The Performance of the Pivotal Voter Model in Small Scale Elections: Evidence from Texas Liquor Referenda

Andrea Moro (with Stephen Coate and Michael Conlin)

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Question

- How well does the pivotal voter model of electoral turnout perform in small scale elections ?

- Ledyard (1984): rational voters motivated by the chance they might swing the election in a strategic environment + incomplete information

- We estimate the parameters of the model using data from Texas liquor liberalization referenda

Motivation

- Palfrey and Rosenthal (1985): as the number of eligible voters goes to infinity, only those with negative or zero cost vote

- This results is often used to dismiss the model as a reasonable explanation of voter turnout in large elections (Green and Shapiro 1984, Feddersen 2004)

- However this does not mean that it is in not a good model in small scale elections

Other theories of voter turnout

- The Group-based models : groups coordinate their turnout
  - "Ethical" models (Feddersen and Sandroni 2002): everybody follows the rule maximizing the groups' aggregate payoff
  - "Mobilization" models (Shachar and Nalebuff): leaders organize followers

- Expressive voting theories
  - The intensity model: voters are more likely to vote if they feel more strongly about the issue

Empirical regularities regarding turnout

- Strong relationship to sociodemographic variables (Ashenfelter and Kelly 1975, Rosenstone 1980)
  - Used to argue that cost considerations matter
- Likelihood of being decisive: conflicting evidence
  No: Ashenfelter and Kelly (1975);
  Yes: Silberman and Durden (1975), Rosenthal and Sen (1973)
- More direct test: Hansen, Palfrey and Rosenthal (1987) assume a symmetric pivotal model and use only "close" elections
- Technical difficulties + Hard to find the right data

The data we have

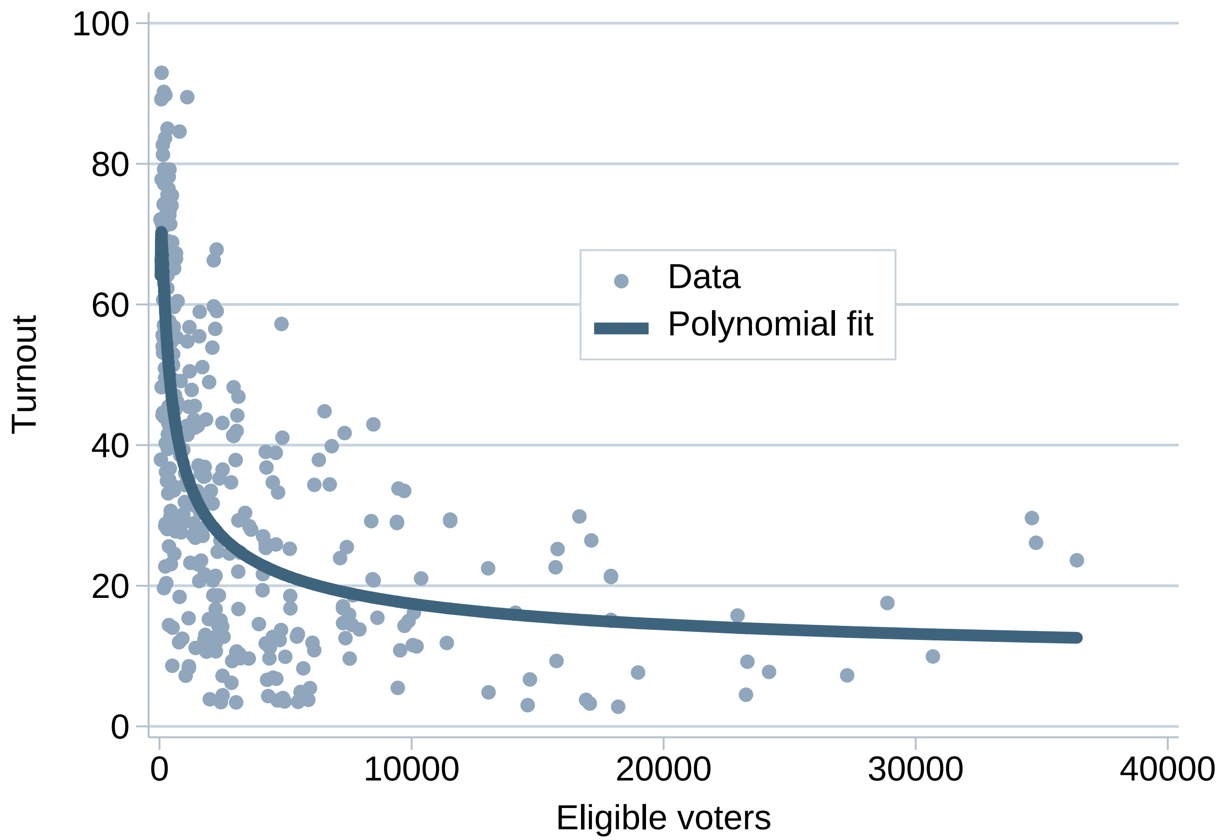
- Coate and Conlin (2004) assembled data on 366 local liquor referenda in Texas between 1976 and 1996.
  Prior to the referendum the local jurisdiction prohibited the sale of alcohol

