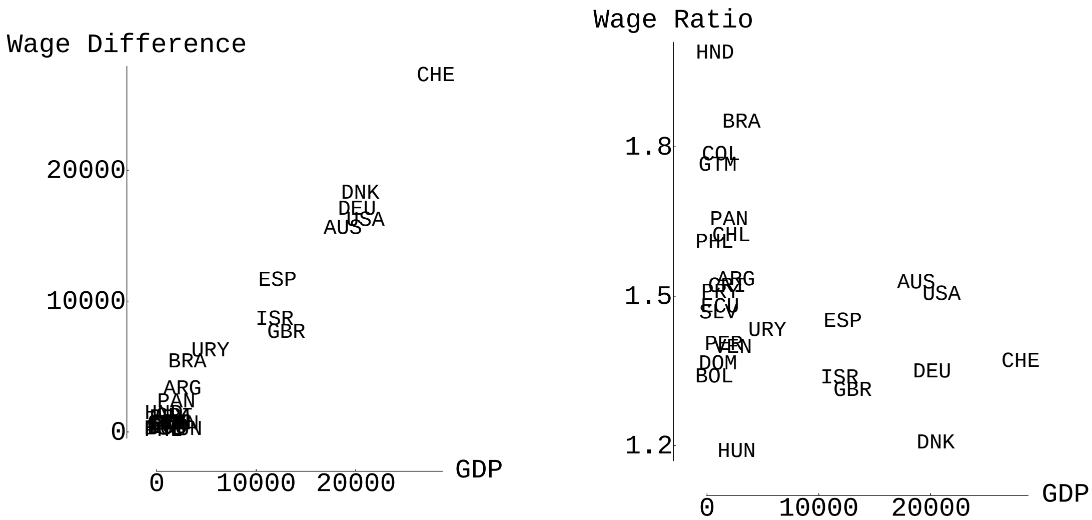

Figure 1: Education wage differential and wage ratio against GDP (1989 PPP-Adjusted dollars)

utilities is what is relevant.[3,4]

B.2 The relationship between Export/Domestic sector wage ratios and GDP

Another robust implication of our theory is that, in a rich country, the wage of workers in the export sector should exceed the wage of workers in the sector where the country is a net importer. The opposite relation should hold in a poor country.

This implication is also consistent with data. Table 1 summarizes information about how wages in exporting industries compare to the overall average wages in different countries. We computed the average wage in the "export sector" as a weighted average of the (average) wage in each industry, weighted by the industry's value of exports. The table reports the ratio of the average export sector wage to the average wage. A comparison of column (2) and (3) reveals that in rich countries exporting industries pay higher wages than average, while in poor countries, exporting industries pay less than average. This fact is consistent over time. The ratio of the average in the export sector to the average wage in each country is also a significantly

[3] This feature is true regardless of the number of available signals.

[4] It may appear that is critical that the preferences in (15) induce risk-neutrality over money lotteries. However, little change if we instead assume that $u(x_1, x_2) = x_1^{\frac{\alpha}{\gamma}} x_2^{\frac{1-\alpha}{\gamma}}$, in which case $\frac{\gamma-1}{\gamma}$ is the (constant) relative risk aversion of the induced preferences over income lotteries. The only difference in terms of the model is that incentives now are proprtional to $w_g^{\frac{1}{\gamma}} - w_b^{\frac{1}{\gamma}}$ rather than $w_g - w_b$. For $\gamma = 2$, the exact same allocation as in the example in Table 4 is an equilibrium if c is uniform over $[0.0417, 0.0481]$.

$$\frac{\text{Average wage in exporting industries}}{\text{Average wage in manifacturing}}$$

Year	Average of 10 least productive countries	Average of 10 most productive countries	Average of all countries	Correlation with output
(1)	(2)	(3)	(4)	(5)
1980	0.796	1.008	0.940	0.643
1985	0.838	1.076	0.953	0.446
1990	0.895	1.022	0.942	0.257
1995	0.928	1.019	0.954	0.294

Table 1: The wage in exporting industries divided by the average wage: a comparison between rich and poor countries

correlated with output per employee.

