A Game-theoretical Interpretation of Guaranteed Renewability in the Health Insurance Market

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Abstract

A game-theoretical model is developed here that can explain why premiums of health insurance contracts in the individual health insurance market do not vary that much over time, even in absence of any legal restriction on premiums. In this model the insurer and the purchaser interact for an infinite number of periods, and the threats of punishments (raising premium, not purchasing from the insurer) forces both insurer and the purchaser to stay on a constant premium path. This model, unlike other models of guaranteed renewability, does not presume commitment of the insurer.

1 Introduction:

A recently much discussed topic in the health insurance literature is how consumers can be protected against fluctuation in premiums for individual health insurance with the beginning of high cost chronic conditions. Health insurance typically covers risk for a certain period. At a subsequent period the factors that determine the level of premium an insurance purchaser is charged may change .So a person is subject to the risk that the premium at the beginning of next year may be higher or lower than in previous year. A risk averse insurance purchaser would like to avoid this uncertainty about future premiums.

A good number of papers have recently addressed this issue. Cutler (1993) suggested that intertemporal variability in the cost of health care results in failure of the private market to fully insure long-term care. However, Pauly, Kunreuther and Hirth(1995) (PKH) has come with some affirmative results .They suggest that under the assumption of absence of informational asymmetry between the insurer and insured, there are ways to design health insurance system where premiums do not vary over time in response to change in health status. That is, irrespective of realization of health status, the premium sequence takes a pre-determined path. Their proposed premium scheduled is also immune to the possibility of defection on the side of the purchaser at any period of time. They call such premium policy as Guaranteed Renewable (GR) premium schedule. PKH(1998) extends similar results for group insurance policy.

However, in their model, though they assume that the insurance purchasers are unable to commit, they implicitly assume commitment of the insurance provider. In PKH(1995) the proposed premium schedule is a declining sequence, where in the earlier periods the premium has to be high enough to permit persons who become high risk in those periods to continue to pay the same sequence of premiums in future periods as those who have yet to suffer a loss. Intuitively, in such a premium schedule, if otherwise there is no legal recourse, a firm has an incentive to charge, for example in the last period, a higher premium to a purchaser who turned out to be a high risk. PKH(1995) calls this a lock-in problem.

Indeed there are some legal restrictions on changing an individual's health insurance policy over time, especially due to change in health status risk. The Health Insurance Portability and Accountability Act (HIPAA), which became effective on 1997, requires that insurers have to renew coverage to individuals. But this regulation does not state anything about limiting the rates that an insurer may charge an individual at the time of renewal (Patel And Pauly (2002)). In fact an insurer can slip out from such regulations in many ways, for example, by charging a rate to an individual, who has turned out to be a high risk, so high that it is effectively same as denial of coverage (Patel And Pauly (2002)).

Also guaranteed renewable insurance policies do exist in the individual health insurance market and are relatively stable, from well before this act has been effective (Pauly, Percy and Herring (1999), Feldman (1987)).This suggests that there must be some other mechanisms that are sustaining this guaranteed renewability of health insurance rather than legal restrictions.

The aim of this paper is to investigate whether a market can sustain a guaranteed renewable premium policy when there is lack of commitment both on the insurer's and purchaser's side. Throughout the paper, I assume that there is no informational asymmetry between the insurer and purchaser, that is, both of them have the same knowledge about the probability of health status realization at any point of time and both of them can observe health status realizations perfectly. First, it is shown that in a finite period model, given lack of commitment, no guaranteed renewable premium schedule could be sustained in the market. Then it is shown that in an infinite period model, under some assumption on the health risk redefinition process, a GR premium schedule can be sustained. This is possible by a threat of punishment for both insurer and purchaser in case they do not abide by the GR premium path.

It turns out that the risk redefinition process, that is, how the perceived health risk (both of the insurer and purchaser, since there is no informational asymmetry, they both have the same expectation) of the individual changes in response to health status, is an important determinant of whether a GR premium can be sustained or not. In a finite period model, GR premium cannot be sustained irrespective of whatever the risk redefinition process is. In an infinite period model, a GR premium schedule can be supported as an equilibrium only if the risk redefinition is temporary, that is, an incidence of loss (an occurrence of bad health or disease) of the individual does not lead to a perception of that person being a high risk for ever.

Also, in my model there is only one firm in the individual insurance market that offers a long-term health insurance contract, and all other firms offers only one-period contracts. The number of firms offering one-period contract is quite large, so that the spot-market premium is offered at an actuarially fair rate at equilibrium. Anyway, it is shown that in a finite period model, GR premium schedule cannot be sustained no matter how many firms offer long-term health insurance contracts. In an infinite period model, GR premium schedule may be sustained if the firms offering long-term contracts are not identical (they differ in discount factor).

This model has some interesting policy implications. It has long been debated in the health insurance literature whether public intervention in the health insurance market is desirable or not. A typical form of public intervention in health insurance market in U.S. is Medicare. Medicare is a partial health insurance coverage that is provided to the disabled and elderly people. In particular, all people above age 65 are eligible for Medicare. Medicare program is heavily subsidized, the coverage and premiums are far more generous than in a privately provided health insurance contracts. However, they do not provide coverage for all types of health care costs, in fact they pay for only less than half of the health care cost for the elderly (Finkelstein(2004)). For example, they do not provide coverage for prescription drugs and long-term care.

It is interesting to see how this kind of partial public intervention affects the market equilibrium in my model. It is shown that a partial intervention in the market that provides coverage for a limited portion of loss at a low cost (or no cost at all) after certain age period can only contribute at unraveling the equilibrium with GR premium schedule, and thereby can only lower welfare. Basically, the market has its own way of achieving an optimal outcome, and public intervention can only contribute at worsening the outcome.

