DETECTING COLLUSION IN PROCUREMENT AUCTIONS

PATRICK BAJARI
GARRETT SUMMERS*

I. INTRODUCTION

Collusion is an agreement among a group of firms, called a cartel, designed to limit competition among the participants. If all firms in the cartel follow the agreement, buyers will face higher prices, giving the cartel members profits above the normal competitive level. State and federal statutes have proscribed such agreements to protect consumers from collusive behavior, and considerable resources are expended every year on the investigation and prosecution of colluders.

To prosecute and, by so doing, deter future collusion, we need to know how to detect collusive behavior. Certainly we would like to be able to catch cartels red-handed in "smoke-filled" rooms, but this is extremely difficult to do in practice. Alternatively, we need a set of tools that can help indicate when behavior is collusive, in order to point prosecutors in the right direction. While collusive behavior exists in all types of markets, we focus here on tests developed by Patrick Bajari and Lixin Ye to detect collusive bid rigging by firms competing in sealed-bid auctions.[1] Our goal is to explain these tests to non-economists, using a

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* Patrick Bajari is Assistant Professor, Department of Economics, Stanford University, and Faculty Research Fellow, National Bureau of Economic Research. Garrett Summers is a Ph.D candidate in economics at Harvard University. This article is based on Prof. Bajari's own research on collusion, as well as joint research with Lixin Ye. Prof. Bajari would like to thank seminar participants at Harvard, MIT, Stanford, Northwestern, Penn, Michigan, Princeton, Wisconsin, Berkeley, and Yale for feedback on this research. John Geweke, Andy McLennan, and Bob Marshall assisted Prof. Bajari at an early stage with this research. Paul Milgrom, Steve Tadelis, and Steve Brenner also provided very useful comments on various parts of this work. Pueo Keffer provided outstanding research assistance. The National Science Foundation, grants SES0112106 and SES0122747, and the Hoover Institution's National Fellows Program, provided generous research support. All remaining errors are our own.

[1] For a more in-depth discussion of the issues discussed in this paper, see Patrick Bajari & Lixin Ye, Competition Versus Collusion in Procurement Auctions: Identification and Testing (Stanford University Working Paper), and Deciding Between Competition and

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minimum of technical jargon, so that the key concepts may be understood without advanced training in economics or statistics.

Bajari and Ye begin by describing some models of competitive bidding for procurement contracts that have recently been developed by leading theorists, such as Maskin and Riley.[2] An analytically innovative feature of these models is that, before bidding begins on a project, market participants expect certain firms to have a relative cost advantage (or disadvantage) to complete that project. Maskin and Riley refer to their model as an "asymmetric" model of bidding. A number of recent applied papers, including those by Bajari and Ye; Pesendorfer; Porter and Zona; and Baldwin, Marshall, and Richard, have found that cost asymmetries are important for explaining observed patterns of bidding.[3]

Bajari and Ye apply the theory of competitive bidding with asymmetric bidders to distinguish between competitive and collusive bidding. They identify a set of conditions that are necessary and sufficient for an observed set of bids to be generated from their model of competitive bidding. They refer to these conditions as conditional independence and exchangeability. If the assumptions of their competitive model hold, these conditions are necessary and must be observed in the data. The conditions also are sufficient in the sense that, if these conditions are observed in the data, then it is possible to "reverse-engineer" a model of competitive bidding that would generate the observed bids. In other words, when bidding satisfies these conditions, it is always possible that a competitive bidding process generated the observed pattern of bidding. Therefore, if conditional independence and exchangeability hold, an analysis of bidding data cannot reject the hypothesis that the firms in question behaved competitively. Conversely, bidding that does not satisfy these conditions was not generated by the fully competitive process modeled by Bajari and Ye, indicating the possibility of collusion. Bajari and Ye develop a set of statistical tests to determine whether bid data satisfy these conditions and apply these tests to a data set of bidding by construction firms in the Midwest.

The first condition implied by competitive bidding is conditional independence. The bids of various competing firms should not be correlated, *after* adjusting for the impact on their bids of all publicly observed

Collusion (Stanford University Working Paper). All subsequent references to Bajari and Ye are to these papers.

[2] Eric Maskin & John Riley, *Asymmetric Auctions*, 67 REV. ECON. STUDS., 413 (2000).

[3] *See* Martin Pesendorfer, *A Study of Collusion in First-Price Auctions*, 67 REV. ECON. STUDS., 381 (2000); Robert H. Porter & J. Douglas Zona, *Detection of Bid Rigging in Procurement Auctions*, 101 J. POL. ECON. 518 (1993); Robert H. Porter & J. Douglas Zona, *Ohio School Milk Markets: An Analysis of Bidding*, 30 RAND J. ECON. 263 (1999); Laura H. Baldwin,

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information about the project, such as the distances of the firms to the project. On the other hand, if a subset of the firms in the industry are members of a cartel and submit phony bids to give the outward appearance of competition, the bids may be correlated in a manner that can be detected using appropriate statistical techniques.

The second condition implied by competitive bidding is exchangeability. All competing firms behave in the same way when faced with the same cost structure for themselves and rival firms. If the publicly observed factors affecting costs or other information that firms use to compute their bids is permuted or exchanged among the firms, then the bids should permute among the firms in the same way when exchangeability holds. Such exchanges in costs may be observed when, for example, one project is close to one set of firms while the next is close to a different group of firms. Bajari and Ye demonstrate that exchangeability can fail when firms collude and they develop statistical tests to identify such failures of exchangeability.

There are at least two limitations to testing for collusion using conditional independence and exchangeability. First, when bids are conditionally independent and exchangeable, we know it is possible to "reverse-engineer" a model of competitive bidding that could have generated the bidding data and, therefore, we cannot rule out from this evidence that the observed bids were generated by competitive bidding. However, it is also possible in theory that the same pattern of bids could result from collusion. If a cartel is clever in how it colludes, it can submit bids that are both conditionally independent and exchangeable and therefore consistent with the implications of the Bajari and Ye model of competitive bidding. It is worth noting, however, that in all case studies of collusion of which we are aware, failures of conditional independence or exchangeability accompanied collusion.

A second and, in our opinion, more serious limitation of using conditional independence and exchangeability to test for collusion is that, in practice, statistical tests may incorrectly reject the hypothesis that the bids are conditionally independent and exchangeable. This can occur if the economist does a poor job of modeling firms' cost structure and fails to control for important information about costs that is publicly available to the bidders. These tests can also be misleading if the structure of the Bajari and Ye model of competitive bidding is not sufficiently general to capture the main factors that influence bidder behavior.

Robert C. Marshall, & Jean-Francois Richard, *Bidder Collusion at Forest Service Timber Sales,* 105 J. POL. ECON. 657 (1997).

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Because the first two tests may not be conclusive, Bajari and Ye suggest supplementing them with a third test. The first two tests can identify one or more specific potential cartels, namely the particular firms that submitted bids that are not conditionally independent or exchangeable. The third test runs a "horse race" between a model of competition and models of collusion with the specific potential cartels identified by the first two tests.