B.2.1 Construction of Table 1

Construction of an "export sector" average wage requires firm-level data which is difficult or impossible to obtain for many developing countries. We used instead aggregate data from the World Bank Trade and Production database which reports manufacturing industry data disaggregated at the 3-digit ISIC rev. 2 level (a total of 28 industries) from several countries. For each country and industry i we have the following variables: $Wag_i, Lab_i, Out_i, Exp_i$, i.e. total Wages paid, Employment, Total Output and Total Exports in each industry. The computation of the average export sector wage requires a variable we don't have : Lbx_i (labor employed to produce export goods in industry i). If we had Lbx_i the average export sector wage could be computed as follows (recall that Wag_i report total wages paid, not averages)

$$\frac{\Sigma_i \frac{Wag_i}{Lab_i} Lbx_i}{\Sigma_i Lbx_i} \tag{B1a}$$

To approximate Lbx_i we used the value of exports Exp_i, that is we have computed the average wage in each industry, weighted by the value of the industry's export.

$$\overline{w}^i_{\exp} = \frac{\Sigma_i \frac{Wag_i}{Lab_i} Exp_i}{\Sigma_i Exp_i} \tag{B2}$$

if Lbx_i/Exp_i is constant across i then there is no distortion in the approximation. The average wage in manufacturing is similarly computed by weighting the average wage in each industry by the value of the industry's total output:

$$\overline{w}^i = \frac{\Sigma_i \frac{Wag_i}{Lab_i} Out_i}{\Sigma_i Out_i} \tag{B3}$$

which is how one would have computed the average wage had we not had employment data but only average wage in each industry. We computed the ratio $\overline{w}^i_{\exp}/\overline{w}^i$ and reported in the table averages of such ratio in columns (2), (3), and (4), and its correlation with output per employee in column (5). To construct columns (2) and (3) countries were ranked by the size of manufacturing output per employee. When countries are ranked using other indicators (Manufacturing Value Added per employee, GDP per capita) similar results are obtained. Industries with missing values of wages or output were ignored, whereas industries with a missing value of exports (but reporting output and wages) were treated as having zero exports. Finally, we ignored countries that have more than 5 industries with value of export greater than the value of output. These are rare cases of "entrepot" countries that mostly export goods that have not been manufactured locally such as Hong Kong, Macao and Singapore. The exclusion of such countries does not significantly affect the reported results. After such exclusion the number of countries for which we could compute an average export sector wage ranges from 35 (1980) to 52 (1990).

C Derivation of Autarky Equilibria in Section 4.2

Here we provide the details of the equilibrium characterization under autarky in the parametrized version of the model discussed in Section 4.2.

C.1 Equilibria of Type A

This type of equilibrium has all b workers are allocated to the simple-sector and g workers are allocated to the high tech sector. The associated outputs are thus

$$\begin{aligned} x_1 &= c = \pi\eta \\ x_2 &= s = \pi(1-\eta) + (1-\pi)\,\eta, \end{aligned} \tag{C4}$$

and for the goods market clearing conditions to hold with individual demands given in Page 12 in the main paper it follows that

$$\frac{x_1}{x_2} = \frac{\alpha}{(1-\alpha)}\frac{1}{p_1}, \tag{C5}$$

so by combining (C4) and (C5) we get that the equilibrium price in this type of equilibrium must satisfy the expression for the price in the main paper:

$$p_1 = \frac{\alpha}{1-\alpha} \cdot \frac{\pi(1-\eta) + (1-\pi)\,\eta}{\pi\eta}. \tag{C6}$$

and for firms to earn zero profit the wages must be:

$$\begin{aligned} w_b &= 1 \\ w_g &= p_1 \underbrace{\frac{\pi\eta}{\pi\eta + (1-\pi)(1-\eta)}}_{P(g,\pi)}. \end{aligned} \tag{C7}$$

Hence all prices and quantities are determined under the assumption that the hypothesized allocation of workers constitutes an equilibrium. What remains to be checked is that there are no incentives to reallocate any workers given these prices and wages. For there to be no incentives to reallocate workers with signal g to sector 2 it must be that

$$\begin{aligned} p_1 \frac{\pi\eta}{\pi\eta+(1-\pi)(1-\eta)} &\underset{/(\mathrm{C6})/}{=} \frac{\alpha}{1-\alpha}\frac{\pi(1-\eta)+(1-\pi)\,\eta}{\pi\eta}\frac{\pi\eta}{\pi\eta+(1-\pi)(1-\eta)} \\ = \; & \frac{\alpha}{1-\alpha}\frac{\pi(1-\eta)+(1-\pi)\,\eta}{\pi\eta+(1-\pi)(1-\eta)} \geq 1 \Leftrightarrow \alpha\left(\pi(1-\eta)+(1-\pi)\,\eta\right) \geq (1-\alpha)\left(\pi\eta+(1-\pi)(1-\eta)\right) \\ \Leftrightarrow \; & \alpha\left(\pi(1-\eta)+(1-\pi)\,\eta+\pi\eta+(1-\pi)(1-\eta)\right) = \alpha \geq \left(\pi\eta+(1-\pi)(1-\eta)\right) \\ \Leftrightarrow \; & \pi\,(2\eta-1) \leq \alpha+\eta-1 \Leftrightarrow \pi \leq \frac{\alpha+\eta-1}{2\eta-1} \text{ (which can only hold if } \alpha \geq 1-\eta) \end{aligned} \tag{C8}$$