- Until 2001 liquor referenda were held on special dates, different than standard election days

- Additional information about jurisdictions from the Census (more details later)

Data - turnout

Voter turnout as a percent of eligible voters: $(Yes + No) / Eligible$



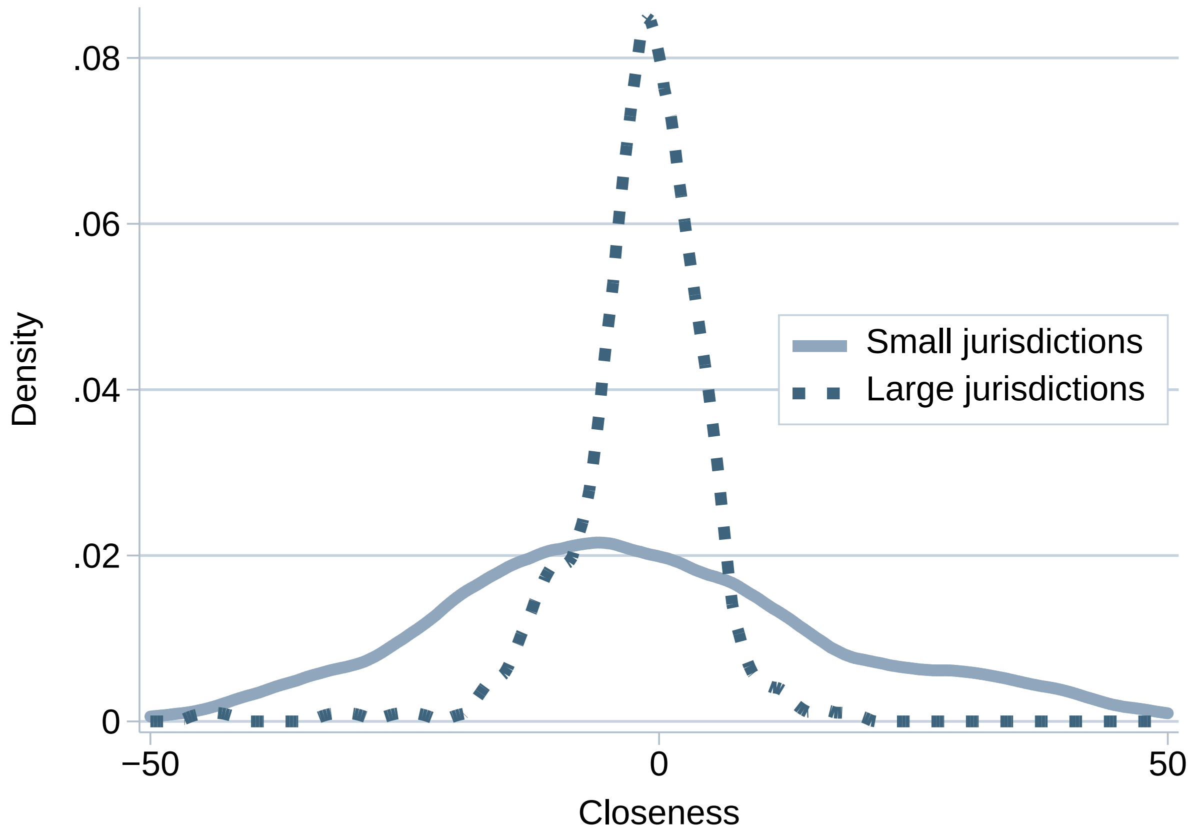
Paper: Figure 1, page 11

Data - turnout

	Eligible voters n	N. of obs.	Perc. turnout
"Small" jurisdictions	$n < 247$	48	0.62
	$247 < n < 434$	48	0.55
	$434 < n < 900$	48	0.43
"Large" jurisdictions	$900 < n < 2245$	72	0.32
	$2245 < n < 5170$	72	0.23
	$5189 < n < 30000$	72	0.18
	$n > 30000$	6	0.26

Data - closeness

The distribution of the percent vote difference: $(Yes - No)/Eligible$



The pivotal voter model

- Citizens, indexed by $i \in \{1, ..., n\}$ vote to relax liquor restrictions
- μ : Probability citizen i is a supporter
- b : Supporters' willingness to pay for the relaxation
- x : Opposers' willingness to pay to avoid the relaxation
- $c_i \sim G = U[0, c]$: Cost of voting for citizen i
- Each citizen knows her cost, but only knows the distribution of costs of the other citizens

The pivotal voter model (cont.)