The third test uses information on the structure of costs and typical markups observed in the industry. Possible sources of such information include firms not suspected of collusion, consulting engineers who prepare cost estimates for similar projects, and the internal accounts of bidders obtained by prosecutors. For their empirical analysis, Bajari and Ye collect information from knowledgeable industry sources. They then run a "horse race" between the competing models by using statistics to compute the probability that each model explains how bidding firms behaved, given the observed bids and the prior information about industry costs and markups.

Bajari and Ye apply their tests for collusion to a data set of bidding by construction firms in Minnesota, North Dakota, and South Dakota during the years 1994–1998 for a type of highway repair called seal coating. In order to demonstrate how these methods can be applied in practice, we will summarize how they conduct their empirical tests.

The Bajari-Ye work is consistent with and extends the analysis in several recent empirical papers on the subject of bid rigging. Porter and Zona, in both of their papers, and Pesendorfer analyze data sets where it is known that bid rigging has taken place.[4] In each case, these papers identify bidding patterns that indicate a failure of conditional independence and exchangeability, consistent with the analysis of Bajari and Ye.

Porter and Zona propose tests for collusion, some of which are similar to the tests proposed in Bajari and Ye. Bajari and Ye extend the important analysis of Porter and Zona in several ways. First, Porter and Zona do not specify an equilibrium model of either competitive or collusive bidding in their analysis. It is therefore unclear in Porter and Zona under what conditions, if any, their proposed tests for collusion will be valid outside of the specific cases they examine. By relating their analysis to an economic model of bidding, Bajari and Ye clarify the circumstances under which the Porter-Zona tests for collusion are valid.

Second, Bajari and Ye introduce new tests for collusion that are not present in Porter and Zona. In their empirical work, Bajari and Ye

[4] Porter & Zona, *supra* note 3; Pesendorfer, *supra* note 3.

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conduct forty-six separate tests of the theoretical model of competitive bidding, which allow them to address the problem of identifying which bidders are members of the cartel. The testing in Porter and Zona, on the other hand, takes the identity of the cartel members as given. Furthermore, the Bajari-Ye method for detecting collusion by running a statistical "horse race" between competitive and collusive models of industry equilibrium is not found in Porter and Zona.

We believe that, taken together, the three tests proposed by Bajari and Ye are useful diagnostic tools for detecting suspicious bidding behavior. No method for detecting collusion is likely to be infallible and these tests have their limitations. Furthermore, while these tests can help to detect departures from competitive behavior, they do not directly establish that a departure is the result of an illegal agreement, rather than a form of tacit collusion resulting from firms acting independently while aware of the effects of their behavior on each other. Nonetheless, these methods can be used as a first step to determine whether suspicious bidding has occurred and whether further investigation and analysis is warranted.

II. TESTING FOR COLLUSION

A. PROCUREMENT AUCTIONS

A large body of economic theory has demonstrated that both competitive and collusive bidding strategies depend on the rules of the auction and the cost structure of the bidders. Bajari and Ye begin their analysis with a model of competitive bidding that specifies the rules of the auction and the resulting bidding strategies of competing firms. Their tests for collusion are based on searching for patterns in the bidding data that are not consistent with bidding strategies implied by their model of competitive bidding. The tests suggested by Porter and Zona are also valid if the modeling assumptions used in Bajari and Ye hold. If the circumstances in a market do not fit the assumptions of this competitive model, then the results of the tests proposed by both Bajari and Ye and Porter and Zona may be misleading.

First, the Bajari and Ye model assumes that firms submit sealed bids for a procurement contract and that the contract is awarded to the lowest bidder. These rules are common practice in the vast majority of public sector procurements, and are also used in many auctions in the private sector as well.

Second, Bajari and Ye assume that each firm has private information about its costs. In the model discussed in Bajari and Ye, before the

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bidding begins, each firm estimates its cost to complete the project. This cost estimate is private information known to each firm; firm A knows its own cost estimate but does not know the precise cost estimate of B, just as B does not know A's cost estimate.[5]

Third, Bajari and Ye assume that the firms' bidding strategies are a Bayes-Nash equilibrium. In equilibrium, firms are rational and submit bids that maximize their expected profits, taking into account all possible information about themselves and their rival firms. Obviously, costs are a major determinant in a firm's calculation of its profit-maximizing bid. Bidding below cost increases the chances of winning but insures the firm loses money on the contract if it wins. One would not expect a firm following such a strategy to last long in the industry. Bidding exactly at cost insures that no money is lost; however, no profit is made either. Firms attempting to maximize profit will include a markup over costs in their bids, but the larger the markup the less likely the bid will win. A profit-maximizing firm must trade off the benefits of increasing its bid (a higher profit if it wins) against the costs of increasing its bid (a lower probability of winning the contract).

Fourth, Bajari and Ye assume that bidders' costs for a project differ and that the bidding firms know their costs differ. In the terminology of auction theory, the firms have asymmetric costs. There are many reasons for observable cost differences among firms. One reason is location. Firms that are closer to a construction project generally have lower transportation costs and, thus, tend to submit lower bids than firms located farther away, all else equal. A second reason for cost differences is variation in available capacity. A firm choosing its bid for a project today must take into account that winning this contract will limit its available capacity to complete other contracts in the future, including some that might promise greater profits. The less capacity the firm has available when a project is being bid, the greater the likelihood that winning this bid will preclude it from winning a later, more profitable contract.

If the modeling assumptions listed above hold, Bajari and Ye identify two conditions, conditional independence and exchangeability, that are both necessary and sufficient for a set of bids to be considered "competi-

[5] As we discuss later, even though each firm's precise cost is known only to itself, there is public information about factors that affect (although they do not completely determine) various firms' costs. Thus firms will have some information about the approximate level of their costs relative to that of other firms.

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tive" bids.[6] If these assumptions hold, we cannot reject the hypothesis that the observed bids were generated competitively by testing for these conditions. These conditions are implied by the Bajari and Ye model of competitive bidding and if these conditions hold, it is always possible to reverse-engineer a model of competitive bidding that generates these bids as an equilibrium.

B. CONDITIONAL INDEPENDENCE

Bajari and Ye demonstrate that a first implication of their model of competitive bidding is the conditional independence of bids. Conditional independence implies that, after we have adjusted for or conditioned on all of the information about the project and firms' costs that is publicly available to the bidders, the bids should not be correlated. This result has a simple and intuitive explanation. Before submitting a bid for a contract, a firm typically computes cost estimates for itself and other firms. The bid takes into account all of the public information expected to affect its own costs and those of other firms. In addition, the firm takes into account private information about its own costs that only it has and that can affect only its own bid. If there is no collusion, each firm independently arrives at its cost estimate and bid. Since the cost estimates were arrived at independently, the bids should be independent after taking account of the publicly observable cost information.

This does not mean that competitive bids will be uncorrelated before adjusting for publicly observed factors that affect bids. For example, say that two firms are located close together. The two firms will tend to bid high and low on the same projects because, as the public information on distance indicates, they will tend (all else equal) to have relatively high or low transportation costs for the same projects.

In collusive bidding, by contrast, firms will coordinate on how to bid before the auction. The cartel may designate a firm to win the contract and have other cartel members submit "phony" bids crafted to give the appearance of competition. If the cartel members have coordinated on how to bid before the auction, their bids will typically be correlated and this can be detected through the careful application of econometric methods.