Finally, for there to be no incentives to reallocate workers with signal b to sector 1 it must be that

$$\begin{aligned} w_b &= 1 \geq p_1 \underbrace{\frac{\pi\,(1-\eta)}{\pi\,(1-\eta)+(1-\pi)\eta}}_{P(b,\pi)} = \\ &= \frac{\alpha}{1-\alpha}\frac{\pi(1-\eta)+(1-\pi)\,\eta}{\pi\eta}\frac{\pi\,(1-\eta)}{\pi\,(1-\eta)+(1-\pi)\eta} = \\ &= \frac{\alpha}{1-\alpha}\frac{1-\eta}{\eta} \Leftrightarrow \alpha \leq \eta, \end{aligned} \tag{C9}$$

which completes the derivation summarized on page 4.2 in the main text.

C.2 Equilibria of Type B

In this type of equilibrium, a fraction $\gamma \in (0,1)$ of g workers are in sector 1 and all b workers and the remaining g workers are allocated sector 2. In this case firms in sector 2 must be indifferent between which type of worker to hire, so $w_g = w_b = 1$. Firms in sector 1 must make zero profits, that is

$$w_g = 1 = p_1 \frac{\pi\eta}{\eta\pi+(1-\eta)\,(1-\pi)} \Leftrightarrow p_1 = \frac{\eta\pi+(1-\eta)\,(1-\pi)}{\eta\pi}, \tag{C10}$$

so the equilibrium price of good 1 is determined from this indifference condition. The associated outputs are

$$\begin{aligned} x_1 &= \gamma\eta\pi \\ x_2 &= (1-\gamma)\left(\eta\pi+(1-\eta)\,(1-\pi)\right)+(1-\eta)\pi+\eta\,(1-\pi)\,, \end{aligned} \tag{C11}$$

and by substituting (C11) into (C5) we find that goods market clearing requires that there is some $\gamma \in (0,1)$ such that

$$\begin{aligned} p_1 &= \frac{\eta\pi+(1-\eta)\,(1-\pi)}{\eta\pi} \\ &= \frac{\alpha}{1-\alpha}\frac{(1-\gamma)\left(\eta\pi+(1-\eta)\,(1-\pi)\right)+(1-\eta)\pi+\eta\,(1-\pi)}{\gamma\eta\pi} \Leftrightarrow \\ (1-\alpha)\,\gamma\left(\eta\pi+(1-\eta)\,(1-\pi)\right) &= \alpha\left((1-\gamma)\left(\eta\pi+(1-\eta)\,(1-\pi)\right)+(1-\eta)\pi+\eta\,(1-\pi)\right) \Leftrightarrow \\ \gamma\left(\eta\pi+(1-\eta)\,(1-\pi)\right) &= \alpha \Leftrightarrow \gamma = \frac{\alpha}{\left(\eta\pi+(1-\eta)\,(1-\pi)\right)} \end{aligned} \tag{C12}$$

Since workers with signal g are equally valuable in sector 1 and sector 2 the condition that there are no incentives to reallocate workers with signal b to sector 1 is automatically satisfied. Hence it only remains to check that γ derived in (C12) is a number on $(0,1)$. Obviously $\gamma > 0$, so we need to check that

$$\begin{aligned} \gamma &= \frac{\alpha}{(\eta\pi + (1-\eta)(1-\pi))} < 1 \Leftrightarrow \alpha < (\eta\pi + (1-\eta)(1-\pi)) \\ &\Leftrightarrow \alpha + \eta - 1 < \pi(2\eta - 1) \Leftrightarrow \pi > \frac{\alpha + \eta - 1}{2\eta - 1}. \end{aligned} \tag{C13}$$

For $\alpha < \eta - 1$ this is satisfied for all $\pi \in [0,1]$, for $\alpha > \eta$ this is never satisfied, while for $1 - \eta \leq \alpha \leq \eta$ this is a condition that says that for π above a threshold (given by $\frac{\alpha+\eta-1}{2\eta-1}$) the equilibrium is of this type.