The structure of this paper is as following: In section 2, I introduce the basic model and the equilibrium concepts used. I also define precisely what I would refer to as a GR premium schedule in my model. In section 3, A finite period model is considered. It is shown that the GR premium schedule cannot be supported as an equilibrium. In section 4, an infinite period model is developed. It is shown that the GR premium schedule can be supported as an equilibrium when the risk redefinition process is temporary. In section 5, the effect of introducing partial coverage at a low cost after a certain age on the equilibrium of section 4 is discussed. A brief discussion concludes the paper.

2 The Model

There is a risk neutral insurer (the firm) and a strictly risk averse agent (the insurance purchaser).The agent's von-Neuman-Morgensten utility function is $u : R_+ \rightarrow R$, which is strictly increasing and strictly concave.There are $T \geq 1$ periods.Both the insurer and the agent discounts future at a constant rate $\beta \leq 1$.The agent is assumed to have a costant income of y (> 0) each period. This assumption is not necessary for the conclusions drawn in this paper, this is just for simplification.

The risk redefiniiton process that is considered here is similar to PKH(1998).Before the first period, the agent is low risk.At the beginnig of each period the low risk agent turns into a high risk with probability $0 < \varphi < 1$.The expected loss of a high risk agent is assumed to be L, where $0 < L < y$.The expected loss of a low risk agent is assumed to be 0.If a person becomes high risk, he remains high risk for $n \geq 1$ periods, and then becomes a low risk again.That is ,for example,if $n = 1$, he remains high risk for that period and becomes a low risk in the next period, when he would again face a probability of φ of becoming a high risk .If $n = 2$, he remains high risk in that period and also the next period.In the following period , he becomes a low risk again.

Now if $n = 1$, then actually there is no risk redefinition: an incidence of high risk does not change a person's probability of being a high risk next period.The shock process then is simplly i.i.d over time.Whenever $n \geq 2$,then becoming high risk (from low risk)in any period means that the person will be high risk in (at least) the next period with probability one;therefore ; it changes his risk status.Therefore shock processes with high risk period length of $n \geq 2$ are more interesting for our process.[1]

There are many other firms in the market who only offer one-period contracts.So the spot market for health insurance is perfectly competitive and therefore ,as Arrow(1963) has pointed out,would offer an actuarially fair premium rate (and full coverage) .These contracts are offered at the beginning of each period, before the occurence of change of risk status.Therefore the actuarially fair premium rates for high risk and low risk persons are L and φL respectively.

It is also assumed that the only choice variable for the insurer is the premium rate for each period,and it has to provide full coverage(A more sophisticated model would take both premium rate and coverage as decision variable, however this additional degree of freedom of choice is not going to help the insurer that much as long as he has to compete with the spot market).On the other hand , the choice variable for the insured in each period is either to accept or reject the premium rate offered by the insurer for that period.In case of rejection the agent gets insurance from the spot market, at an actuarially fair rate.

Given the assumptions above, one can describe the situation as a game.There are two players: the insurer and the purchaser.I would only consider pure strategies in this model.A strategy for the insurer is a schedule of premium rates for all periods and for all contingencies of risk status over

[1] Actually a shock process with high risk period lenth of n represents an nth order Markov Process.

time.A strategy for a purchaser is a schedule of choice of either acceptence or rejection for each level of premium ofered, for all periods and for all contingencies.The payoffs for the insurer and the agent are the expected discounted profit and expected discounted utility , respectively.

Let 0 denote the low risk state and 1 denote the high risk state. Health state realization of period $t \leq T$ can be denoted by a h_t,where $h_t \in \{0, 1\}$.History upto period t can be denoted by a vector $h^t = (h_1, h_2, ..., h_t)$.Denote the space of History upto period t as H^t.Now denote the premium offered by insurer at period t as P_t,where $P_t \in R$.Also denote the choice of the purchaser,given the premium offered P_t, as c_t, where $c_t \in \{accept, reject\}$.Note that the choice"reject " implies choice of purchasing health insurance contract from spot market for that period.Let's denote the premium chosen by the insurer and choice made by the purchaser upto period t as $P^t = (P_1, P_2, ..., P_t)$ and $c^t = (c_1, c_2, ..., c_t)$ respectively.So a strategy for a insurer is a set of functions $P = \{P_t : H^{t-1} \times R^{t-1} \times \{accept, reject\}^{t-1} \to R\}_{t=1}^{T}$.Similarly, a strategy for the agent is a set of functions $C = \{c_t : H^{t-1} \times R^t \times \{accept, reject\}^{t-1} \to \{accept, reject\}\}_{t=1}^{T}$.Lets denote the payoff of the insurer and purchaser as $ER(P, C)$ and $EU(P, C)$ respectively.

Given that there is no informational asymmetry between the insurer and the agent, subgame perfect equilibrium seems to be the appropriate equilibrium notion to use.In what follows , I am considering pure strategy subgame perfect equilibria of the games.

Also what needs to be clarified here is the definition of a Guranteed Renewable premium sequence.According to Patel and Pauly(2002), " Guaranteed renewablility is a contractual feature in which the insurer agrees both to sell another policy to the insured person(if that person wishes to buy) at the end of the term of the current policy period and to charge a premium for that policy that is not affected by any individual loss experience or change in the insured person's circumstances during the term of the current policy." . SInce in our model pre-commitment in future premium rates is not possible, an equilibrium sequence of premium ,P^T,will be said to have a GR feature if it is independent of history, that is, if in all periods $t \leq T$,the premium P_t is independent of history h^{t-1}.Therefore, for example, a level premium sequence has GR feature.Also, a premium sequence that increases at a fixed rate over time also has GR feature.

In the next section I look at the equiliria of this game when $T < \infty$.

3 Eqilibrium in a Finite period Game:

Let's find out the equilibria of this game when $T < \infty$.First, I consider the case of $T = 2$ and $n = 2$.Since this is a finite period game with perfect information, it can be solved using backword induction.