To test whether bids are conditionally independent, the econometrician first gathers all publicly available information about factors that

[6] In addition to conditional independence and exchangeability, there are a few "technical" conditions that are required. Interested readers can consult Bajari & Ye, *supra* note 1, for a complete discussion.

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would affect each firm's bid: each firm's location, available capacity, previous experience in the market, etc. This information is called "public" because it is available to everyone, including each rival firm.

Second, the econometrician estimates how the bids of each firm change as each of these factors varies from firm to firm and project to project. For example, say that each firm's cost of completing a project increases with the distance from the firm to the project location. The econometrician would collect information on the bid and distance to the work site for each firm and for each project and run a regression estimating how much on average each firm's bid changes with the distance of each firm from the project location.

Finally, the econometrician will test to see if the bids of firms are correlated or independent, after adjusting for all public information (in this example, the information on distances). In simple terms, the econometrician tests for a persistent pattern that when firm A bids more than one would predict from public information, another firm, B, also usually bids more (or less) than one would predict.[7]

C. EXCHANGEABILITY

A second condition implied by the competitive model of bidding is exchangeability between bids. Exchangeability means that all firms behave identically when faced with the same cost structure for both themselves and rival firms.

A simple example will illustrate the concept. Suppose there are three firms in the industry, A, B, and C; the only costs in the industry are transportation costs; and a firm's cost is $100,000 times its distance in miles to the project site. A, B, and C have distances and costs as listed in Table 1 for a particular project.

Given these costs, how should the firms bid? Firm A has the lowest cost but knows that it will win the project only if it bids less than firm

Table 1
Costs for Firms A, B, and C

Firm Identity	*Distance*	*Cost*
A	10 miles	$1.0 million
B	12 miles	$1.2 million
C	13 miles	$1.3 million

[7] Conditional independence may fail because of either positive or negative correlation.

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B, the firm with the next-lowest cost. Firm A should not bid more than $1.2 million because, if firm B expected A to bid more than $1.2 million, B could undercut A's bid and win the contract while still covering its costs. Obviously, the non-collusive equilibrium outcome in this case is for A to bid just under $1.2 million and win the contract. Firm A is the only firm willing to bid this low because it is the only firm that can do so and still make a profit. Firm B should bid $1.2 million and C should bid $1.3 million in equilibrium. Neither will bid less, lest it win the contract and lose money, and neither will bid more in order to maximize the (unlikely) possibility of winning.

How do the equilibrium bids change if A and B are in a cartel? If A and B can make side payments to each other, it is obviously in their joint interests to let A win the project.[8] Since A has lower costs than B, the cartel makes more profits if A rather than B wins the project. Since firm A does not have to worry that its bid will be undercut by firm B, the equilibrium outcome is for firm A to bid just under $1.3 million, for firm C to bid $1.3 million, and for B to submit a phony bid above $1.3 million.

Now, let's consider how firms A, B, and C would bid on a second project with distances and costs as in Table 2. In the second project, B and C have exchanged costs; B's distance is now 13 miles while C's distance has decreased to 12 miles.

If bidding is competitive, firms B and C simply exchange or permute bids, just as they have exchanged costs. Firm C now bids $1.2 million and B bids $1.3 million because neither will bid lower than its costs. Firm A continues to bid just under $1.2 million in order to win the

Table 2
Costs for Firms A, B, and C on Second Project

Firm Identity	*Distance*	*Cost*
A	10 miles	$1.0 million
B	13 miles	$1.3 million
C	12 miles	$1.2 million

[8] Most economic models of collusion assume that collusion only takes place if collusion raises expected profits. A side payment from A to B could eliminate B's incentive to undercut a bid by A that is higher than the competitive level. A variety of other real-world mechanisms to facilitate collusion can also be used, including the simple possibility that over a series of bids each cartel participant will profit more by continuing to be a cartel member than by breaking discipline.

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contract from the firm with the next-lowest cost (now firm C rather than firm B).

If firms A and B collude, they again should award the project to firm A because A still has lower costs than B (and indeed firm B now will not be able to underbid C). With the exchange of costs by B and C, however, firm A will not make the same bid as before. Firm A now should bid just under $1.2 million as C, which is not part of the cartel, will bid $1.2 million. We summarize the costs and the bidding for the two projects in Table 3.

Table 3 shows that the competitive bids are exchangeable. When two competing firms have the same cost and face the same costs for rival firms, they will bid the same. The competitive bids submitted by A are identical for the first and the second project because the cost of A's closest rival (firm B in the 1st project and C in the 2nd project) is the same. The competitive bids of B and C permute as their costs are permuted between the two projects. In the first project, B has cost of $1.2 million and the costs of its rivals are $1.0 million and $1.3 million. In response to this situation, B bids $1.2 million. When we go from the first project to the second project, the positions of B and C are "exchanged" and firm C bids $1.2 million.

Table 3 also shows that the exchangeability of costs and bids breaks down when bidding is collusive. Firm A bids $1.29 million on the first project and $1.19 million on the second project, even though the cost of A's closest "rival" remains unchanged. What has changed, of course, is whether the rival with the closest cost is in the cartel. In addition, with collusive bidding, the bids of firms 2 and 3 no longer exchange when their costs are exchanged.

Bajari and Ye demonstrate that the property of exchangeability generalizes to much more complicated environments than the one studied above. In models with private information, bids will no longer appear

Table 3
Failure of Exchangeability with Collusive Bidding

Firm Identity	*Cost for 1st Project*	*Competitive Bid for 1st Project*	*Collusive Bid for 1st Project*	*Cost for 2nd Project*	*Competitive Bid for 2nd Project*	*Collusive Bid for 2nd Project*
A	$1.0 Mil.	$1.19 Mil.	$1.29 Mil.	$1.0 Mil.	$1.19 Mil.	$1.19 Mil.
B	$1.2 Mil.	$1.2 Mil.	Phony bid> $1.29 Mil.	$1.3 Mil.	$1.3 Mil.	$1.3 Mil.
C	$1.3 Mil.	$1.3 Mil.	$1.3 Mil.	$1.2 Mil.	$1.2 Mil.	$1.2 Mil.

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deterministic either to the econometrician or to rival firms. For given values of publicly observed cost information (such as distance in the previous example), the analyst and other firms will be able to predict only the distribution or range of likely bids, rather than a single determined bid, because unobserved private costs also affect bids. In these more general settings, it is the distribution of non-collusive bids, rather than the specific bids themselves, that must exchange when publicly observed cost conditions exchange. Bajari and Ye describe statistical techniques that can be used to test for exchangeability in the more general setting.

III. CASE STUDIES

In this section, we describe some recent case studies of collusion in procurement auctions. We first discuss some case studies in which the economists were fairly certain from prosecutorial evidence that collusion occurred and had good information about the identity of the cartel. We then turn to the Bajari and Ye case study and describe, in some detail, how they applied their tests for conditional independence and exchangeability.

A. CASE STUDIES OF CARTELS

Porter and Zona, in their 1993 *Journal of Political Economy* paper,[9] present a case study of bidding behavior by firms competing for state highway construction contracts on Long Island in the early 1980s. Collusion was known to have occurred in this industry. Prior to the study, one of the large firms was convicted in federal court of rigging bids on a Long Island highway construction project, and several other firms were named as unindicted co-conspirators.

