

Auctions (applications)

We survey some recent empirical work concerning the analysis of auctions. We begin by describing a two-step nonparametric approach for estimating bidding models that is commonly used in the applied literature. Two applications of this approach are considered: empirical work on bidding in Treasury markets, and empirical tests for collusion in auctions.

In this article, we survey some recently developed methods for the econometric analysis of auction data and related applications. Since the mid-1990s, auctions have been an active area of research in empirical industrial organization. Auctions are an attractive setting for empirically testing game theory, for three reasons. First, real-world auctions have well-defined rules, which often correspond closely to game forms in economic theory. The mapping between the data and economic theory is typically less ambiguous in auctions than in other applications in empirical industrial organization. Second, the theoretical literature on auctions is well developed and offers many testable implications. Third, there are many high quality, easily accessible data sets. For example, detailed data sets from public sector procurements or online auctions can easily be collected from the Internet.

In this survey, we shall describe the estimation strategy proposed in Guerre, Perrigne and Vuong (2000) (henceforth GPV) and two substantive applications. The empirical literature in auctions is diverse. Numerous useful alternative approaches have been proposed, so it is impossible to cover all of them in a short survey. However, the work of GPV and related extensions is widely viewed as one of the most important recent additions to the literature. This survey will omit many of the technical details which are required to correctly implement these estimators. Instead, we discuss the estimators somewhat informally, focusing on what we believe is the key intuition behind these methods. Fortunately, there are several excellent surveys that discuss these estimators and related applications in considerable detail. See, in particular, Athey and Haile (2007), Hendricks and Porter (2007) and Hong and Paarsch (2006).

1. The first-price auction

Following GPV, consider a first-price sealed-bid auction with independent private values. There are $i = 1, \dots, N$ bidders. Bidder i 's valuation for winning the auction is denoted by v_i and is private information. The bidders are symmetric in the sense that each bidder's valuation is an i.i.d. draw from a distribution $F(v)$, which is common knowledge. After learning their valuations, each bidder independently and simultaneously submits a bid b_i . Bidders are risk neutral, and bidder i receives utility $v_i - b_i$ if i is the high bidder and zero otherwise. The equilibrium bid function is symmetric and strictly increasing under fairly mild regularity conditions. Let $b = \mathbf{b}(v)$ denote the equilibrium bid function and $\phi(b) = \mathbf{b}^{-1}(v)$ denote the inverse bid function.

Bidder i 's expected utility from bidding b_i is equal to

$$(v_i - b_i)F(\phi(b_i))^{N-1}. \quad (1)$$

Bidder i wins the auction when the other $N - 1$ bidders bid less than b_i . Bidder $j \neq i$ bids less than b_i when j 's valuation is less than $\phi(b_i)$. The probability of this event is $F(\phi(b_i))$. Therefore the probability that bidders $j \neq i$ bid less than b_i is $F(\phi(b_i))^{N-1}$. Expected utility is the product of the surplus bidder i receives conditional on winning, $(v_i - b_i)$, times the probability that i wins the auction. Given v_i , the first-order condition for utility maximization is

$$(v_i - b_i)(N - 1)f(\phi(b_i))\phi'(b_i) - F(\phi(b_i)) = 0. \quad (2)$$

Suppose that the econometrician observes $t = 1, \dots, T$ independent repetitions of the auction described above. For each auction t , the econometrician observes all of the bids $b_{i,t}$. The object that GPV wish to estimate is the distribution of bidder valuations, $F(v)$. GPV's approach is structural in the sense that they attempt to recover the economic primitives of the model. As we shall discuss in our applications, structural estimation of the model may allow the economist to answer a number of substantive questions. For example, we can assess the efficiency of the observed auction mechanism or test between competing models, such as competition versus collusion).

GPV note that an econometric approach based directly on evaluating eq. (2) may be difficult. This equation involves the inverse bid function, ϕ , and its derivative, ϕ' , which in turn are complicated, nonlinear functions of the unknown $F(v)$. In principle, it is

possible to estimate parametric auction models based on eq. (2), as in Paarsch (1992), Donald and Paarsch (1993), Hong and Shum (2002) and Bajari and Hortaçsu (2003). However, these methods rely on restricting attention to carefully chosen parametric distributions or require the use of reasonably sophisticated numerical methods. (Despite these limitations, it is worth noting that many parametric approaches generate superconsistent estimators, which converge much more quickly than the nonparametric rate of convergence as in GPV. This may be useful when the sample size available to the econometrician is limited. See Donald and Paarsch, 1993, and Hirano and Porter, 2003, for a discussion.)