Consider the decision of the agent at the end of the second period, if he became high risk in the first period.Since he is high risk ,he will be offered a premium of L form the spot market .Therefore he is going to aaccept a premium offer P_2 from the insurer if and only if $P_2 \leq L$.(we assume, throughout the paper that when the agent is indifferent between accepting and rejecting an offer, he always chooses to accept it).Given this, to maximize its profit, the insurer is going to offer $P_2 = L$.

Now consider the decision of the agent at the end of the second period, if he did not become high risk in the first period.In case he becomes high risk in second period,he will be offered a premium of L form the spot market .Therefore he is going to aaccept a premium offer P_2 from the insurer if and only if $P_2 \leq L$.(we assume, throughout the paper that when the agent is indifferent between accepting and rejecting an offer, he always chooses to accept it).Given this, to maximize its profit, the insurer is going to offer $P_2 = L$..Now if the agent remains low risk in the next period , in the spot market he will be offered 0 premim.So he will accept an offer P_2 from the insurer if and only if $P_2 \leq 0$.The only feasible offer for the insurer is then $P_2 = 0$.

Given these unique Nash Equilibria of the subgames of the second period, Let's consider the decision of the agent at the end of first period .First consider the case where he has turned out to be a high risk in the first period.His next period utility, whether or not he accepts an offer from the insurer ,is $\beta u(y - L)$.Now, if he accepts an offer from the insurer ,P_1,his total expected utility is $u(y - P_1) + \beta u(y - L)$.On the other hand, if rejects insurer's offer, his premium from spot market for first period would be L,Therefore he will get a total expected utility of $u(y - L) + \beta u(y - L)$.So he will accept an offer from te insurer only if $P_1 \leq L$.Given this,the insurer is going to offer$P_1 = L$.The insurer's expected profit from these offers equal to zero, because premim offers equal expected payments in each case.

Next, consider the case where he has turned out to be a low risk in the first period.His next period utility, whether or not he accepts an offer from the insurer, is $\beta((1-\varphi)u(y) + \varphi u(y-L))$. Now, if he accepts an offer from the insurer, P_1, his total expected utility is $u(y - P_1) + \beta((1 - \varphi)u(y) +$

$\varphi u(y{-}L))$. On the other hand, if rejects insurer's offer, his premium from spot market for first period would be 0, therefore he will get a total expected utility of $u(y) + \beta((1-\varphi)u(y) + \varphi u(y-L))$. So he will accept an offer from the insurer only if $P_1 \leq 0$.Given this, the insurer is going to offer$P_1 = 0$.Again, insurer's total expected profit is equal to zero, since its premium offers equal expected payments in each case.

From the discussion above, it is seen that this game has a unique outcome:- every period , the insurer offer premium rates same as what the spot market would offer.As a result of that , the agent remains vulnerable to the risk of being charged high premium in case of a change in risk status.Since this outcome is unique,obviously no GR premium schedule is feasible.Equilibrium Premium offers are always history dependent.

Also, the resulting premium schedules and payoffs of insurer and agent will remain the same even if there are more than one insurer.In the second period , to a high risk agent,an insurer cannot offer $P_2 < L$, because in that case his expected profit would be $P_2 - L < 0$.Similarly,to a low risk agent, the insurer cannot offer $P_2 < 0$, because in that case his expected profit would be $P_2 < 0$.So all insurers are going to make the same offer,and the agent can randomly pick any offer from any insurer,which also is not going to change the expected profit of the insurers(it is equal to zero, anyway).Similarly, the insurers are going to make the same offer in first period, too.

Now, the results from this game of $T = 2$ and $n = 2$,that is, the uniqueness of equilibrium and that the equilibrium premium schedule is the same as spot market premium schedule, directly extends to any game of $2 \leq T < \infty$ and $2 \leq n \leq \infty$.At the last period T (the subgame that starts from period T), the agents will only accept premium offers that are equal to or less than their expected loss given their risk status. So the premiums offered at equilibrium are equal to the expected losses for each type, that is $P_T = L$ if the agent is high risk and $P_T = 0$ if the agent is low risk. This is the unique Nash equilibrium for this subgame starting at period T. Since this outcome of period does not depend on whatever actions are chosen a tperiod $T-1$, therefore at period $T-1$ also, the agent and the insurer behave as if they are in the last period. So you have the same (and unique)equilibrium premium offers at period $T-1$ also. Then the same premiums are offered at period $T-2$, and so on. So, in all periods, premiums offered by the insurer at eqilbrium is unique,and equals to the spot market rate under each risk status. Clearly, this equilibrium sequence of premium offers do not have GR feature, as they are history dependent. Note also that this result does not depend on the risk redefiniton process, i.e. how many periods the agents remains high risk once he becomes high risk. The fact that the interaction between the insurer and the agent ends after finite number of periods uniquely determines the outcome of the game.

The results from this section can be summarized in the following proposition:

Proposition 3.1: *In the model described in section 2, when $2 \leq T < \infty$ and $2 \leq n \leq \infty$, then there exists only one subgame perfect equilibrium of this game. At this equilibrium, the premiums offered are the same as the spot market equilibrium. Therefore, no equilibrium with GR premium feature exists.*

4 Infinite Period Model

Now suppose that the insurer and the purchaser interacts over infinite number of periods, that is $T = \infty$. In this case, whether or not equilibrium with GR premium exists depends on value of parameters of the model (and utility function of the agent) as well as the value of n. In section 4a , It will be shown that when $n = \infty$, then, no equilibrium with GR premium exists. In section 4b, we will show that when $n < \infty$, then under some conditions on parameters, some equilibria with GR premium schedule exists. In particular, sufficent and necessary conditions on parameters for existence of a level premium schedule will be shown.