Porter and Zona gathered data on all firms (cartel and non-cartel) and ran regressions to test how certain characteristics of a firm, like distance from the project, affected the bids of firms in the cartel on the one hand and of firms not in the cartel on the other hand. They found the patterns one would expect for the competitive group—for example, greater distance increased the bid and a greater backlog of work decreased the bid—but discovered counterintuitive bidding behavior for firms in the cartel. Many factors that increased the bid for non-cartel firms actually decreased it for cartel members. The ranking of non-cartel firms by bid matched their ranking by cost—firms with higher costs submitted higher bids. However, the ranking of cartel firms by bid and

[9] *Supra* note 3.

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costs did not match. Information obtained from cartel insiders suggests that the cartel members submitted "phony" bids meant to give the outward appearance of competition, which is consistent with the ranking of firms by bids not matching their ranking by cost. According to Porter and Zona, the insiders described the process as follows:

> We all sat at the conference table . . . one of the contractors would have a list of upcoming contracts . . . they'd talk about the contract . . . how much money . . . who won the last one . . . who should get this one . . . the contractors who were tagged to be the low bidders would work out their bid figures . . . The rest of the contractors would then come up with higher bids.

This discrepancy between cartel and non-cartel bids can be interpreted as a failure of exchangeability, in the language of Bajari and Ye.

Porter and Zona also document that the residuals to estimated bid functions for firms in the cartel are much more highly correlated with each other than the residuals for the non-cartel firms. Finding correlation in the bids should not be surprising because the cartel members coordinated their actions before bidding began. As discussed in Part II.B., this is a failure of conditional independence because the residuals measure the differences between firms' actual bids and the bids that would be predicted based on public information.

Porter and Zona, in their 1999 *RAND Journal of Economics* paper, examine bidding for contracts to supply school milk in Ohio, which is also a market with known collusion.[10] Guilty pleas to price fixing have been entered in at least a dozen states with total levied fines of over $90 million. In Ohio, thirteen dairies were charged with collusion in school milk auctions from 1980 to 1990. Porter and Zona analyze data collected for that court case and compare the bidding of colluders and non-colluders.

Porter and Zona find that competitive firms were less likely to submit bids on Ohio school milk contracts for distant work than for nearer work, presumably because of rising transportation costs. By contrast, firms believed to be colluding bid on distant contracts more frequently than one would have predicted from the behavior of the competitive firms. Also, cartel firms submitted bids that were unusually high for contracts closer to them and unusually low for contracts far away from them—exactly the opposite of what competitive theory dictates we should see in the data. Once again, this is a failure of exchangeability. Porter and Zona also test for correlation among the decisions by cartel firms

[10] *Supra* note 3.

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of whether to bid. They found that if one ring member bid, then other members were more likely to bid; this is called complementary bidding. Porter and Zona also document failures of conditional independence.

To summarize, previous case studies of collusion, such as Porter and Zona, document patterns of bidding that suggest the tests proposed by Bajari and Ye would be able to detect collusive behavior. Bajari and Ye extend the work of authors, such as Porter and Zona, by establishing a more rigorous link to economic theory and by precisely stating the conditions under which their tests for collusion will be valid.

B. THE BAJARI AND YE CASE STUDY

Bajari and Ye apply their tests for detecting collusion to a unique data set of bidding by highway contractors in Minnesota, North Dakota, and South Dakota. They focus on a particular activity in road construction called seal coating, which is a low-cost alternative to resurfacing a highway. Seal coating adds oil and aggregate (such as sand, crushed rock, gravel, or pea rock) to the surface of a road. This gives the road a new surface to wear and adds oil that will soak into the underlying pavement and slow the development of cracks. The data set was purchased from Construction Market Data (CMD) and contains detailed bidding information for nearly all of the public and private seal-coating projects in these three states during the years 1994–1998.[11] The data set records the project location, the deadline for bid submission, bonding requirements, the identities of all bidders, an extremely detailed project description, and many other variables.

The vast majority of contracts for seal coating are in the public sector. In the Bajari and Ye data set, 230 of the contracts (46.5%) are awarded by cities, 195 (39.3%) are awarded by states, 68 (13.7%) are awarded by counties, and only 2 are awarded by the federal government. Firms submit sealed bids for public-sector seal-coating contracts and, if awarded, contracts must go to the lowest responsible bidder.[12] Firms have strong financial incentives to honor their contractual obligations when they are the low bidder. Contractors typically submit a bid bond

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[11] Construction Market Data sells information to general contractors about upcoming construction projects. Many of the general contractors in the market subscribe to *Construction Bulletin*, a weekly periodical published by CMD, to search for work. *Construction Bulletin* also reports bids for contracts that were awarded in previous weeks. Based on conversations with DOT officials, general contractors, and CMD, Bajari and Ye assert that almost all public and private road construction projects exceeding $10,000 are contained in the data set.

[12] Public procurement officials have the right to reject all bids but do so infrequently in practice.

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of 5–10 percent of their total bid, guaranteeing that they will not withdraw their bid after the public reading of all bids. After the contract is awarded, the low bidder must submit both a performance bond and a pay bond to guarantee the completion of the contract and payment of all subcontractors.

The combined value of contracts awarded in the data set is $92.8 million.[13] Fifty-five firms in the data set won at least one job, of which only eighteen had a revenue share of contracts exceeding 1 percent.[14] Table 4 summarizes the revenue shares of these eighteen firms (identified by their ID number in the dataset) and how frequently each bid for contracts in the data set. The largest 7 firms captured 65.6% of contract revenues, led by firm 2, which alone accounted for 21.1% of contract revenues and participated in 66.9% of the auctions conducted.

Table 4
Revenue Shares and Participation by Firm

Firm ID No.	*No. of Wins*	*Avg. Bid*	*Revenue Share %*	*No. Participation*	*% Participation*
1	92	82,790	8.2	145	29.3
2	102	191,953	21.1	331	66.9
3	20	363,565	7.8	69	14.0
4	35	241,872	9.1	114	23.0
5	29	283,323	8.9	170	34.3
6	40	77,423	3.3	84	17.0
7	45	62,085	3.0	121	24.4
8	16	87,231	1.5	134	27.1
9	10	237,408	2.6	14	2.8
11	4	328,224	1.4	28	5.7
12	3	317,788	1.0	8	1.6
14	4	754,019	3.2	25	5.1
17	5	1,018,578	5.5	8	1.6
20	13	355,455	5.0	38	7.7
21	2	903,918	1.9	5	1.0
22	2	903,953	2.0	8	1.6
23	2	439,619	1.0	4	0.8
25	3	382,012	1.2	13	2.6
Total	427		87.7		

[13] The size of contracts varied greatly. Of the 495 contracts in the data set, 7 contracts were awarded for more than $1 million, 256 contracts were awarded for less than $1 million but more than $100 thousand, and 232 contracts were awarded for less than $100 thousand. A total of 98 firms bid on at least one of these 495 contracts.

[14] A firm's revenue share is defined as the total value of the firm's winning bids as a percent of the total value of winning bids for all contracts.

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Bid rigging has occurred in these auctions in the past. The owner of the largest firm, firm 2, received a one-year prison sentence for bid rigging in the late 1980s. The owners of two other firms, firms 4 and 5, paid treble damages for bid rigging with firm 2. The owners of all three firms were, at one time, banned from bidding for public-sector seal coat contracts.