C.3 Equilibria of Type C

Now, a fraction β of b workers and all g workers are allocated to the high tech sector, and the remaining b workers are allocated to the simple sector. This implies that the equilibrium wages must be

$$\begin{aligned} w_g &= p_1 \frac{\pi\eta}{\pi\eta + (1-\pi)(1-\eta)} \\ w_b &= p_1 \frac{\pi(1-\eta)}{\pi(1-\eta) + (1-\pi)\eta} = 1, \end{aligned} \tag{C14}$$

and the second equation nails down the candidate equilibrium price as

$$p_1 = \frac{\pi(1-\eta) + (1-\pi)\eta}{\pi(1-\eta)}. \tag{C15}$$

The corresponding outputs are

$$\begin{aligned} x_1 &= \pi\eta + \pi(1-\eta)\beta \\ x_2 &= (\pi(1-\eta) + (1-\pi)\eta)(1-\beta) \end{aligned} \tag{C16}$$

It follows directly from (C14) that $w_g > 1$, so there are no incentives to reallocate workers. The one condition that remains to be checked is that there exists $\beta \in (0,1)$ such that the goods market clears.. Substituting from (C16) into (C5) and using (C15) we get that this requires that

$$\begin{aligned} p_1 &= \frac{\pi(1-\eta) + (1-\pi)\eta}{\pi(1-\eta)} = \frac{\alpha}{1-\alpha} \frac{(\pi(1-\eta) + (1-\pi)\eta)(1-\beta)}{\pi\eta + \pi(1-\eta)\beta} \Leftrightarrow \\ \frac{1}{(1-\eta)} &= \frac{\alpha(1-\beta)}{(1-\alpha)(\eta + (1-\eta)\beta)} \Leftrightarrow (1-\alpha)(\eta + (1-\eta)\beta) = \alpha(1-\beta)(1-\eta) \Leftrightarrow \\ \beta(1-\eta) &= \alpha(1-\eta) - (1-\alpha)\eta \Leftrightarrow \beta = \frac{\alpha - \eta}{1-\eta}. \end{aligned} \tag{C17}$$

Hence, this type of equilibrium exists if and only if $\alpha > \eta$.

D Derivation of Benefits from Investment Under Autarky for the Model in Section 4.3

Here we provide the derivations of the expression (20) in the main document.

In terms of p_1, w_g and w_b the gross incentives to invest are (expression (18) in main text)

$$E\left(v\left(w,p\right)|\text{inv}\right) - E\left(v\left(w,p\right)|\text{ don't inv}\right) = \frac{(2\eta-1)(w_g-w_b)}{\left(p_1\right)^{\alpha}}\alpha^{\alpha}(1-\alpha)^{1-\alpha} \tag{D1}$$

and since we can solve for p_1, w_g and w_b in terms of π and the parameters of the model we can derive a closed form expression for the gross benefits of investment as a function of π.

D.1 When $\alpha \leq \eta$

If $\alpha \leq \eta$ we showed that equilibria must be of type A or type B. When $\pi > \frac{\alpha+\eta-1}{2\eta-1}$ we then showed that the equilibrium must be of type B, implying that $w_g = w_b = 1$, so the benefits to invest are zero in this case. Hence

$$B\left(\pi\right) = 0 \text{ for all } \pi > \frac{\alpha+\eta-1}{2\eta-1} \tag{D2}$$

Hence, it remains to derive $B\left(\pi\right)$ for $\pi \leq \frac{\alpha+\eta-1}{2\eta-1}$ in which case the equilibrium is of type A and w_g and w_b are given by the expressions in (C7) and p_1 by (C6). From (C7) we have that

$$\begin{aligned} w_g - w_b &= p_1\frac{\pi\eta}{\pi\eta+(1-\pi)(1-\eta)} - 1 = \frac{\alpha}{1-\alpha}\frac{\pi(1-\eta)+(1-\pi)\eta}{\pi\eta}\frac{\pi\eta}{\pi\eta+(1-\pi)(1-\eta)} - 1 \qquad \text{(D3)} \\ &= \frac{\alpha}{1-\alpha}\frac{\pi(1-\eta)+(1-\pi)\eta}{\pi\eta+(1-\pi)(1-\eta)} - 1 \\ &= \frac{1}{1-\alpha}\frac{\alpha\left(\pi(1-\eta)+(1-\pi)\eta\right)-(1-\alpha)\left(\pi\eta+(1-\pi)(1-\eta)\right)}{\pi\eta+(1-\pi)(1-\eta)} \\ &= \frac{1}{1-\alpha}\frac{\alpha\left(\pi(1-\eta)+(1-\pi)\eta+\pi\eta+(1-\pi)\left(1-\eta\right)\right)-\left(\pi\eta+(1-\pi)(1-\eta)\right)}{\pi\eta+(1-\pi)(1-\eta)} \\ &= \frac{1}{1-\alpha}\frac{\alpha-\left(\pi\eta+(1-\pi)(1-\eta)\right)}{\pi\eta+(1-\pi)(1-\eta)} \end{aligned}$$