- Strategy: $f : [0, c] \times \{\text{supporter,opposer}\} \to \{\text{vote,abstain}\}$

- Focus on symmetric equilibria, where all supporters and opposers use the same strategy

- W.l.o.g assume they use a "cutoff" strategy:

$$\begin{aligned} \text{supporter } i \text{ votes if } c_i &\leq \gamma_s^* \\ \text{opposer } i \text{ votes if } c_i &\leq \gamma_o^* \end{aligned}$$

The probability of an election outcome

- $P(s)$: probability that s of the other $n-1$ voters are supporters

$$P(s) = \binom{n-1}{s} \mu^s (1-\mu)^{n-1-s}$$

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- $\rho(Y, N; \gamma_s^*, \gamma_o^*)$: probability Y vote yes, N vote no

$$= \sum_{s=Y}^{n-1-N} P(s) \quad \binom{s}{Y} \left(G(\gamma_s^*)\right)^Y \left(G(\gamma_s^*)\right)^{s-Y}$$

$$\binom{n-1-s}{N} \left(G\left(\gamma_o^*\right)\right)^N \left(1 - G\left(\gamma_o^*\right)\right)^{n-1-s-N}$$

(Go to: likelihood function)

The pivotal voter model (equilibrium)

Equilibrium conditions (n even, supporters win when outcome is tied)

$$\overbrace{\sum_{v=1}^{n/2} \rho(v-1, v; \gamma_s^*, \gamma_o^*) \cdot b}^{\text{Expected benefit to a supporter}} = \gamma_s^*$$

$$\overbrace{\sum_{v=0}^{n/2-1} \rho(v, v; \gamma_s^*, \gamma_o^*) \cdot x}^{\text{Expected benefit to an opposer}} = \gamma_o^*$$

The data

- 366 local liquor elections in Texas between 1976 and 1996 where prior to the election the voting jurisdictions prohibited the retail sale of all alcohol.

Jurisdiction	N	Voters	Supporters win	Close elections*
Small	144	< 900	65	28
Large	222	> 900	87	64

* $<$ 10% margin of victory

- Additional information from the U.S. Census and Churches & Church Membership in the U.S.

The data: additional info

	Small jurisdictions	Large jurisdictions
Number of referenda	144	222
Jurisdiction characteristics		
Voting age population	370 (200)	6,539 (8,742)
Fraction of baptists	52% (11)	46% (14)
Located in an MSA	44% (50)	43% (50)
Incorporated city or town	95% (22)	42% (50)
Referendum characteristics		
Beer/wine	46% (50)	37% (48)
Off-premise	40% (49)	39% (49)
Off- and on-premise	15% (35)	24% (43)
More liberal than county	42% (49)	28% (45)
Held on weekend	68% (47)	72% (45)

Paper: Table 1, page 10

Identification

4 parameters: b, x, μ, c

- Only relative prices matter $c = 1$
- The magnitude of b, x affect turnout
- $b - x$ and μ are separately identified because their effect varies with the size of the jurisdiction
  - e.g. when turnout is high, the vote share is close to μ, the fraction of supporters, and $b - x$ has not much effect
  - when turnout is low, then both μ and $b - x$ affect the vote share.

Estimation

- For each jurisdiction j, we assume:

$$\begin{array}{rrcl} \text{supporter's benefit} & b_j & = & \exp\left(\boldsymbol{\beta}^b \cdot \mathbf{z}_j^b\right) \\ \text{opposer's benefit} & x_j & = & \exp\left(\boldsymbol{\beta}^x \cdot \mathbf{z}_j^x\right) \\ \text{fraction of supporters} & \mu_j & = & \dfrac{\exp\left(\boldsymbol{\beta}^\mu \cdot \mathbf{z}_j^\mu\right)}{1+\exp\left(\boldsymbol{\beta}^\mu \cdot \mathbf{z}_j^\mu\right)} \\ \text{cost distribution upper bound} & c_j & = & \exp\left(\beta^c \cdot z_j^c\right) \end{array}$$

- Variables used:
  $\mathbf{z}^b, \mathbf{z}^x = 1$, off-premise, off/on-premise, city, more liberal than cty.
  $\mathbf{z}^\mu = 1$, fraction of baptists, MSA
  $z^c =$ election on weekend (c normalized)