The first step in the analysis is to identify characteristics that affect firms' costs of completing contracts and about which there is public information. One such characteristic is each firm's distance from the project; Bajari and Ye calculate the distance from each firm to each project. Bajari and Ye also construct a new variable, "CAP," to measure each firm's capacity utilization level using information on winning bids and bidding dates. A firm's capacity at a particular time is defined as the ratio of the firm's used capacity (measured by the firm's total winning bid for seal coating contracts up to that time) over the firm's total winning bid amount in the entire season.[15] Another important control variable for Bajari's and Ye's analysis is an engineer's cost estimate. The engineer's estimate is supposed to represent a "fair market value" for completion of the project. Estimates were available for 139 out of the 441 projects in the data set and Bajari and Ye restricted their statistical analysis to these projects.

Table 5 provides summary statistics on bidding for these contracts. The average winning bid is $175,000, with bids ranging from around $4,000 to over $1.7 million. The second row of the table suggests that the engineer's cost estimate is a reasonable estimate of the fair market value for the contract. On average, the winning bid divided by the engineer's estimate is quite close to 1.

In Table 5, "Money on the Table" is defined as the difference between the second lowest bid and the lowest bid. The average value of money on the table is $16,000. This is consistent with the presence of private information about costs. If there were no private information about costs, as in our example in Part II.C., firms could estimate their rivals' costs and bid with precision and the low bidder would shade his bid to just under the second-lowest bidder. Clearly, that is not happening in this industry. Finally, Table 5, shows that the winning bidder is, on average, closer to the project and has more free capacity than the second lowest bidder.

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[15] For purposes of this calculation, a season starts on September 1 and ends on August 31 of the following calendar year. This measure of capacity was computed using the entire data base of bidding information even though the Bajari and Ye econometric analysis is run on a subset of these projects.

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Table 5
Bidding Summary Statistics

Variable	*No. of Observations*	*Mean*	*Standard Deviation*	*Min. Value*	*Max. Value*
Winning Bid	441	175,000	210,000	3893	1,732,500
Winning Bid/Estimate	139	1.0031	0.1573	0.6662	1.5421
Money on the Table: 2nd Bid-1st Bid	134	15,748	19,241	209	103,481
Normalized Money on the Table: (2nd Bid-1st Bid)/ Estimate	134	0.0776	0.0888	0.0014	0.5099
Number of Bidders per Project	139	3.280	1.0357	1	6
Distance of Winning Bidder	134	188.67	141.51	0	584.2
Distance of Second Bidder	134	213.75	152.01	0	555
Capacity of Winning Bidder	131	0.3376	0.3160	0	0.9597
Capacity of Second Bidder	131	0.4326	0.3435	0	1

Another important determinant of firms' success in winning contracts is familiarity with local regulators and material suppliers. Such familiarity presumably lowers the costs of preparing a potentially successful bid and of arranging for necessary supplies. Table 6 presents, for each state and for each firm, the percentage of the firm's total dollar volume done in that state. Most firms in the data set work primarily in one state, and this pattern persists even after controlling for distance. For instance, firm 3 is located near the boundaries of Minnesota, North Dakota, and South Dakota, yet it performs over 70 percent of its dollar volume of seal coating in South Dakota. Firm 6 is located near the Minnesota-South Dakota border, but it won no contracts in South Dakota. This suggests that the concentration by state is not explained by distance alone.

1. *Bid Function Regressions*

The theoretical model of auctions in Bajari and Ye implies that a firm's bid should be a function of factors that influence both its own and its rivals' costs. Tables 5 and 6 suggest that the engineer's cost estimate, firms' distance to the project, available capacity, and previous experience in the state are important factors about which there is public information that influences costs. Bajari and Ye use regression analysis to model firms' bids as a function of the following variables.

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Table 6
Concentration of Firm Activity by State

Firm ID No.	*MN. Concentration*	*ND. Concentration*	*SD. Concentration*
1	1	0	0
2	0.2781	0.7218	0
3	0	0.2377	0.7623
4	0	1	0
5	0.1246	0.5338	0.3414
6	0.8195	0.1804	0
7	0.9572	0.0427	0
8	0.7290	0.2709	0
11	0	0	1
14	0	1	0
20	0	1	0

- $BID_{i,t}$: The amount bid by firm *i* on project *t*.
- EST_t: The engineer's cost estimate for project *t*.
- $DIST_{i,t}$: Distance from firm *i*'s headquarters to the midpoint of project *t*.
- $LDIST_{i,t}$: $log(DIST_{i,t}+1.0)$.
- $CAP_{i,t}$: Used capacity measure of firm *i* at the time it bids on project *t*.
- $MAXP_{i,t}$: Maximum percentage of free capacity of all firms on project *t*, excluding firm *i*.
- $MDIST_{i,t}$: Minimum of distances for all firms on project *t*, excluding firm *i*.
- $LMDIST_{i,t}$: $log(MDIST_{i,t}+1.0)$.
- $CON_{i,t}$: The proportion of work done (by dollar volume) by firm *i* in the state where project *t* is located prior to the auction.
- DUM_t: A dummy variable for project *t*, that is, the dummy variable is equal to one if the project is *t* and is zero otherwise.
- DUM_i: A dummy variable for firm *i*, that is, the dummy variable is equal to one if the bidder is firm *i* and is zero otherwise.

Bajari and Ye begin to explore the empirical relationship between costs and bids by estimating the following regression using ordinary least squares and information in the database for the bids of all firms on all contracts.

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$$\frac{BID_{i,t}}{EST_t} = \beta_0 + \beta_1 LDIST_{i,t} + \beta_2 CAP_{i,t} + \beta_3 MAXP_{i,t} + \beta_4 LMDIST_{i,t} + \beta_5 CON_{i,t} + \varepsilon_{i,t}\,. \quad (1)$$

The coefficients, β_1, β_2, and so forth, measure the average effect on bids (relative to the engineering estimate of cost) of each factor.[16] The residual, $\varepsilon_{i,t}$, captures the variation in bids not explained by the factors included in the regression. Some of this unexplained variation in bids is due to private cost information not observable to either the econometrician or other firms.

In equation (1), there are too few data points to include all the variables that measure factors affecting the costs of all competing firms. The example in Part II.C. suggests that the lowest-cost rival will play in the most important role in determining the market prices. Using advanced computational techniques, Bajari and Ye find that this is also true in more general models of bidding. Therefore, they include the maximum free capacity among all rival firms and the distance of the closest rival firm as controls. Their estimates for this regression model are displayed in Table 7; t-statistics are contained in parentheses.[17]

All of the estimated coefficients in Table 7 have intuitively plausible signs. The coefficient on $LDIST_{i,t}$ is positive—on average, the greater the distance of the firm to a project, the higher its bid—which makes sense because increasing distance from the job increases transportation costs. The coefficient on $CAP_{i,t}$ is also positive. As a firm utilizes more of its available capacity, the opportunity cost of winning a job today increases

Table 7
Estimated Bid Functions

Variable	*Coefficient*
Constant	.68 (5.95)
$LDIST_{i,t}$	.040 (3.45)
$CAP_{i,t}$	.17 (8.51)
$MAXP_{i,t}$	.026 (.71)
$LMDIST_{i,t}$	.24 (1.81)
$CON_{i,t}$	–.59 (–1.87)
Sample Size	450
R^2	.85

[16] Bajari and Ye divide by the engineer's estimate to correct the regression for what econometricians refer to as the heteroskedasticity problem.