4.1 The case of $n = \infty$

In our model, $n = \infty$ implies that once a person becomes high risk, he remains high risk for all subsequent periods,that is, he faces a permanent risk redefinition. Therefore this is similar to the assumption of risk redefinition in PKH(1995). In their model , under the assumption of full commitment , an equilibrium with a GR premium feature exists (The equilibrium GR premium schedule of their model satisfies our definition of GR premium schedule here). However, in our model, where no commitment is possible, no equilibrium with GR premium sequence exists.More precisely, the following proposition holds:-

Proposition4.1: *In the model described in section 2, when $T = \infty$ and $n = \infty$,then there doesnt exist any subgame perfect equilibrium such that the equilibrium premium sequence has GR feature, i.e.,independent of past history.*

Proof: To see this , suppose to the contrary, that there exists an equilibrium with a GR premium sequence ,$\{P_t\}_{t=1}^{t=\infty}$.Then, in any period t,in the subgames which starts from the point where the agent has become high risk , the premium schedule from that perid on,$\{P_j\}_{j=t}^{j=\infty}$,must be an equilibrium in those subgames as well .In particular , $\{P_t\}_{t=1}^{t=\infty}$must be an equilibrium in the subgame that starts from the point where the agent has turned into a high risk in the first period.

Let's focus on this this subgame now.Since the agent will remain high risk forever, he will be always offered a premium of L at the spot market.Also the expected loss of the insurer is L in each period.

Now obviously, The eqilibrium defined by the combination of strategies,

Insurer : offering $P_t = L$ each period,

Agent: accepting a premium offer from the firm,at every period , if and only if $P_t = L$,

constitutes a subgame perfect equilibrium of this game ,as it is the unique Nash Equilibrium of the stage game.The agent's expected utility from this strategy is $\frac{u(y-L)}{1-\beta}$, which is equal to the payoff that the agent would get if he always accepts a premium offer from the spot market.So, this this is the worst equilibrium payoff for the agent.Similarly, the firm's payoff from this startegy is zero, therfore this is also the worst equilibrium payoff for the insurer.

Therefore,a premium sequence$\{P_t\}_{t=1}^{t=\infty}$will be a subgame perfect equilibrium of this subgsame if and only if the following is satisfied:-for any $j \geq t$

$$\sum_{\tau=j}^{\tau=\infty} \beta^{\tau-1}(P_\tau - L) \geq 0 \qquad \text{(1a)}$$

$$\sum_{\tau=j}^{\tau=\infty} \beta^{\tau-1}u(y - P_\tau) \geq \frac{u(y-L)}{1-\beta}, \qquad \text{(1b)}$$

Now I argue that the Equilibrium defined above is the only subgame perfect equilibrium of this game .The key is the strict concavity of the utilty function of the agent .

For any premium sequence $\{P_\tau\}_{\tau=j}^{\tau=\infty}$, define $\overline{P} = (1-\beta)\sum_{\tau=j}^{\tau=\infty} \beta^{\tau-1}P_\tau$.Given the strict concavity of utility function of the agent , it must be that

$\frac{u(y-\overline{P})}{1-\beta}, \geq \sum_{\tau=j}^{\tau=\infty} \beta^{\tau-1}u(y - P_\tau).$, with equality only if $\{P_\tau\}_{\tau=j}^{\tau=\infty}$ is a level premium sequence (1c)

From (1b)and (1c), it is clear that $\frac{u(y-\overline{P})}{1-\beta} > \frac{u(y-L)}{1-\beta}$, or equivalently $\overline{P} < L$ if $\{P_\tau\}_{\tau=j}^{\tau=\infty}$is not a level premium sequence.But in that case $\sum_{\tau=j}^{\tau=\infty} \beta^{\tau-1}(P_\tau - L) = (\ \overline{P} - L)/(1-\beta) < 0$, which contradicts (1a).Since this is true for any $j \geq t$,therefore a premium sequence that is not constant over time cannot be an equilibrium of this subgame.

Also , given (1a) and (1b) it is clear that the only level premium sequence that satisfies both (1a) and (1b) is that $P_t = L$ each period, which is the same as the equilibrium defined above.Therefore, uniqueness of the equilibrium defined above is proved.

Now let's go back to our original game.Given our discussion above ,for this game, the combination of strategies,

Insurer : offering $P_t = L$ each period,

Agent: accepting a premium offer from the firm,at every period , if and only if $P_t = L$,

is the only candidate for an equilibrium with GR feature.Now I argue that that this combination of strategies is not an equilibrium of this game.

To show this, first I establish that the combination of strategies:

Insurer:In each period ,offer $P_t = L$ if the agent is high risk , offer $P_t = 0$ if agent is low risk,

Agent :If low risk ,accept a premium offer from firm if and only if $P_t \leq 0$

If high risk ,accept a premium offer from firm if and only if $P_t \leq L$

constitutes a subgame perfct equilibrium of this game.

Consider the point where the agent is become high risk .Given agent's strategy ,one of the best responses of the insurer, is $P_t = L$.Because if it offers $P_t < L$,its profit would be negative, and if it offers$P_t > L$, the offer is not accepted, in which case its profit is zero.On the other hand, given insurer's strategy, the agent would accept any offer less than or equal to L.

Now consider the point where the agent is low risk at the begining of a period.Given that the agent follows the strategy defined above, one of the best responses of the insurer, is $P_t = 0$.Because if it offers $P_t < 0$,its profit would be negative, and if it offers$P_t > 0$, the offer is not accepted, in which case its profit is zero.On the other hand,given insurer's strategy, the agent cannot gain by accepting a premium which is higher than the spot market premium,after any realization of health status.