A key insight of GPV is that the econometric analysis of the first-price auction is greatly simplified by a change of variables. Let $G(b) = F(\phi(b_i))$ denote the equilibrium distribution of the bids. If we substitute $G(b)$ into (1), we can write expected utility as

$$(v_i - b_i)G(b_i)^{N-1}.$$

The first-order conditions now become

$$(v_i - b_i)(N-1)g(b_i) - G(b_i) = 0 \quad (3)$$

$$v_i = b_i + \frac{G(b_i)}{(N-1)g(b_i)}. \quad (4)$$

The right-hand side of (4) involves the bid, b_i , the distribution of the bids, G , and the density of the bids, g . GPV observe that if we have access to a large number of independent repetitions of the same auction, then both G and g can be consistently estimated using standard techniques. Given estimate \hat{G} and \hat{g} of G and g , we can form an estimate $\hat{v}_{i,t}$ of bidder i 's private information $v_{i,t}$ in auction t by evaluating the empirical analogue of equation (4):

$$\hat{v}_{i,t} = b_{i,t} + \frac{\hat{G}(b_{i,t})}{(N-1)\hat{g}(b_{i,t})}. \quad (5)$$

To summarize, the estimator proposed by GPV is as follows:

1. Given bids $b_{i,t}$ for $i = 1, \dots, N$ and $t = 1, \dots, T$, estimate the distribution and density of bids $\hat{G}(b)$ and $\hat{g}(b)$.

2. Compute $\hat{v}_{i,t}$ for $i = 1, \dots, N$ and $t = 1, \dots, T$ using eq. (5). Use the empirical cdf of the $\hat{v}_{i,t}$ to estimate F .

This procedure is attractive for three reasons. First, it does not impose parametric assumptions on F during estimation. Since the economist is likely to have poor a priori information about the distribution of values, this is desirable for empirical work. Second, the procedure described above is computationally simple to implement since it does not require evaluation of ϕ and ϕ' . Finally, it is possible to demonstrate that $F(v)$ is nonparametrically identified. The intuition is quite simple. As T grows arbitrarily large, the economist will be able to estimate G and g very precisely under standard regularity conditions. Equation (4) implies that for any given bid b_i we can recover the latent valuation v_i that generates this bid (that is, $v_i = \phi(b_i)$). Since the distribution of b_i is known, it can easily be demonstrated that $F(v)$ is therefore identified.

GPV also demonstrate that the first-price auction model can be tested. Given estimates \hat{G} and \hat{g} , define $\xi(b)$ as

$$\xi(b) = b + \frac{\hat{G}(b)}{(N-1)\hat{g}(b)}.$$

Theoretical models of bidding imply that the bid function should be increasing, that is, bidders with higher valuations should submit higher bids. Therefore, if $\hat{G}(b)/(N-1)\hat{g}(b)$ is sufficiently close to $G(b)/(N-1)g(b)$, $\xi(b)$ should be monotonically increasing if the model is correctly specified. This prediction of the theory could be rejected by the data since \hat{G} and \hat{g} are estimated nonparametrically and do not impose a priori that $\xi(b)$ is increasing.

2. Generalizations and applications

Following GPV, a large number of authors have proposed similar estimators for other auction models. In these papers, a key step is typically to rewrite the first-order conditions in terms of the equilibrium distribution of the bids (for example G and g). Next, as in eq. (4), the economist attempts to isolate private information on the left-hand

side as a function of the bids on the right-hand side. Following GPV, the economist then nonparametrically estimates the distribution of the bids from the data and recovers the latent private information by evaluating the empirical analogue of the first-order condition.

This basic algorithm often needs to be modified for different auctions. However, attempting to follow these steps as a first pass will typically take the economist a long way towards deriving an estimator. Listed in Table 1, in alphabetical order, are some recent papers which build on the insights of GPV in other auction models.

Table 1 Related papers

Paper	Topic
Athey and Haile (2002)	Identification in auctions
Bajari and Ye (2003)	First-price auctions with collusion
Brendstrup and Paarsch (2003)	Dutch and first-price auctions with asymmetric bidders
Campo et al. (2002)	Auctions with risk aversion
Campo, Perrigne and Vuong (2003)	Asymmetric first-price auctions with affiliated values
Flambard and Perrigne (2006)	Asymmetric first-price auctions
Hendricks, Pinkse and Porter (2003)	Common value auction models
Hortaçsu (2002)	Treasury auctions
Li and Perrigne (2003)	Random reserve prices
Li, Perrigne and Vuong (2002)	Affiliated private values
Pesendorfer and Jofre-Bonet (2003)	Dynamic first-price auctions

Next, in order to illustrate how these techniques are used in practice, we briefly summarize Hortaçsu (2002) who analyses bidding in Treasury bill auctions, and Bajari and Ye (2003) who test for collusion in procurement auctions.