[17] One can reject the hypothesis that the coefficient equals zero and the factor had no effect on bids with at least 90% confidence if the t-statistic exceeds 1.65 and with at least 95% confidence if the t-statistic exceeds 1.96.

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because the firm will have less free capacity to bid on profitable jobs in the future. The coefficient on $MAXP_{i,t}$ is also positive. If a firm knows its rivals have less free capacity, it will expect the rivals to bid higher, prompting a strategic increase in the firm's own bid. Similarly, the coefficient on $LMDIST_{i,t}$ is positive. If the distances from a firm's rivals to the project, and thus their costs, are greater, the firm will expect its rivals to bid less aggressively, prompting a strategic increase in the firm's own bid. Finally, the coefficient on $CON_{i,t}$ is negative, indicating that a firm's previous bidding success in the same state is associated with lower bids, which is consistent with the hypothesis that a higher concentration of work in a state lowers costs. All of the estimated coefficients, except for $MAXP_{i,t}$, are statistically significant at conventional levels.

To test for exchangeability and conditional independence, Bajari and Ye generalize the model in equation (1). In the data, it was clear that firms could be naturally categorized into two groups. The first group is a set of eleven "large" firms that win contracts frequently. The second group is a set of "fringe" firms that are struggling to break into the industry and that won few, if any, contracts. Bajari and Ye therefore estimate a regression model based on the following two equations:

$$\frac{BID_{i,t}}{EST_t} = \beta_0 + \beta_{1,i}LDIST_{i,t} + \beta_{2,i}CAP_{i,t} + \beta_{3,i}MAXP_{i,t} + \beta_{4,i}LMDIST_{i,t} + \beta_{5,i}CON_{i,t} + \varepsilon_{i,t} \quad (2)$$

if i is one of the 11 largest firms.

$$\frac{BID_{i,t}}{EST_t} = \alpha_0 + \alpha_1 LDIST_{i,t} + \alpha_2 CAP_{i,t} + \alpha_3 MAXP_{i,t} + \alpha_4 LMDIST_{i,t} + \alpha_5 CON_{i,t} + \varepsilon_{i,t} \quad (3)$$

if i is not one of the 11 largest firms.

In equation (2), Bajari and Ye allow the coefficients of the bid functions to be different for each of the eleven largest firms in the market. This allows them to determine whether the bids of different firms respond differently to one or more cost factors. Bajari and Ye also include a set of auction or project dummy variables in equations (2) and (3) to control for factors about the project that are publicly observed by the firms, but not by the econometrician. Finally, Bajari and Ye include a set of dummy variables for the eleven main firms in equation (2). This allows them to control for persistent differences in productivity across these firms that affect all bids because, as the data revealed, certain firms had more success in winning contracts.

To test for conditional independence of firm i's bid and firm j's bid, Bajari and Ye test whether the correlation between the residuals of the bid functions for firms *i* and *j*, $\varepsilon_{i,t}$ and $\varepsilon_{j,t}$ is zero. The residuals measure the extent to which actual bids diverge from the bids one would predict based on public information. As described above, if bids are conditionally independent, there should be no persistent pattern or correlation in the divergence between the actual and predicted bids of different firms. This test is carried out for each pair of the eleven largest firms. They find, using standard confidence levels, that the bids are conditionally independent for all but four pairs of firms: (firm 1, firm 2), (firm 2, firm 4), (firm 5, firm 14), and (firm 6, firm 7). Only one of these pairs, firm 2 and firm 4, however, bid against one another more than a handful of times. The other pairs—firm 1 and firm 2, firm 5 and firm 14, and firm 6 and firm 7—bid against each other, on average, no more than two or three times a year in the data set.[18] The correlations of the residuals for the bids of the remaining pairs of firms could not be distinguished from zero at statistically significant levels.

To test for exchangeability between firms *i* and *j*, Bajari and Ye test the hypothesis that the coefficient $\beta_{i,k}$ is equal to $\beta_{j,k}$ for $k = 1, \ldots, 5$. These tests reveal whether firms *i* and *j* respond in the same way to changes in variables that affect their bidding strategies. As explained above, exchangeability means that any competitive firm faced with a given set of cost conditions for itself and its rivals should bid in the same way. All firms, except for the pair firm 2 and firm 5, pass the exchangeability test.

Overall, Bajari and Ye conduct forty-six separate tests for the presence of conditional independence and exchangeability. Forty-one of these tests fail to reject the hypothesis of competitive bidding at conventional levels of statistical significance. Therefore, Bajari and Ye conclude that they cannot reject the hypothesis that most of the bidders in the market were behaving competitively. The bidding patterns of firm pairs 2 and 4 and 2 and 5, however, do not appear to be consistent with those predicted by the Bajari and Ye model of competitive bidding. It is worth noting, as mentioned above, that the owners of these three firms were previously sanctioned for bid rigging.

C. Potential Problems

Testing for collusion using statistical tests for conditional independence and exchangeability can provide valuable information but will not

[18] We discount these results for two reasons. The first is that our tests for zero correlation are based on what econometrians refer to as asymptotic approximations, which can be incorrect in small samples. Second, collusion among firms that bid against each other

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always point the econometrician in the right direction when investigating a set of bids. A first potential problem is that, for the statistical tests to yield reliable results, the econometrician must account for all of the information that is publicly available to the firms before they bid. For instance, suppose that the econometrician fails to take account of capacity utilization and the capacity utilization of both firms A and B is low compared to that of other firms in the industry. Then we would expect, after controlling for the factors that are included by the econometrician, that firms A and B typically would bid more aggressively than other firms in the industry. As a result of the omitted variable, these firms would fail the statistical tests for conditional independence and exchangeability.

In equations (2) and (3), Bajari and Ye attempt to control for omitted variables by including dummy variables for each project and the eleven largest firms in the industry. Project dummy variables will capture the effect on bidding of costs for completing the project that are publicly observed by the firms, but not by the econometrician. A firm dummy variable will account for the effect on bidding of persistent differences in productivity across firms. If there are omitted variables that are not firm-specific or project-specific, however, testing for conditional independence and exchangeability using equations (2) and (3) can be misleading.

If tests for conditional independence and exchangeability fail, the economist should consult knowledgeable industry sources to make sure that he has taken account of all relevant publicly observable cost information. The first time Bajari and Ye estimated equations similar to (2) and (3), they found many failures of exchangeability and conditional independence. They contacted industry sources and explained the patterns of correlation they observed. For example, their initial results showed that the bids of firms 2 and 3 were strongly negatively correlated. Industry sources suggested this probably was due to a failure to take account of previous experience bidding in a given state. Bajari and Ye then constructed the variable $CON_{i,t}$ and they found that most of the bids appeared to be consistent with competition.

A second possible problem is that a sufficiently sophisticated cartel could avoid detection using tests for conditional independence and exchangeability. For instance, suppose that the members of the cartel agree to first compute what their bids for the project would be if they behaved competitively, but then each submit bids 1.2 times their competi-

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frequently is a more pressing practical issue than firms that only bid against each other once or twice in a given year.