Also note that the ex-ante payoff from this equilibrium for the insurer is zero, which it gains by not participating in the market anyway.On the other hand , the agents payoff from this strategy is ,$u(y)(1 + \frac{(1-\varphi)\beta}{1-(1-\varphi)\beta}) + u(y-L)(\frac{\varphi\beta}{1-\beta} \cdot \frac{1}{1-(1-\varphi)\beta})$,

which is the same as if he always buys an insurance from the spot market.Therefore these payoffs constitute the worst subgame perfect equilibrium for both insurer and agent.

Given these, the strategy combination,Insurer : offering $P_t = L$ each period,

Agent: accepting a premium offer from the firm,at every period , if and only if $P_t = L$,

will be a subgame perfect equilibrium of this game only if the payoff of the agent , from a point where the agent is low risk,is at least as large as what it would get from the worst subgame perfect equilibrium, that is,

$$u(y-L)\tfrac{1}{1-\beta} \geq u(y)(1+\tfrac{(1-\varphi)\beta}{1-(1-\varphi)\beta}) + u(y-L)(\tfrac{\varphi\beta}{1-\beta}\cdot\tfrac{1}{1-(1-\varphi)\beta}) \qquad \text{(1d)}$$

But straightforward computation suggests that,

$u(y-L)\tfrac{1}{1-\beta} - u(y)(1+\tfrac{(1-\varphi)\beta}{1-(1-\varphi)\beta}) - u(y-L)(\tfrac{\varphi\beta}{1-\beta}\cdot\tfrac{1}{1-(1-\varphi)\beta})$

$=u(y-L)(\tfrac{1}{1-\beta})(1-\tfrac{\varphi\beta}{1-(1-\varphi)\beta}) - u(y)\tfrac{1}{1-(1-\varphi)\beta}$

$=\tfrac{1}{1-(1-\varphi)\beta}(u(y-L)-u(y)) < 0$

which contradicts (1d).Therefore this strategy combination with GR feature cannot be an equilibrium of this game.

Therefore, to conclude , there doesnt exist any equilibrium with GR premium feature if $n = \infty$.

4.2 The Case of $n < \infty$

In this section I show that when $n < \infty$, then, for sufficiently high values of β(discount factor),some equilibria with GR premium feature exists.In particular , I show that for every value of $n < \infty$,there exists an equilibrium with a level premium sequence when β is sufficiently high.The reason I focus on level premium sequence is that given the fact that the agent is strictly risk averse and the insurer is risk neutral,a level premium sequence is Pareto Optimal.

Before proceeding to this proof of existence, note first that the combination of strategies,

Insurer: In each period ,offer $P_t = L$ if the agent is high risk , offer $P_t = 0$ if agent is low risk,

Agent: If low risk ,accept a premium offer from firm if and only if $P_t \leq 0$. If high risk ,accept a premium offer from firm if and only if $P_t \leq L$,

constitutes a subgame perfect equilibrium for any game with $n < \infty$. The logic is the same as we stated in case of $n = \infty$.Also ,the payoff from this equilibrium for the insurer is zero, and the agent's payoff is the same as what he would get if he always chooses to accept premium offers from spot market.Therefore these payoffs are the worst subgame perfect equiibrium for both insurer and agent.From now on, we call this subgame perfect equilibrium as *spot market equilibrium.*

Subcase 1:When $n = 1$

This is the case where essentially there is no risk redefinition.Therefore existence of an equilibrium with a level premium sequence when β is sufficiently high is obvious, as in Thomas and Worrall(1988) .The following proposition holds:-

Proposition 4.2*: For any premium level between φL and P^*, where P^*satisfies*

$$u(y-P^*) = \varphi u(y-L) + (1-\varphi)u(y) \tag{2a}$$

there exists a $\beta^* < 1$ such that for all $\beta \in (\beta^*, 1)$,a level premium sequence with this premium level can be supported as an (subgame perfect) equilibrium.

Proof: The fact that $\varphi L < P^*$ follows from concavity of $u(\cdot)$.

Consider the following path of this game:-

Insurer:In each period ,offer $P_t = P$,after any health status realization of the agent.

Agent :In each period,accept a premium offer from firm if and only if $P_t \leq P$,at any health status realization.

Now it is easy to see that this is an equilibrium path.Consider any subgame that starts from a point where the agent has become high risk.Remember that spot market equilibrium is the worst subgame perfect equilibrium.Following Van Damme(19), Theorem 8.5.8 (pp190), the path defined above will be an equilibrium if and only if

$\frac{P}{1-\beta} - L - \frac{\beta\varphi L}{1-\beta} \geq 0$

or $\frac{P-\beta\varphi L}{1-\beta} - L \geq 0.$ $\qquad (2b)$

and $\frac{u(y-P)}{1-\beta} \geq u(y) + \beta.\frac{\varphi u(y-L)+(1-\varphi)u(y)}{1-\beta}$

or, $\frac{u(y-P)-\beta u(y-P^*)}{1-\beta} \geq u(y)$ $\qquad$ (2c)

Therefore if $P^* > P > \varphi L$, there exists a $\beta' < 1$ such that for all $\beta \in (\beta', 1)$, both inequalities (2b) and (2c) will be satisfied.

Now consider any subgame that starts from a point where the agent has become a low.risk.the path defined above will be an equilibrium if and only if,

$\frac{P}{1-\beta} - \frac{\beta\varphi L}{1-\beta} \geq 0$ $\qquad$ (2d)

and ,$\frac{u(y-P)}{1-\beta} \geq u(y) + \beta.\frac{\varphi u(y-L)+(1-\varphi)u(y)}{1-\beta}$

or $\frac{u(y-P)}{1-\beta} \geq u(y) + \beta.\frac{u(y-P^*)}{1-\beta}$ $\qquad$ (from(2a))

or $\frac{u(y-P)-\beta u(y-P^*)}{1-\beta} \geq u(y)$ $\qquad (2e)$

Therefore if $P^* > P > \varphi L$, there exists a $\beta'' < 1$ such that for all $\beta \in (\beta'', 1)$, the inequalities above will be satisfied.