2.1. Auctions for Treasury bills

Hortaçsu (2002) asks how governments should conduct auctions for Treasury bills. Treasury bill auctions are an example of a multiple unit auction since large numbers of T-bills are typically sold during a single auction. Since there are multiple units, a ‘bid’ in a Treasury auction is a demand curve, instead of a scalar as in the example of Section 1.

Two commonly used mechanisms for conducting a Treasury bill auction are the uniform price auction and the discriminatory auction. In a uniform price auction, the auctioneer begins by aggregating all of the individual demand curves into a market demand curve. The supply curve is vertical, with an intercept equal to the number of T-bills that the government wishes to sell. The market-clearing price is determined by the intersection of the supply and demand curve. Each bidder pays his demanded quantity at the market-clearing price, analogous to a competitive market. By contrast, in a discriminatory auction, the intersection of the supply and market demand curves determines the price for the last unit purchased. Analogous to first-degree price discrimination, bidder i pays the area under his demand curve, so that the price for the first unit purchased will be higher than for the last unit purchased.

There is no general consensus about which auction mechanism should be preferred. Since the equilibria to these auctions are quite complicated, it is difficult to characterize revenue in each auction. Each year, nearly \$4 trillion dollars of securities are sold in T-bill auctions. Given the size of these markets, econometrically modelling the determination of the bids and comparing revenue from alternative auction mechanisms is an interesting public policy question.

The particular market that Hortaçsu examines is the short-term (13-week) market for T-Bills in Turkey. This market is run using a discriminatory auction. Hortaçsu uses the Wilson (1979) auction of shares model as a starting point for his econometric analysis. He assumes that bidders have private values. According to surveys of bidders, 42 per cent purchases in the auctions are to meet reserve requirements imposed by the Turkish Central bank. Thirty-seven per cent of purchases are for resale in the secondary market. Ten per cent are to fulfil customer orders and ten per cent are to fulfil collateral requirements, for investment funds administered by the bank, and for buy-and-hold purposes. Other than those shares purchased for resale, the other sources of demand are probably best modelled as private values.

Let s_i denote bidder i 's private information about her willingness to pay for government debt and $v_i(q, s_i)$ denote bidder i 's valuation for the q th unit. Assume that private information is distributed i.i.d. $s_i \sim F(s)$. Let $y_i(p)$ denote the demand curve submitted by bidder i . Hortaçsu assumes that $y_i(p)$ is strictly decreasing and

differentiable. If there are N bidders and Q units of debt for sale, the market-clearing price p^c will satisfy

$$Q = \sum_i y_i(p^c).$$

The cdf of the market-clearing price, conditional on i 's bid function $y_i(p)$ is

$$\begin{aligned} H(p, y_i(p)) &= \Pr \left\{ y_i(p) \leq Q - \sum_{j \neq i} y_j(p) \right\} \\ &= \Pr \{ p^c \leq p \mid y_i(p) \}. \end{aligned} \quad (6)$$

Equation (6) is analogous to a residual supply curve. The term $H(p, y_i(p))$ is the probability that the market-clearing price will be less than p given i 's own bid, $y_i(p)$. However, unlike a residual supply curve in a model with certainty, the bidder has to take into account her uncertainty about the bids of others.

Given a bid $y_i(p)$, the surplus that a bidder gets, conditional on p^c is equal to

$$\int_0^{y_i(p^c)} v_i(q, s_i) dq - \int_0^{y_i(p^c)} y_i^{-1}(q) dq.$$

There are two terms in the above sum. The first term is the integral of $v_i(q, s_i)$ from 0 to $y_i(p^c)$. This is bidder i 's valuation for the units that she wins. The second term is the integral of i 's inverse demand curve. This determines the total payment that i just made for the units that she won. Therefore, i 's expected profit from submitting a bid of $y_i(p)$ is equal to

$$\int_0^\infty \left\{ \int_0^{y_i(p^c)} \{v_i(q, s_i) - y_i^{-1}(q)\} dq \right\} dH(p^c, y_i(p)).$$

Following Wilson (1979), the first-order condition for maximization implies that

$$v_i(y_i(p), s_i) = p + \frac{H(p, y_i(p))}{\frac{\partial}{\partial p} H(p, y_i(p))}. \quad (7)$$

That is, a bidder's valuation will be equal to the price on the submitted demand curve plus a bid-shading factor, $H(p, y_i(p)) / (\partial / \partial p) H(p, y_i(p))$. Just as in the first-price auction example in Section 1, Hortaçsu notes that $H(p, y_i(p))$ is the cdf of the equilibrium distribution of bids given $y_i(p)$. Given a large number of repetitions of the same, or similar auctions, this object can be estimated from the observed bidding data. And, similar to the first-price auction example above, an estimate of bidder i 's valuation,

$v_i(y_i(p), s_i)$ can be recovered by evaluating the empirical analogue of (7). While the econometric details are somewhat involved, a key economic insight was expressing the first-order conditions in terms of a function of the bids which, in principle, can be recovered from the data.