and from (C6) we have that

$$\frac{1}{p_1^{\alpha}} = \left(\frac{1-\alpha}{\alpha}\right)^{\alpha}\left(\frac{\pi\eta}{\pi(1-\eta)+(1-\pi)\eta}\right)^{\alpha}, \tag{D4}$$

so substituting into (D1) we get

$$\begin{aligned} B\left(\pi\right) &= (2\eta-1)\left(\frac{1-\alpha}{\alpha}\right)^{\alpha}\left(\frac{\pi\eta}{\pi(1-\eta)+(1-\pi)\eta}\right)^{\alpha}\left(\frac{1}{1-\alpha}\frac{\alpha-\left(\pi\eta+(1-\pi)(1-\eta)\right)}{\pi\eta+(1-\pi)(1-\eta)}\right)\alpha^{\alpha}(1-\alpha)^{1-\alpha} \qquad \text{(D5)} \\ &= (2\eta-1)\left(\frac{\pi\eta}{\pi(1-\eta)+(1-\pi)\eta}\right)^{\alpha}\left(\frac{\alpha-\left(\pi\eta+(1-\pi)(1-\eta)\right)}{\pi\eta+(1-\pi)(1-\eta)}\right), \end{aligned}$$

so combing (D5) with (D2) (and observing that the right hand side of (D5) is negative if $\pi > \frac{\alpha+\eta-1}{2\eta-1}$) we get (20) in the main text.

D.2 When $\alpha > \eta$

In this case the equilibrium is of type C and from (C14) we get that

$$\begin{aligned} w_g - w_b &= \frac{\pi(1-\eta)+(1-\pi)\eta}{\pi(1-\eta)} \frac{\pi\eta}{\pi\eta+(1-\pi)(1-\eta)} - 1 \\ &= \frac{\pi(1-\eta)+(1-\pi)\,\eta}{1-\eta} \frac{\eta}{\pi\eta+(1-\pi)(1-\eta)} - 1 \\ &= \frac{\pi+\eta-2\pi\eta}{1-(\pi+\eta-2\pi\eta)} \frac{\eta}{1-\eta} - 1, \end{aligned} \tag{D6}$$

and (C15) implies that

$$\frac{1}{p_1^\alpha} = \left(\frac{\pi\,(1-\eta)}{\pi(1-\eta)+(1-\pi)\,\eta} \right)^\alpha . \tag{D7}$$

Substituting (D6) and (D7) into (D1) we obtain

$$B\,(\pi) = (2\eta-1) \left(\frac{\pi\,(1-\eta)}{\pi(1-\eta)+(1-\pi)\,\eta} \right)^\alpha \left(\frac{\pi+\eta-2\pi\eta}{1-(\pi+\eta-2\pi\eta)} \frac{\eta}{1-\eta} - 1 \right) \alpha^\alpha (1-\alpha)^{1-\alpha}$$

E Derivation of Trade Equilibria in Section 4.5

In this section we derive the equilibrium characterization and the ranges for different forms of equilibria for the model with international trade.

E.1 Conditions that must hold in any equilibrium.

For the same reason as under autarky, market clearing and optimality on the behalf of consumers imply that (recall that $\alpha = 1/2$)

$$p_1 = \frac{x_2}{x_1} \tag{E1}$$

In addition, there will always be some workers producing the low tech good, so we know immediately that $w_b^h = 1$ in any equilibrium since we have labeled the countries so that the workers that have the lowest probability of being productive in the high tech sector are the workers with bad signals in country h.

E.2 Equilibria of type $\mathbf{A}^t$

In both countries g workers are employed in sector 1 and b workers in sector 2 ("according to signals" in both countries). For this to be an equilibrium we have that

$$\begin{aligned} w_b^f &= w_b^h = 1 \\ w_g^f &= p_1 \frac{\eta\pi^f}{\eta\pi^f+(1-\eta)\pi^f} = \Big/ \eta = \frac{2}{3} \Big/ = p_1 \frac{2\pi^f}{1+\pi^f} \\ w_g^h &= p_1 \frac{\eta\pi^h}{\eta\pi^h+(1-\eta)\pi^h} = \Big/ \eta = \frac{2}{3} \Big/ = p_1 \frac{2\pi^h}{1+\pi^h} \end{aligned} \tag{E2}$$

and outputs are given by

$$x_1 = \frac{2}{3}\left(\pi^f + \pi^h\right) \tag{E3}$$

$$\begin{aligned} x_2 &= \frac{1}{3}\pi^f + \frac{2}{3}\left(1-\pi^f\right) + \frac{1}{3}\pi^h + \frac{2}{3}\left(1-\pi^h\right) \\ &= \frac{2-\pi^f+2-\pi^h}{3} \end{aligned} \tag{E4}$$