The likelihood

- observables $\mathbf{z}_j$ determine b_j, x_j, μ_j, c_j for each jurisdiction j
- The equilibrium conditions determine a set of M_j equilibria
- Use an (arbitrary) equilibrium selection rule
  denote the selected equilibrium $(\gamma^*_{sj}, \gamma^*_{oj})$.
- Likelihood of observing an outcome
  conditional on equilibrium thresholds $(\gamma^{m*}_{sj}, \gamma^{m*}_{oj})$

$$L(\Omega) = \prod_j \rho(Y_j, N_j; \gamma^*_{sj}, \gamma^*_{oj})$$

Results: parameters

Parameter/Variable ($\ln L: -5694.21$)		Estimate	Marg. Eff.
μ:	Fraction of baptists	-0.058 (0.188)	-0.015
	Located in an MSA	-0.089 (0.072)	-0.022
	Constant	0.062 (0.097)	
b:	Off-premise	0.182 (0.086)	2.85
	Off- and on-premise	-0.642 (0.232)	-7.89
	Incorporated city or town	1.819 (0.354)	13.68
	More liberal than county	0.199 (0.068)	3.15
	Constant	0.875 (0.405)	
x:	Off-premise consumption	0.097 (0.082)	1.56
	Off- and on-premise	-0.589 (0.253)	-7.58
	Incorporated city or town	1.791 (0.340)	13.97
	More liberal than county	0.361 (0.062)	5.90
	Constant	0.886 (0.370)	
c:	Held on weekend	-0.172 (0.085)	-0.16

Paper: Table 2, page 17

Results: mean estimates

Parameter	Mean estimate
Fraction of supporters μ	0.500 (0.011)
Supporters' benefit b	15.52 (4.81)
Opposers' benefit x	15.90 (5.12)
Upper bound on cost c	0.892 (0.074)
Supporters that vote $\frac{\gamma_s}{c}$	0.516 (0.167)
Opposers that vote $\frac{\gamma_o}{c}$	0.530 (0.174)

An average voting cost ($c/2$) of \$10 implies $b = \$348$ and $x = \$357$

Multiplicity of equilibria not salient.