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tive bid. It can be shown that such a bidding scheme will result in bids that are conditionally independent and exchangeable.

The relationship between conditional independence and exchangeability and competition and collusion is summarized in Figure 1. All competitive bids in auctions covered by the Bajari-Ye analysis must be conditionally independent and exchangeable. Some cartel bids (from a sophisticated cartel) also could be conditionally independent and exchangeable. If bids are not conditionally independent and exchangeable, however, they could not have been generated from the Bajari-Ye model of competitive bidding. Some collusive bids do fail to be conditionally independent and exchangeable.

We personally believe that this second problem is probably not that important in practice. All case studies of collusion of which we are aware find at least some failure of conditional independence and exchangeability. If conditional independence and exchangeability fail to hold, the competitive models described in Bajari and Ye could not have generated the observed bids, and collusion is one of the explanations for this failure. We are unaware of any case where a cartel has behaved in a way that would avoid detection using tests for conditional independence and exchangeability.

A final problem in testing for conditional independence and exchangeability is that these tests alone will not be able to distinguish collusive bidding resulting from an illegal agreement from bidding that departs from the competitive pattern because firms act independently but are aware of their interdependence. The latter behavior, sometimes called tacit collusion, is not illegal under the Sherman Act but may generate bidding patterns that differ from those of the competitive model used by Bajari and Ye. For instance, suppose that two firms, located in different counties, manage to tacitly coordinate their bidding without directly communicating, so that neither bids aggressively for jobs in the rival's home county. Statistical tests might well detect that bidding resulting from such tacit collusion fails to be conditionally independent and exchangeable.

While tests using conditional independence and exchangeability may not be able to distinguish tacit collusion from more overt forms of collusion, it is worth noting that in many procurement markets, tacit collusion may be difficult to sustain. Projects tend to be far from uniform in size and are not distributed in a geographically uniform way in any given year. For instance, if in year 1, the vast majority of projects are in county A and in year 2, the vast majority of projects are in county B, a tacit agreement not to compete aggressively is likely to break down. If

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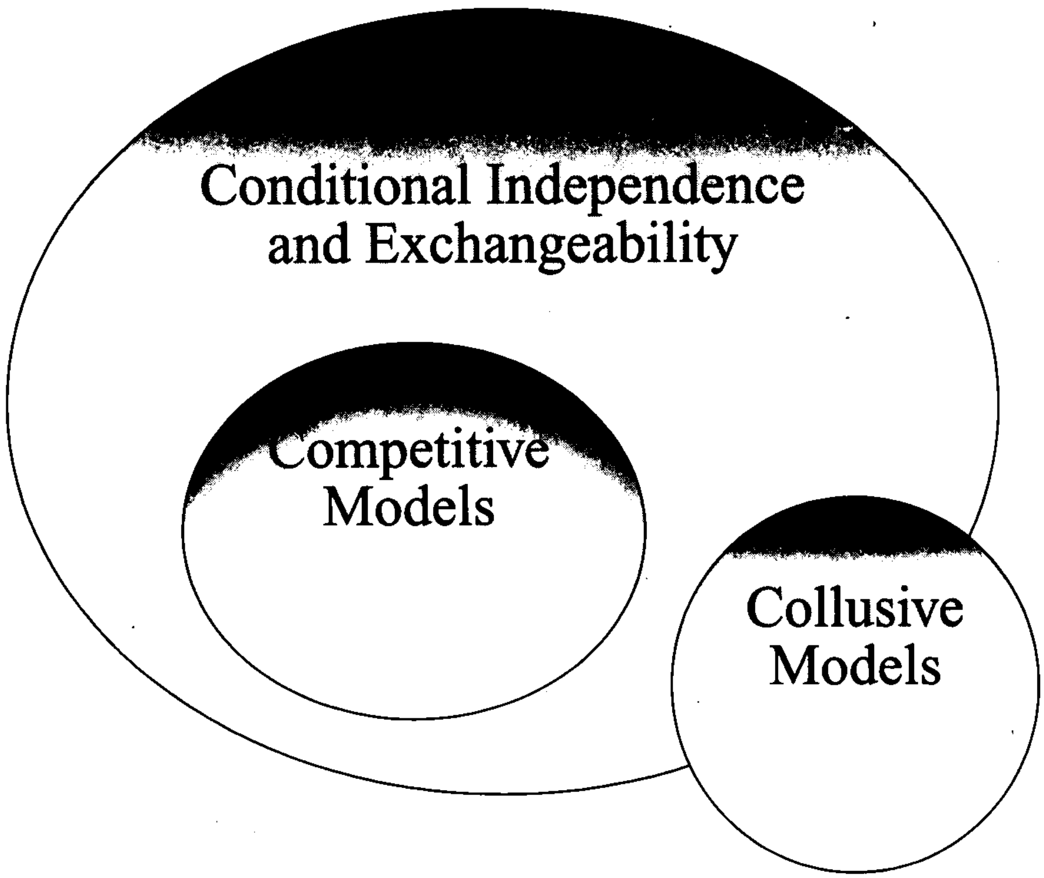

Figure 1

the firms are able to make side payments to each other (which is illegal), they may be able to sustain collusion. Thus, the tests may identify bidding patterns that further analysis, or other evidence, could show was very unlikely to have resulted from independent bidding.

IV. COMPARING MODELS OF COMPETITION AND COLLUSION

The previous section explains that while collusion is certainly one reason why observed bidding patterns may fail statistical tests for conditional independence and exchangeability, it is certainly not the only reason. This section describes a third, complementary test that Bajari and Ye propose for detecting collusion. This test runs a statistical horse race between competitive and collusive models. The description of this test follows the Bajari-Ye empirical application.

The Bajari and Ye tests for the conditional independence and exchangeability of bidding suggest three models of firm behavior that

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are candidates for further analysis by antitrust authorities: model A in which all firms bid competitively and there is no collusion, model B in which firms 2 and 4 collude (suggested by tests rejecting conditional independence of bidding by these firms), and model C in which firms 2 and 5 collude (suggested by tests rejecting exchangeability for bidding by these firms).

The analysis for this statistical horse race begins with the assumption that each firm or cartel (in the case of a collusive model) submits bids intended to maximize its profits. Bajari and Ye demonstrate that it is possible to infer from a firm's observed bid what would be its costs if each of the particular models, A, B, and C, were true. The specification of their competitive model, on which such inferences are based, was discussed earlier. The analysis of collusive models B and C assumes that the cartel member with the lowest cost wins the project while the other cartel member either refrains from bidding or submits a phony bid. Bajari and Ye argue that this specification is appropriate because when collusion on bids for road projects was detected in the past, firms made side payments to each other. With this mechanism for collusion, it is natural to assume that the cartel member with the highest potential profits will be chosen as the serious bidder for the project.

Given a bid $BID_{i,t}$ for a particular firm and project, each model, at its estimated parameter values, implies a value for the markup. For instance, if in auction 5 firm 2 bid $147,000, model A may imply a markup of 2 percent, model B may imply a markup of 5 percent and model C may imply a markup of 4 percent. For each observed bid, Bajari and Ye calculate the markups implied by each model. The implied markups for each model of course vary from bid to bid and, thus, the implied markup of the winning bid varies from project to project. We summarize in Table 8 their findings about the distribution of markups across winning bids that is implied by each of the three models.