so to satisfy (E1) p_1 must solve

$$p_1 = \frac{4-\pi^f-\pi^h}{2\left(\pi^f+\pi^h\right)} \tag{E5}$$

Since $\pi^h \leq \pi^f$ by labeling of the countries the two relevant conditions to check are that there are no incentives to use workers with signal b in the high tech sector in country f and that there are no incentives to use workers with signal g in the low tech sector in country h. The first of these restrictions implies that

$$p_1 \frac{\pi^f}{2-\pi^f} \leq 1 \tag{E6}$$

and the second implies that

$$\begin{aligned} p_1 \frac{2\pi^h}{1+\pi^h} &= \frac{\left(4-\pi^f-\pi^h\right)2\pi^h}{2\left(\pi^f+\pi^h\right)\left(1+\pi^h\right)} \geq 1 \Leftrightarrow \\ \left(4-\pi^f-\pi^h\right)\pi^h &\geq \left(\pi^f+\pi^h\right)\left(1+\pi^h\right) \Leftrightarrow \\ \pi^h\left(4-\pi^h-\left(1+\pi^h\right)\right) &= \pi^h\left(3-2\pi^h\right) \geq \pi^f\left(1+2\pi^h\right) \Leftrightarrow \\ \pi^f &\leq \frac{\pi^h\left(3-2\pi^h\right)}{\left(1+2\pi^h\right)}. \end{aligned} \tag{E7}$$

Finally, to see that (E6) is redundant we note that $\pi^f \geq \pi^h$ implies that

$$\begin{aligned} p_1 &= \frac{4-\pi^f-\pi^h}{2\left(\pi^f+\pi^h\right)} \leq \frac{4-2\pi^f}{4\pi^f} = \frac{2-\pi^f}{2\pi^f} \Rightarrow \\ p_1\frac{\pi^f}{2-\pi^f} &\leq \frac{2-\pi^f}{2\pi^f}\frac{\pi^f}{2-\pi^f} = \frac{1}{2} < 1, \end{aligned} \tag{E8}$$

so (E6) is implies by the other equilibrium conditions.

E.3 Equilibria of type B^t

Now, a fraction γ of g workers in country h and all g workers in country f are employed in sector 1 and a fraction $1-\gamma$ of g workers and all b workers in country h and all b workers in country f are employed in sector 2 ("mixing the good" in h and "according to signals" in f). Such an equilibrium must satisfy

$$\begin{aligned} w_b^f &= w_b^h = 1 \\ w_g^f &= p_1 \frac{\eta\pi^f}{\eta\pi^f + (1-\eta)\pi^f} = \Big/ \eta = \frac{2}{3} \Big/ = p_1 \frac{2\pi^f}{1+\pi^f} \\ 1 = w_g^h &= p_1 \frac{\eta\pi^h}{\eta\pi^h + (1-\eta)\pi^h} = \Big/ \eta = \frac{2}{3} \Big/ = p_1 \frac{2\pi^h}{1+\pi^h} \end{aligned} \tag{E9}$$

and the associated outputs are

$$x_1 = \frac{2}{3}\pi^f + \gamma\frac{2}{3}\pi^h = \frac{2\left(\pi^f + \gamma\pi^h\right)}{3} \tag{E10}$$

$$\begin{aligned} x_2 &= \underbrace{\frac{1}{3}\pi^f + \frac{2}{3}\left(1-\pi^f\right)}_{\text{bad signals in } f} + \underbrace{\frac{1}{3}\pi^h + \frac{2}{3}\left(1-\pi^h\right)}_{\text{bad signals in } h} + (1-\gamma)\underbrace{\left(\frac{2}{3}\pi^h + \frac{1}{3}\left(1-\pi^h\right)\right)}_{\text{good signals in } h} \\ &= \frac{2-\pi^f+2-\pi^h+(1-\gamma)\left(1+\pi^h\right)}{3}. \end{aligned}$$