Paper: Table 3, page 18

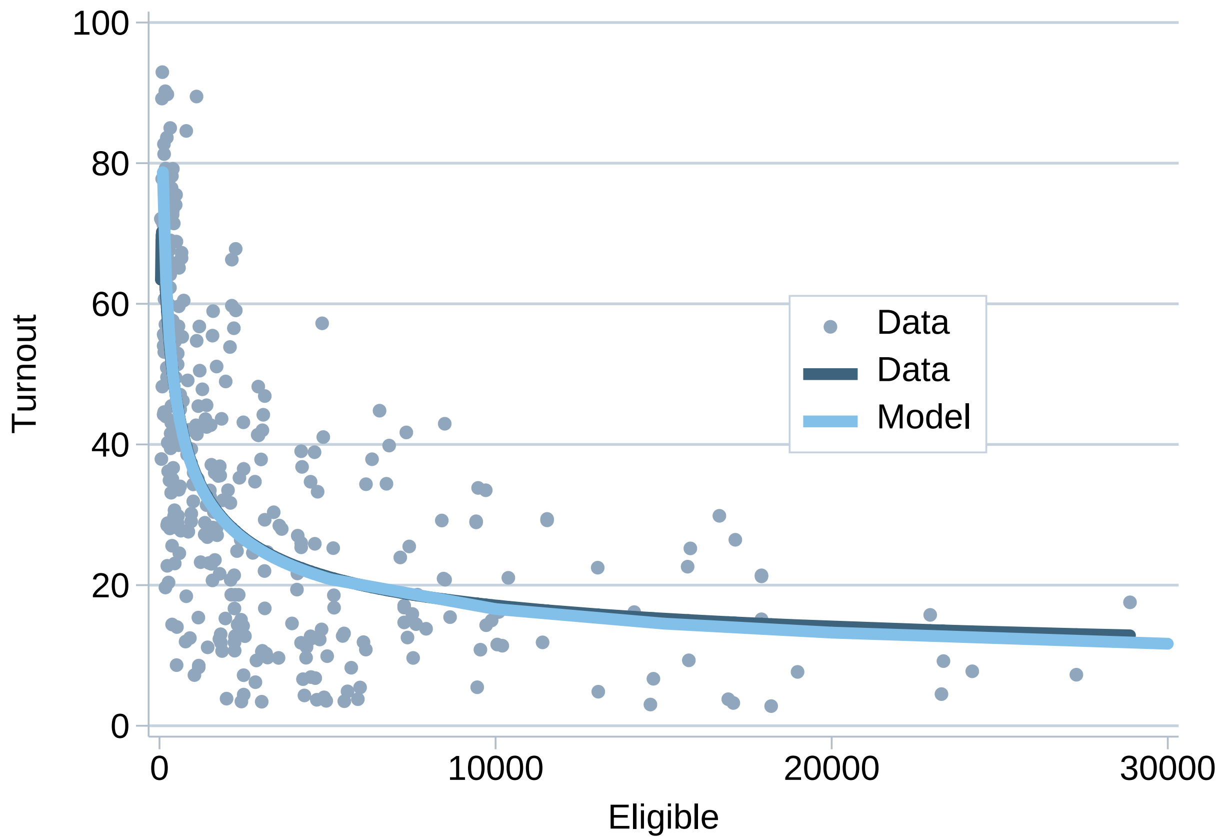
Goodness of fit, turnout

Eligible voters n	N. of obs.	Data	Pivotal-voter model
$n < 247$	48	0.62	0.65
$247 < n < 434$	48	0.55	0.51
$434 < n < 900$	48	0.43	0.40
All $n < 900$	144	0.54	0.52
$900 < n < 2245$	72	0.32	0.19
$2245 < n < 5170$	72	0.23	0.11
$5189 < n < 30000$	72	0.18	0.08
$n > 30000$	6	0.26	0.06