Now define $\beta^* = \max(\beta', \beta'')$ .Since condition(6) and (7) are the necessary and sufficient conditions for a level premium sequence with premium level P to be a subgame perfect equilibrium, therefore,for *all* $\beta \in (\beta^*, 1)$,a level premium sequence with premium level P can be supported as an (subgame perfect) equilibrium.

Q.E.D.

Subcase 2:When $n = 2$

This is the first case in temporary risk redefinition, where an agent who becomes high risk in one period remains high risk in the next period with probability 1.In this case the following proposition holds:-

Proposition 4.3:For any premium level between $\frac{2\varphi}{1+\varphi}L$ *and* P^**, where* P^**satisfies*

$$u(y - P^*) = \frac{2\varphi}{1+\varphi}u(y-L) + (1 - \frac{2\varphi}{1+\varphi})u(y) \tag{3a}$$

there exists a $\beta^* < 1$ *such that for all* $\beta \in (\beta^*, 1)$ *,a level premium sequence with this premium level can be supported as an (subgame perfect) equilibrium.*

Proof:-The fact that $\frac{2\varphi}{1+\varphi}L < P^*$follows from concavity of $u(\cdot)$.

Before going on to the proof of this proposition , I need to proof the following Lemma.

Lemma1:-When $n = 2$, the expected loss of the insurer converges to $\frac{2\varphi}{1+\varphi}L$.Expected payoff of the agent from spot market converges to $\frac{2\varphi}{1+\varphi}u(y-L) + (1 - \frac{2\varphi}{1+\varphi})u(y)$.

Proof:-I need to show that the high (and low) risk probabilities converges to some number over time.First I start from low risk probabilities.

Probability of low risk in the first period=$1 - \varphi$

Probability of low risk in the second period=$(1 - \varphi)(1 - \varphi)$

Probability of low risk in the third period=$[1 - (1 - \varphi)\varphi](1 - \varphi) = (1 - \varphi + \varphi^2)(1 - \varphi)$

Probability of low risk in the fourth period=$(1 - \varphi + \varphi^2 - \varphi^3)(1 - \varphi)$

Probability of low risk in the n-th period=$(1 - \varphi + \varphi^2 - \varphi^3 + ... + (-1)^{n-1}\varphi^{n-1})(1 - \varphi)$

It is easy to see that as $n \to \infty$, this number converges to=$\frac{1-\varphi}{1+\varphi}$

Therefore high risk proabilities converges to =$1 - \frac{1-\varphi}{1+\varphi} = \frac{2\varphi}{1+\varphi}$

So expected loss converges to $\frac{2\varphi}{1+\varphi}L$.

And expected payoff of the agent from spot market converges to $\frac{2\varphi}{1+\varphi}u(y-L) + (1 - \frac{2\varphi}{1+\varphi})u(y)$.

Q.E.D.

Now I return to the proof of the proposition.Consider the following path of this game:-

Insurer:In each period ,offer $P_t = P$,after any health status realization of the agent.

Agent :In each period,accept a premium offer from firm if and only if $P_t \leq P$,at any health status realization.

To see that this is an equilibrium path,one needs to consider payoff from it, at three different types of subgame(all the subgames of this game is any one of these three types).

First ,consider a subgame that starts from a point where the agent has turned out to be a low risk.Following Van Damme(19),Theorem 8.5.8(pp190), the path defined above will be an equilibrium if and only if

$$\frac{P}{1-\beta} - \beta\varphi L - \beta^2[\varphi + (1-\varphi)\varphi]L - \beta^3[(1-\varphi)\varphi + (1-(1-\varphi)\varphi)\varphi]L - .... \geq 0 \quad (3b)$$

and $\frac{u(y-P)}{1-\beta} \geq u(y) + \beta.[\varphi u(y-L) + (1-\varphi)u(y)] + \beta^2[(\varphi + \varphi(1-\varphi))u(y-L) + (1-\varphi)^2 u(y)] + \beta^3[((1-\varphi)\varphi + (1-(1-\varphi)\varphi)\varphi)u(y-L) + (1-((1-\varphi)\varphi + (1-(1-\varphi)\varphi)\varphi))u(y)] + ...(3c)$

Note that the lefthandside of the inequality (3b) involves expected loss and we know that expected loss converges to $\frac{2\varphi}{1+\varphi}L$,according to Lemma 1.Therefore ,if $P > \frac{2\varphi}{1+\varphi}L$,then there exists a finite integer T such that at period $T$and at all later periods , the expected loss is smaller than P.Let the expected loss at period $t \geq 1$ be denoted by $EL(t)$.Then (3b) could be rewritten as

$$[P + \beta(P - EL(1)) + \beta^2(P - EL(2)) + ... + \beta^{T-1}(P - EL(T-1))] + [\beta^T(P - EL(T)) + \beta^{T+1}(P - EL(T+1)) + ...] \geq 0 \qquad (3b')$$

Now the first part of the lefthandside of (3b′) may be positive ,or negative, depending on the values of β, φ, and P,but is finite.On the other hand , the second part is always positive,and increasing in β.Therefore there exists a $\beta' < 1$ such that for all $\beta \in (\beta', 1)$, the inequality above will be satisfied.Note that if $P \leq \frac{2\varphi}{1+\varphi}L$,this inequality may not be satisfied.