Hence, to also satisfy market clearing (E1) $\gamma \in [0,1]$ must satisfy

$$\begin{aligned} p_1 &= \frac{1+\pi^h}{2\pi^h} = \frac{x_2}{x_1} = \frac{2-\pi^f+2-\pi^h+(1-\gamma)\left(1+\pi^h\right)}{2(\pi^f+\gamma\pi^h)} \Leftrightarrow \quad \text{(E11)} \\ (1+\pi^h)(\pi^f+\gamma\pi^h) &= \pi^h\left(2-\pi^f+2-\pi^h+(1-\gamma)\left(1+\pi^h\right)\right) \Leftrightarrow \\ \pi^f\left(1+\pi^h\right)+\gamma\left(\pi^h\left(1+\pi^h\right)\right) &= \pi^h\left(5-\pi^f\right)-\gamma\pi^h(1+\pi^h) \\ \gamma\left(2(1+\pi^h)\pi^h\right) &= \pi^h(5-\pi^f)-\left(1+\pi^h\right)\pi^f = 5\pi^h-\left(1+2\pi^h\right)\pi^f \Leftrightarrow \\ \gamma &= \frac{5\pi^h-\pi^f\left(1+2\pi^h\right)}{2(1+\pi^h)\pi^h} \end{aligned}$$

In order for $\gamma \leq 1$ it must be that

$$\begin{aligned} \frac{5\pi^h-\pi^f\left(1+2\pi^h\right)}{2(1+\pi^h)\pi^h} &\leq 1 \Leftrightarrow 5\pi^h-\pi^f(1+2\pi^h) \leq 2(1+\pi^h)\pi^h \Leftrightarrow \quad \text{(E12)} \\ \pi^f(1+2\pi^h) &\geq 5\pi^h-2(1+\pi^h)\pi^h = \pi^h(3-2\pi^h) \\ \pi^f &\geq \frac{\pi^h(3-2\pi^h)}{(1+2\pi^h)} \end{aligned}$$

In order for $\gamma \geq 0$ it must be that $\pi^f \leq \frac{5\pi^h}{1+2\pi^h}$, but this condition turns out to be redundant. The final condition for equilibrium is that there are no incentives to use workers with bad signals in f in the high tech sector, which implies that

$$p_1 \underbrace{\frac{\pi^f}{2-\pi^f}}_{P(b,\pi^f)} = \frac{1+\pi^h}{2\pi^h}\frac{\pi^f}{2-\pi^f} \leq 1 \Leftrightarrow \frac{2-\pi^f}{\pi^f}\frac{2\pi^h}{(1+\pi^h)} \geq 1 \tag{E13}$$

but this is just reversing the inequality in (E19), so this holds whenever $\pi^f \leq \frac{4\pi^h}{1+3\pi^h}$ and since

$$\frac{5\pi^h}{1+2\pi^h} - \frac{4\pi^h}{1+3\pi^h} > 0 \tag{E14}$$

we conclude that the condition for $\gamma \geq 0$ is automatically satisfied whenever there are no incentives to use bad signals in the high tech sector in f, so the relevant range for this equilibrium is when $\frac{\pi^h(3-2\pi^h)}{(1+2\pi^h)} \leq \pi^f \leq \frac{4\pi^h}{1+3\pi^h}$.

E.4 Equilibria of type C^t

In this case, all workers in country h are employed in sector 2, all g workers and a fraction β of b workers in country f are employed in sector 1.

Let w_g^j and w_b^j denote the wages for high and low signal workers in country $j = h, f$. Given that all workers are in sector 1 in country h and that all workers are in sector 2 in country f it must be that

$$\begin{aligned} w_g^h &= w_b^h = 1 \\ w_g^f &= p_1 \frac{\eta \pi^f}{\eta \pi^f + (1-\eta)\pi^f} = \Big/ \eta = \frac{2}{3} \Big/ = p_1 \frac{2\pi^f}{1+\pi^f} \\ 1 = w_b^f &= p_1 \frac{(1-\eta)\pi^f}{(1-\eta)\pi^f + \eta(1-\pi^f)} = \Big/ \eta = \frac{2}{3} \Big/ = p_1 \frac{\pi^f}{2-\pi^f} \end{aligned} \tag{E15}$$

These are simply zero profit conditions conditional on the hypothetical allocation of workers. Since all workers in h and a fraction $(1-\beta)$ of the $\frac{1}{3}\pi^f + \frac{2}{3}(1-\pi^f)$ workers in f with bad signals are in the low tech sector, so the output in that sector is

$$x_2 = 1 + (1-\beta)\left(\frac{1}{3}\pi^f + \frac{2}{3}(1-\pi^f)\right) = 1 + \frac{(1-\beta)(2-\pi^f)}{3}. \tag{E16}$$

The output in the high tech sector is

$$x_1 = \frac{2}{3}\pi^f + \beta\frac{1}{3}\pi^f = \frac{(2+\beta)\pi^f}{3} \tag{E17}$$