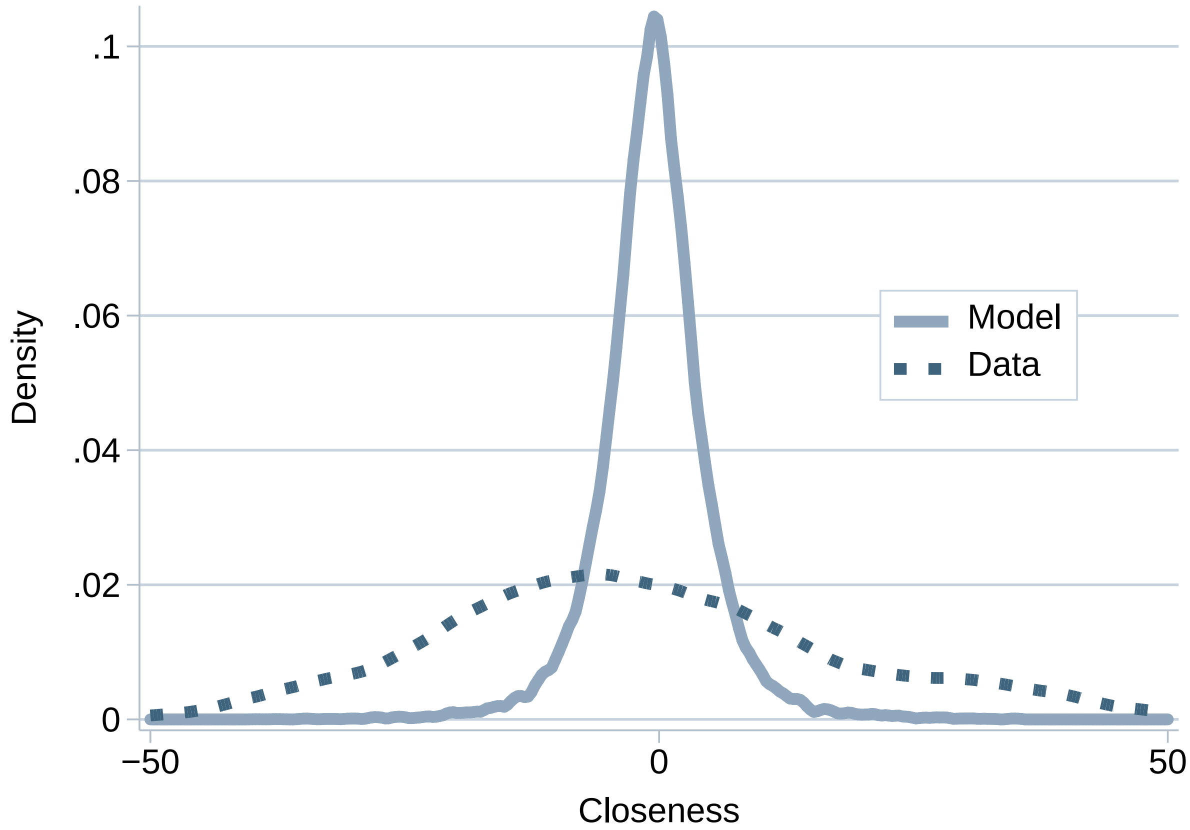
Paper: first part from Table 4, page 19

Turnout, mean estimates

The model is, in principle, capable of generating “high” turnout

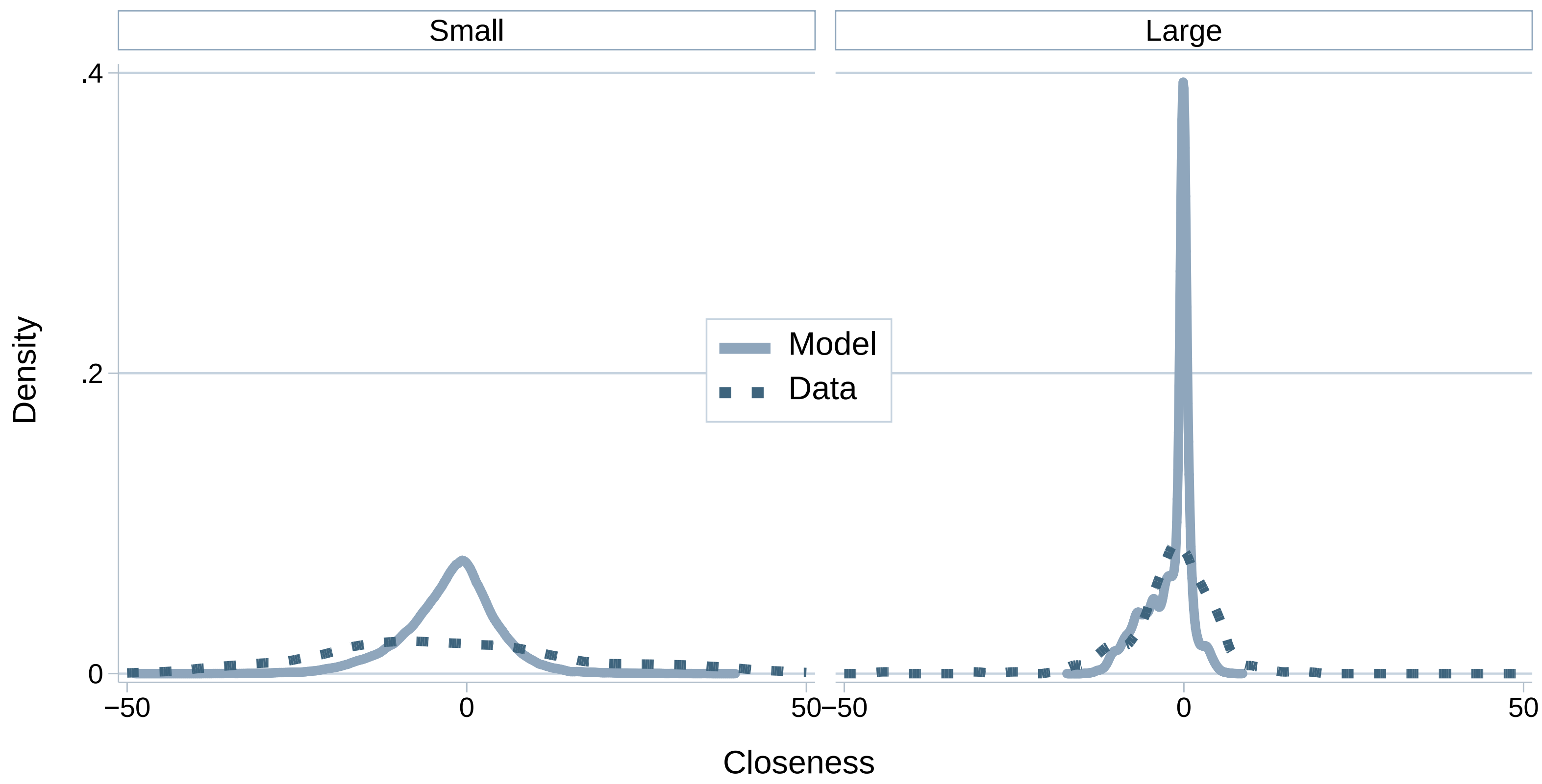


Goodness of fit, closeness



Paper: Figure 3, page 20

Goodness of fit, closeness by size



The intensity model

- α = strength of voters' desire for policy.
- As before: cost $c_i \sim U[0, c]$,
  fraction of supporters μ,
  benefit to supp. b,
  benefit to opp. x.
- Voter i votes if

$$
\begin{aligned}
c_i &\leq \alpha b \\
c_i &\leq \alpha x
\end{aligned}
$$

$\alpha = \text{(eligible voters)}^{\beta}$

Intensity model, parameter estimates

Parameter / Variable (ln L: -4298.0)		Estimate	Marginal Effect
μ:	Fraction of baptists	-0.647 (0.128)	-0.157
	Located in an MSA	-0.362 (0.025)	-0.088
	Constant	0.180 (0.095)	
b:	Off-premise	0.163 (0.019)	0.406
	Off- and on-premise	-0.305 (0.032)	-0.670
	Incorporated city or town	0.504 (0.039)	0.992
	More liberal than county	0.057 (0.018)	0.141
	Constant	0.359 (0.069)	
x:	Off-premise	0.554 (0.015)	0.118
	Off- and on-premise	-0.631 (0.032)	-1.065
	Incorporated city or town	0.032 (0.031)	0.067
	More liberal than county	0.247 (0.016)	0.532
	Constant	0.659 (0.092)	
c:	Held on weekend	0.007 (0.012)	0.007
α:	Eligible voters	-0.252 (0.010)	-7.62