Now consider inequality (3c).The righthandside of it involves expected payoff of the agent in the spot market.This expected payoff converges to $\frac{2\varphi}{1+\varphi}u(y-L) + (1 - \frac{2\varphi}{1+\varphi})u(y)$,or $u(y - P^*)$.Therefore ,if $P < P^*$,then there exists a finite integer T' such that at period $T$and at all later periods , the expected payoff is smaller than $u(y - P)$.Let the expected loss at period $t \geq 1$ be denoted by $EP(t)$.Then (3c) could be rewritten as

$$[u(y-P) - u(y) + \beta(u(y-P) - EP(1)) + \beta^2(u(y-P) - EP(2)) + ... + \beta^{T'-1}(u(y-P) - EP(T'-1))] + [\beta^{T'}(u(y-P) - EP(T')) + \beta^{T'+1}(u(y-P) - EP(T'+1)) + ...] \geq 0 \quad (3c')$$

Now the first part of the lefthandside of (3c′) may be positive ,or negative, depending on the values of β, φ, and P,but is finite.On the other hand , the second part is always positive,and increasing in β.Therefore there exists a $\beta'' < 1$ such that for all $\beta \in (\beta'', 1)$, the inequality above will be satisfied.Note that if $P \geq P^*$,this inequality may not be satisfied.

Define $\beta_1^* = \max(\beta', \beta'')$.Then for all $\beta \in (\beta_1^*, 1)$, both ineqalties will be satisfied.

Secondly,consider the subgame that starts from a point where the agent has become high risk,in the current period, and was low risk in the last period.Since this period and the next period he will be high risk with certainity, for the path defined will be an equilibrium if and only if

$(P - L) + \beta(P - L) + \beta^2(P - EL(1)) + \beta^3(P - EL(2)) + \text{.............} \geq 0$ (3d)

and,$[(u(y - P) - u(y - L)) + \beta(u(y - P) - u(y - L)) + \beta^2(u(y - P) - EP(1)) + \beta^3(u(y - P) - EP(2)) + \text{.......} \geq 0$ (3e)

By the same logic as in the case of previous subgame,if$P^* > P > \frac{2\varphi}{1+\varphi}L$,there exists β_2^* such that for all $\beta \in (\beta_2^*, 1)$, both ineqalties will be satisfied.

Secondly,consider the subgame that starts from a point where the agent has become high risk,in the current period, and was low risk in the last period.Since this period and the next period he will be high risk with certainity, for the path defined will be an equilibrium if and only if

$(P - L) + \beta(P - L) + \beta^2(P - EL(1)) + \beta^3(P - EL(2)) + \text{.............} \geq 0$ (3d)

and,$[(u(y - P) - u(y - L)) + \beta(u(y - P) - u(y - L)) + \beta^2(u(y - P) - EP(1)) + \beta^3(u(y - P) - EP(2)) + \text{.......} \geq 0$ (3e)

By the same logic as in the case of previous subgame,if there exists β_2^* such that for all $\beta \in (\beta_2^*, 1)$, both ineqalties will be satisfied.

Thirdly,consider the subgame that starts from a point where the agent is a high risk,in the current period, and was high risk in the last period as well.The path defined will be an equilibrium if and only if

$(P - L) + \beta(P - EL(1)) + \beta^2(P - EL(2)) + \text{.............} \geq 0$ (3d)

and,$[(u(y - P) - u(y - L)) + \beta(u(y - P) - EP(1)) + \beta^2(u(y - P) - EP(2)) + \text{.......} \geq 0$ (3e)

By the same logic as in the case of previous subgame,if $P^* > P > \frac{2\varphi}{1+\varphi}L$,there exists β_3^* such that for all $\beta \in (\beta_3^*, 1)$, both ineqalties will be satisfied.

Define $\beta^* = \max(\beta_1^*, \beta_2^*, \beta_3^*)$.Then, if$P^* > P > \frac{2\varphi}{1+\varphi}L$, for *all* $\beta \in (\beta^*, 1)$,a level premium sequence with premium level P can be supported as an (subgame perfect) equilibrium.

Q.E.D.

The key to the proof of proposition 4.3 is that the high risk(and the low risk) probabilities converges to a number strictly less than 1.So the expected loss of insurer and expected payoff of the agent also converges to some limit.Therefore, on a path with constant premium, where the premium is strictly larger than this limit expected loss and strictly smaller than the premium that would give the limit expected payoff to the agent , both agent and insurer has no incentive to deviate from that path if β is sufficiently high.

So the results from proposition 3 holds actually for any $n < \infty$.

The general case: $n < \infty$

Proposition 4.4: *In a game where the number of periods for which the agent remains sick is* $n < \infty$,*For any premium level between* $\frac{n\varphi}{1+(n-1)\varphi}L$ *and* P^*, *where* P^**satisfies*

$$u(y - P^*) = \frac{n\varphi}{1+(n-1)\varphi}u(y - L) + (1 - \frac{n\varphi}{1+(n-1)\varphi})u(y) \quad (3a)$$

there exists a $\beta^* < 1$ *such that for all* $\beta \in (\beta^*, 1)$ *,a level premium sequence with this premium level can be supported as an (subgame perfect) equilibrium.*

Proof is similar to the case of $n = 2$.The value of β that satisfies all the 2(n+1) necessary and sufficient conditions is the desired value of β^*.

5 Effect on Equilibrium of Introducing Partial Public Insurance:The case of Medicare

A typical form of public intervention in health insurance market in U.S. is Medicare.Medicare is a partial health insurance coverage that is provided to the the disabled and elderly people.In particular, all people above age 65 are eligible for Medicare.Medicare program is heavily subsidized,the coverage and premiums are far more generous than in a privately provided health insurance contracts.

However,they do not provide coverage for all types of health care costs, in fact they pay for only less than half of the health care cost for the elderly(Finkelstein(2004)).For example, they do not provide coverage for prescription drugs and long-term care.Actually 85% of the people who are currently under Medicare coverage also have some form of insurance from the private

Now the question is how can I analyze the effct of introducing Medicare at a certain age.Since Medicare is heavily subsidized ,he way I capture the effect of medicare is that it provides ,after a certain nuimber of period T(The value of T is the difference of 65 and the age a person begins to be covered by private health insurance), coverage for some portion of the loss in case illness for free.Let us assume that after period T ,the portion of the loss that remains to be insured from the market is L'.Then $0 \leq L' < L$.