The price is now already determined by the last condition in (E15) and given this price, the consumers must rationally be willing to consume the quantities in (E16) and (E17), which by use of (E1) implies that $\beta \in [0,1]$ must satisfy

$$\begin{aligned} p_1 &= \frac{2-\pi^f}{\pi^f} = \frac{1 + \frac{(1-\beta)(2-\pi^f)}{3}}{\frac{(2+\beta)\pi^f}{3}} = \frac{3 + (1-\beta)(2-\pi^f)}{(2+\beta)\pi^f} \Leftrightarrow \\ (2-\pi^f)(2+\beta) &= \left(3 + (1-\beta)(2-\pi^f)\right) \Leftrightarrow \\ (2-\pi^f)((2+\beta)-(1-\beta)) &= 3 \Leftrightarrow (2-\pi^f)(1+2\beta) = 3 \Leftrightarrow \\ 2\beta(2-\pi^f) &= 3 - (2-\pi^f) = 1 + \pi^f \Leftrightarrow \\ \beta &= \frac{1+\pi^f}{2(2-\pi^f)} \end{aligned} \tag{E18}$$

Since $\frac{1+\pi^f}{2(2-\pi^f)}$ is always positive, strictly increasing and equal to 1 when $\pi^f = 1$ we conclude that there is always some $\beta \in [0,1]$ such that the conditions in (E15) and (E1) are satisfied. The only condition that remains to be checked is that there are no incentives to employ workers with good signals in h to the complex task, that is that

$$\begin{aligned} p_1 \underbrace{\frac{2\pi^h}{2\pi^h + (1-\pi^h)}}_{P(g,\pi^h)} &= \frac{2-\pi^f}{\pi^f}\frac{2\pi^h}{(1+\pi^h)} \leq w_g^f = 1 \Leftrightarrow \\ (2-\pi^f)\,2\pi^h &\leq \pi^f(1+\pi^h) \Leftrightarrow \frac{4\pi^h}{1+3\pi^h} \leq \pi^f, \end{aligned} \tag{E19}$$

which is satisfied in the region marked A^t in Figure 6).

F Stability Analysis in section 4.5

We show here that $\partial G(B^f(\pi))/\partial\pi^f > 1$ is a sufficient condition for instability. Consider the Jacobian of the difference equation system evaluated at the autarky equilibrium $\pi = (\pi^A, \pi^A)$:

$$\begin{bmatrix} G'(B(\pi)) \left.\frac{\partial(B^f(\pi))}{\partial\pi^f}\right|_{\pi=(\pi^A,\pi^A)} & G'(B(\pi)) \left.\frac{\partial(B^f(\pi))}{\partial\pi^h}\right|_{\pi=(\pi^A,\pi^A)} \\ G'(B(\pi)) \left.\frac{\partial(B^h(\pi))}{\partial\pi^f}\right|_{\pi=(\pi^A,\pi^A)} & G'(B(\pi)) \left.\frac{\partial(B^h(\pi))}{\partial\pi^h}\right|_{\pi=(\pi^A,\pi^A)} \end{bmatrix}. \tag{F20}$$

At the autarky equilibrium both the "cross derivatives" and the "own derivatives" are identical, so (dropping the common factor $G'(B(\pi))$ the characteristic polynomial can be written as:

$$\begin{aligned} & \left(\left.\frac{\partial(B^f(\pi))}{\partial\pi^f}\right|_{\pi=(\pi^A,\pi^A)} - \lambda \right)^2 - \left(\left.\frac{\partial(B^f(\pi))}{\partial\pi^h}\right|_{\pi=(\pi^A,\pi^A)} \right)^2 \\ = \quad & \lambda^2 + 2\lambda \left.\frac{\partial(B^f(\pi))}{\partial\pi^f}\right|_{\pi=(\pi^A,\pi^A)} + \left(\left.\frac{\partial(B^f(\pi))}{\partial\pi^f}\right|_{\pi=(\pi^A,\pi^A)} \right)^2 - \left(\left.\frac{\partial(B^f(\pi))}{\partial\pi^h}\right|_{\pi=(\pi^A,\pi^A)} \right)^2 \end{aligned} \tag{F21}$$

with roots:

$$\lambda_{1,2} = \left.\frac{\partial(B^f(\pi))}{\partial\pi^f}\right|_{\pi=(\pi^A,\pi^A)} \pm \left.\frac{\partial(B^f(\pi))}{\partial\pi^h}\right|_{\pi=(\pi^A,\pi^A)} \tag{F22}$$

The system is unstable if at least one of the roots is greater than one, therefore a sufficient condition is that the own derivative is greater than one.

References

[1] Hendricks, Lutz, "How Important is Human Capital for Development? Evidence from Immigrant Earnings", *American Economic Review* 2002, 92(1), 2002, 198-219

[2] Prescott, Edward C. "Needed: A Theory of Total Factor Productivity," *International Economic Review,* 39, August 1998, 525-552.