Paper: Table 5, page 22

Intensity model, mean estimates

Parameter	Mean estimate
Fraction of supporters μ	0.423 (0.043)
Supporters' expressive benefit αb	0.585 (0.138)
Opposers' expressive benefit αx	0.504 (0.137)
Upper bound on cost c	1.005 (0.003)
Supporters that vote $\frac{\gamma_s}{c}$	0.583 (0.138)
Opposers that vote $\frac{\gamma_o}{c}$	0.501 (0.137)

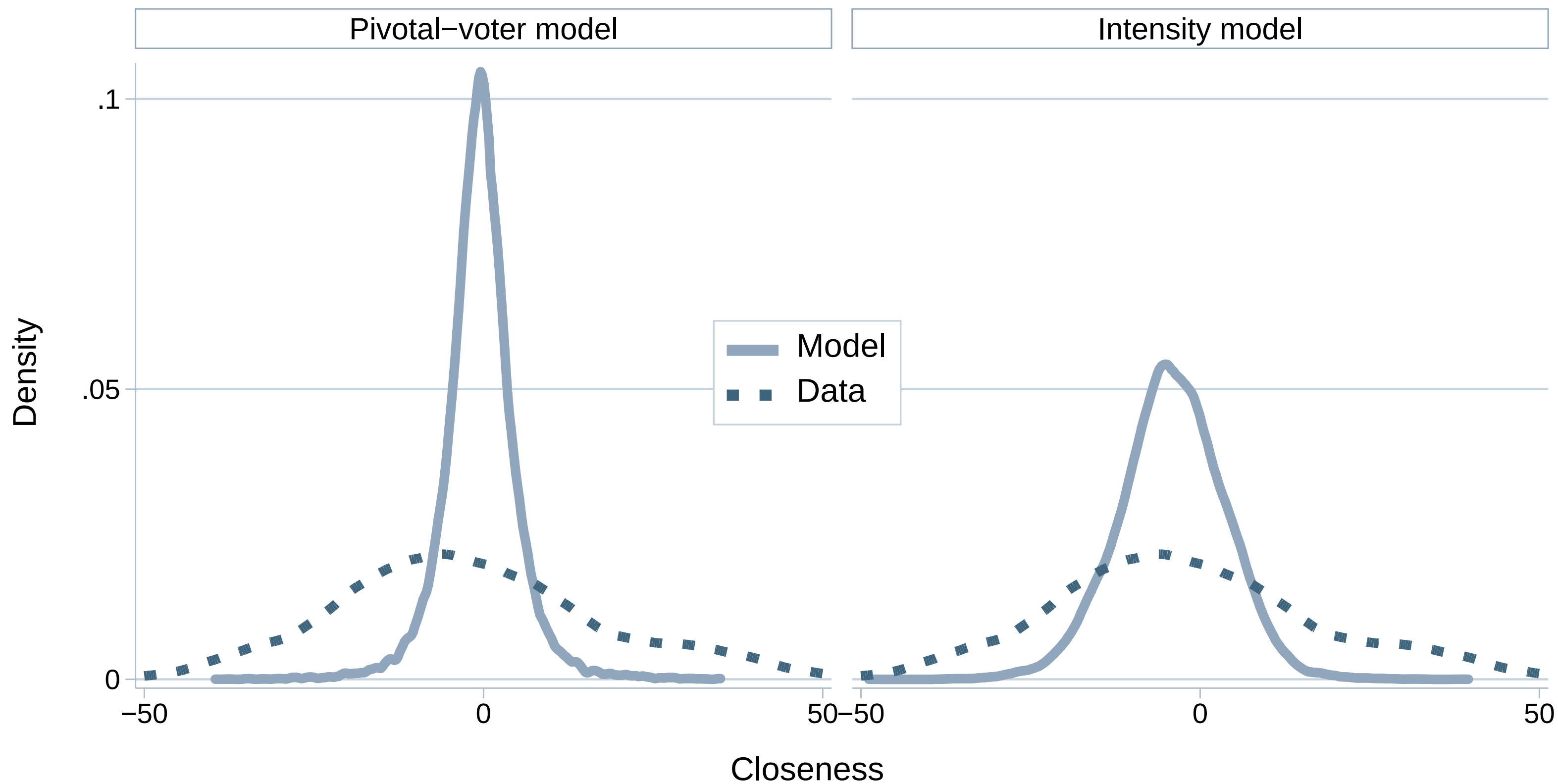
Paper: Table 6, page 23

Intensity model, goodness of fit, turnout

Eligible voters n	N. of Obs.	Data	Intensity model
$n < 247$	48	0.62	0.63
$247 < n < 434$	48	0.55	0.53
$434 < n < 900$	48	0.43	0.45
All ($n < 900$)	144	0.54	0.54
$900 < n < 2245$	72	0.32	0.32
$2245 < n < 5170$	72	0.23	0.24
$5189 < n < 30000$	72	0.18	0.17
$n > 30000$	6	0.26	0.15