I start from the extreme case:$L' = 0$

This is the case when Medicare provides full coverage for loss in case of sickness, for free.So buying insurance from private market is not at all necessary.This means that the agent needs to buy insurance only for a finite number of periods,T. But then essentially we have a finite period model ,and as we have seen in sectrion3, in that case no equilibrium with GR premium feature exists,for any value ofβ.Therefore introduction of Medicare actually can only worsen the welfare of the risk averse agent-for sufficiently high value of β, where he could have been fully insured against change of risk status for first T periods, introduction of medicare now makes that impossible.

The case when$0 < L' < L : -$

In this case , the agent still needs to buy insurance from the private insurance market.But introduction of medicare can unravel the equilibrium of the private insurance market.For a given level of β,for which an equilibrium with level premium exists(as we have seen in section4) ,it might be that after the introduction of medicare, that equilibrium will cease to exist.In particular, the following result holds:-

Proposition 5.1: *In a game with $n < \infty$, for any given level of β and L and integer T,There exists a value $L' < L$ such that if the amount of loss that is to be insured from private insurance market till period T is L and after period T is L',Then there doesnt exist any equilibrium of the game with GR premium feature.*

Proof:-We will only show the proof for the case of $n = 2$.The proof for the cases of higher values of n would be similar.

Suppose on the contrary that an equilibrium with GR premium feature existed , for all $0 \leq L' < L$,and let that premium sequence be $\{P_t\}_{t=1}^{t=\infty}$.Consider the subgame that starts from the Tth period ,and where the agent has become high risk in the last period.The following inequalities have to be satisfied for the insurer :-

$$[P_T - L] + [\beta(P_{T+1} - [\varphi + (1-\varphi)\varphi]L') + \beta^2(P_{T+2} - [(1-\varphi)\varphi + (1-(1-\varphi)\varphi)\varphi]L') + ...]. \geq 0$$

(5a)

First we argue that first part of the lefthandside of this inequality (5a) must be negative if L' is sufficently small .To see this ,first consider the subgame that starts from the Tth period, where the agent is low risk .The following inequality has to be satisfied for the agent:-

$[u(y - P_T) - u(y)] + [\beta(u(y - P_{T+1}) - \varphi u(y - L') - (1-\varphi)u(y)]) + \beta^2(u(y - P_{T+1}) - (\varphi + \varphi(1 - \varphi))u(y - L') - (1-\varphi)^2 u(y)) + ...] \geq 0$ (5b)

If $P_T \geq L$ then the first part of inequality(5b) is negative, and it is the loss of the agent in the current period by accepting P_T.It must be that the gain from this premium sequence in future ,which is the second part of this inequality ,is large enough to compensate for this loss.

But the extent of this gain is determined by the amount of loss that needs to insured in future, that is L'.In fact,an upper bound of this gain can be given by $B = \frac{\beta}{1-\beta}(u(y) - u(y - L'))$.

Why?the second part of the LHS of the inequality above , which is the gain from future,is equal to expected discounted utility from the premium sequence minus expected discounted utility from the spot market .Now consider the value of gain ,$\overline{B} = \beta(\frac{u(y)}{1-\beta}) -$ expected discounted utility .

The most cost efficient way for the insurer to provide this gain is to set a premium of 0 in each period from T, which is clearly not feasible because the expected payoff of the insurer would then be negative.Therefore $\overline{B}$ is an upper bound of the gain from future.But since $B > \overline{B}, B$ is also an upper bound of the gain.Clearly the value of B is incresing in L',so as L' decreases, the potetial gain from future also decreases.

So for L' sufficiently small enough, it must be that the the loss in the current period is also small enough so that inequality (5b) is satisfied .So it must be that that if L' is sufficiently small, then $P_T < L$,

So,we see that if $L'$is sufficiently high ,then the first part of the LHS of inequality (5a) is negative .In order for inequality (5a) to hold , it must be that the the second part of the LHS of (5a) , which is the expected discounted payoff from this premium sequence in future for the insurer, must be positive and at least as large as $L - P_T$.But the extent of this gain is also determined by the value of L'.In fact,an upper bound of this gain can be given by

$B = \frac{\beta L'}{1-\beta}$

Why? the second part of the LHS of inequality (5a) , which is the gain from future,is equal to expected discounted payoff from the premium sequence minus expected discounted loss .Now consider the value of gain ,$\overline{B} = \frac{\beta L'}{1-\beta}-$ expected discounted loss.

The most cost efficient way for the insurer to attain this gain is to set a premium of L' in each period from T, which is clearly not feasible because the agent can gain a higher payoff than that from the spot market.Therefore $\overline{B}$ is an upper bound of the gain from future.But since $B > \overline{B}, B$ is also an upper bound of the gain.Clearly the value of B is incresing in L',so as L' decreases, the potetial gain from future also decreases.

So for sufficently small values of L' , the inequality (5a) would be violated.

Q.E.D.

The logic behind the proof of proposition 5.1 is straightforward.In a world of uncertain health status, a level premium can only be sustained if the present period loss of the involved parties is compensated by the gain in future from maintaining this level premium sequence.But since future gain from the policy is proportionate to the loss in case of future ,L or L'.Therefore, if L' is too small compared to L , then in the period prior to beginning of medicare coverage,it may not be possible to compensate a loss in the current period with future gains.

6 Conclusion:

In this paper it is established that the health insurance market can sustain guaranteed renewable health insurance policies even in absence of any legal restriction. Unlike other models of guaranteed renewability, this model does not presume commitment of the insurer. In future, empirical tests should be conductedon which of the competing models of guaranteed renewability fits better with the data.

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