The next stage of the statistical horse race is to determine, given the implied distribution of markups (and other implications of the models), which model is the most likely to have generated the markups that firms experience. To make this determination, the analyst needs information from external sources about the distribution of markups in the industry that can then be compared with the distributions of markups implied by each of the models.

Bajari and Ye consulted two leading bidders in the industry (neither of which were in a candidate cartel) and elicited their beliefs about the distribution of markups in the seal coating industry. The industry sources Bajari and Ye consulted were among the largest firms and between them

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Table 8
Distribution of Percentage Markups of Winning Bids Implied by Models A, B, and C

Percentile	*A: Competitive Model*	*B: Cartel (2,4)*	*C: Cartel (2,5)*
10	1.229	1.273	1.14
20	1.597	1.818	1.82
30	2.077	2.422	2.56
40	2.536	3.201	3.43
50	3.329	4.126	4.47
60	4.227	5.434	5.84
70	5.692	7.54	9.30
80	10.0	16.21	17.56
90	23.81	33.54	58.26

have fifty years of experience in the industry. Since many firms do not survive in this industry, Bajari and Ye argue that the market survival of these sources indicates they probably have good insights into the overall distribution of costs and markups.[19] The two industry insiders had quite similar views, and Bajari and Ye averaged their beliefs to get the following distribution over markups.

Table 9
Beliefs of Industry Sources About the Distribution of Markups

Percentile	*Percentage Markup*
25th Percentile	3%
50th Percentile	5%
75th Percentile	7%
99th Percentile	15%

In the final stage of the statistical horse race, Bajari and Ye used methods from Bayesian statistics to calculate the probabilities that models

[19] Bajari and Ye report that the industry sources, in their own language, seemed to understand many of the intricacies of the Bajari and Ye model of competitive bidding. In fact, they found that the sources had insights into the competitive bidding model garnered through experience in the real world that had escaped the attention of leading economic theorists with whom they had spoken! The bidders had also spent a great deal of time thinking about their competitors' costs and monitoring the prices of inputs because this is an important strategic variable in determining their optimal bid. One of the industry experts with whom they spoke said, "I think some guys in the industry spend more time thinking about their competitors' costs than their own costs!"

A, B, and C generated the distribution of markups in the industry reported by the industry sources. Bayes's theorem states that the probability that a particular model or hypothesis generated a particular set of outcomes can be calculated from the likelihoods that those outcomes would be observed if they really were generated by each model and prior probabilities that each model is true. In this case, generally speaking, Bajari and Ye use their estimated information on the distribution of markups implied by each model to calculate the likelihood that the observed distribution of markups was generated by each of the three models of how bids were formed. This information is then combined with a prior probability for each model (each was assigned an equal prior probability of being true) to calculate the probabilities that each model accurately describes how bids were generated, given the observed distribution of markups.

The results of this analysis indicate that the competitive model has a probability close to 1, given the actual markups in the industry (as reported by industry sources), while the collusive models each had probabilities close to zero. These results should not be surprising in light of the information in Tables 8 and 9. The collusive models can imply huge markups when the cartel is bidding for a project and the overall number of bidders is small. For instance, Table 8 shows that collusive model C implies that on average one would expect 10 percent of the markups to be greater than 58 percent and 20 percent of the markups to exceed 10 percent. In contrast, the industry sources reported that only 1 percent of markups exceed 15 percent. More generally, the information from the industry insiders suggests markups much lower than those predicted by collusive models B and C. The likelihood is small that observed markups would depart this much from the expected distribution if firms were colluding. Comparing Tables 8 and 9, it is clear that the markups implied by competitive model A match the beliefs of industry experts much more closely than those implied by models B and C.

While Bajari and Ye use the beliefs of sources from leading firms to compare the likelihood of various models, other external sources of information on markups or costs may be available. For instance, in the construction industry, sureties (bonding companies) frequently require firms to submit daily profit-and-loss statements. If this information were available to a prosecutor, it could be used to determine a distribution of observed markups that would form the basis for a calculation of model probabilities. Alternatively, consulting engineers are often used to compute a cost estimate for procurement projects. In the road construction industry, the state department of transportation engineers carefully follow industry cost patterns and observe the bidding of these firms

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over many years. These engineers could be a source of information on costs which in turn could be used to decide which model is the most likely.[20]

The tests described in Part II and in Part IV are complementary. The tests in Part II suggested which specific models of collusion, out of the many possible combinations of firms that could act as a cartel, should be compared to the competitive model. Without such screening, there could be too many different collusive models for the statistical horse race to be feasible. The additional test of Part IV complements and checks the tests of conditional independence and exchangeability. Indeed, Bajari and Ye found the competitive model was by far the most likely, even though the bidding patterns of certain pairs of firms failed the statistical tests for conditional independence or exchangeability.[21]

V. CONCLUSION

A major step in curtailing cartel activity is detecting it, but this can be difficult in practice. This paper describes methods for detecting collusion identified by Bajari and Ye. As collusive behavior is a deviation from competitive behavior, their approach is to first understand competitive behavior and its implications. Once characteristics of competitive behavior by firms are clearly identified, it is much easier to detect deviations from this type of behavior among firms.

Bajari and Ye identify two conditions that must exist in the data if the data has been generated by competitive bidding in procurement auctions. These conditions are conditional independence and exchangeability between firms, and the econometrician can test statistically if these conditions are satisfied. If tests indicate that bidding by a group of firms violates these necessary and sufficient conditions, then we know either that the execution of the statistical tests is flawed or that firms have behaved according to some model of behavior other than competition. The potential for flaws in this statistical testing is not different in kind from the potential that always exists when hypotheses are tested statistically, and there are well-established methods for investigating these problems. If the statistical tests are judged reliable, collusion becomes a possible reason for bidding that deviates from the competitive pattern.

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[20] Such a calculation would use estimates of the costs, rather than markups over costs, implied by the various models.

[21] The tests of Part II also force the econometrician to think hard about the industry cost structure and can provide a mechanism for him to learn about cost variables that he might be omitting. The relevant models suggested by the testing in Part II and the relevant cost variables used in Part II are inputs into the test described in Part IV.

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Having identified which firms fail these two tests, Bajari and Ye propose a second analysis to compute the probabilities that observed outcomes in the industry are the result of competitive models of bidding versus collusive models of bidding. Using information from industry sources about patterns of markups or costs across the industry, the econometrician can compute the probability that each of several specific alternative models of behavior is responsible for what "really" happened. One of these models will be a model of competition between all firms. The other models will specify collusive behavior by the particular firms that fail the tests for conditional independence or exchangeability.

While no empirical techniques for detecting collusion are likely to be flawless and complete by themselves, we believe that the approach described in this paper is a useful first step in detecting suspicious bidding behavior. Additional analysis and evidence will be necessary to establish whether suspicious bidding is the result of an illegal agreement or tacit collusion. The approach described in this paper, however, should help identify cases that justify further investigation. In addition, further analysis of the bidding patterns revealed by these tests might itself shed light on the likelihood that they are the result of an agreement rather than independent behavior.

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