Paper: first part from Table 7, page 24

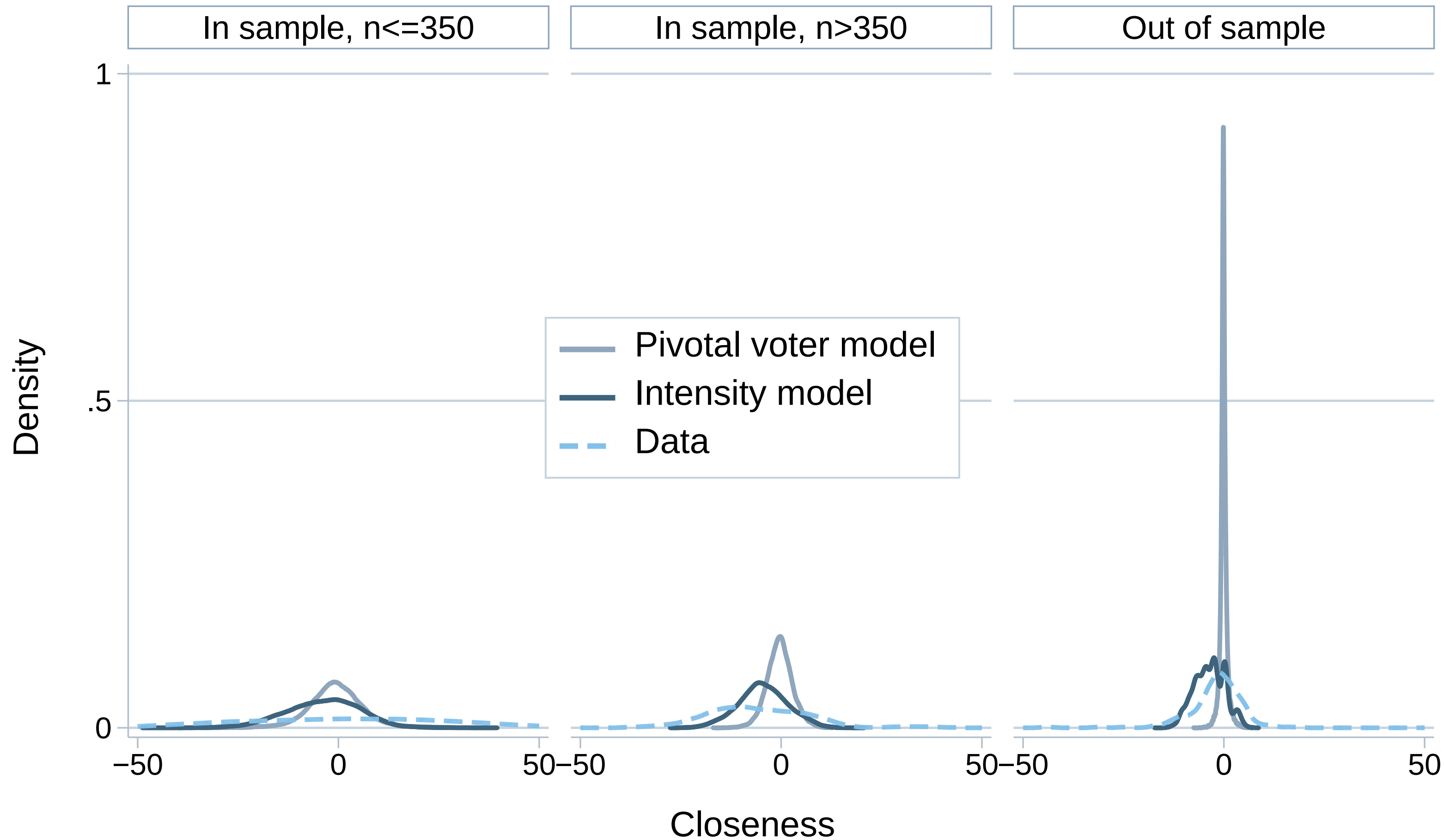
Closeness, comparison between models



A Vuong non-nested models test of the null hypothesis that the two models are equally close to the true dgp

Paper: see Figure 4, page 25

Closeness, comparison, by size



Conclusion

- The pivotal voter model seems to be able to perform well in predicting turnout

- It does not perform well in predicting closeness of the election

- A simple model based on expressive voting does better