Using his estimates of bidder valuations, Hortaçsu examines two applied questions. The first is to explore the impact of reserve requirements on bidding behaviour. He constructs a variable, $\%SHORTFALL_{i,t-1}$, which is the fraction of orders in the previous Treasury auction that were unfulfilled. He finds that when bidders have a large shortfall in previous auctions, they are more likely to bid aggressively in upcoming auctions. Using his survey on bidder demands, he interprets this as derived demand from satisfying reserve requirements to hold a required portfolio of Turkish Treasury notes. For instance, he finds that the R^2 of a regression of the intercept of the submitted bid function on $\%SHORTFALL_{i,t-1}$, $\%SHORTFALL_{i,t-2}$ and an auction fixed effect is 0.61. Bidder-fixed effects only increase R^2 to 0.64.

A second applied question Hortaçsu examines is whether a uniform price auction would generate increased revenue. This is complicated to answer since changing to a uniform price auction would generate an entirely new equilibrium in this market. However, Hortaçsu demonstrates that it is possible to construct a simple upper bound on revenue given estimates of $v_i(q, s_i)$ for $i = 1, \dots, N$. Since bidders typically engage in demand reduction in a uniform price auction, they will bid at most $v_i(q, s_i)$ so that $v_i(q, s_i)$ is an upper bound on i 's bid. Assuming that this upper bound is binding for all bidders, he generates an upper bound on the market-clearing price in the auction. Using his structural estimates, Hortaçsu finds that switching to a uniform price auction would generate a revenue loss of at least 3.8 per cent on average in the auctions in his sample.

Hortaçsu therefore argues that the discriminatory price auction generates higher revenue since bidders are being forced to pay the area under their demand curves. Even after accounting for changes in the strategic incentives to shade bids, discriminatory auctions generate more revenue. However, this conclusion is subtle. Recall that bids are the steepest when shortfalls are the highest. It is hard to argue that forcing banks to hold Turkish Treasury debt is optimal for securing deposits. More likely, this policy was implemented in order to guarantee that there is a constant demand for government debt

even if the government engages in irresponsible fiscal or monetary policies. These results suggest that the reserve requirements plus the discriminatory mechanism may be imposing a burden on the banking sector by forcing banks to hold more than the optimal number of domestic T-bills.

2.2. Collusion application

Next, we briefly discuss an application by Bajari and Ye (2003) that tests for collusive bidding behaviour in procurement auctions. Bid rigging is an important antitrust problem. For instance, Pesendorfer (2000) notes that 55 per cent of the criminal antitrust cases filed by the US Department of Justice involved bid rigging. One well-known example of bid rigging was the ‘concrete club’ in New York where organized crime figures placed an implicit ‘tax’ of two per cent on every ton of concrete used in certain construction jobs in the 1980s. However, the costs of collusion were likely much larger than two per cent. Mafia informer Sammy ‘The Bull’ Gravano, who was involved in bid-rigging in the concrete industry, stated ‘If one of them (contractor) gets a contract for, say, thirteen million, the next thing you know, after he knows he’s got it, he jacks up the whole thing before it’s over to a sixteen- or seventeen-million-dollar job. Now he’s increased the cost thirty-three percent. So our greed (the Mafia) is compounded by the greed of them so-called legitimate guys (contractors)’ (Maas, 1997, p. 271).

While bid-rigging is an important antitrust problem, it can be difficult to detect. Bajari and Ye (2003), expanding on the methods in Section 1, and on the work of Porter and Zona (1993, 1999), propose three statistical tests that can be used to potentially detect bid rigging in procurement auctions. Certainly, no test for bid-rigging can hope to be foolproof. However, it may be a basis for determining which sets of bids are most worrisome and whether further investigation of certain firms is warranted.

Bajari and Ye apply their methods to a set of contracts in the highway construction industry for ‘seal coating’ jobs in Minnesota, North Dakota and South Dakota. Seal coating is a type of highway repair that attempts to extend the life of the road by sealing surface cracks. The surface of the highway is initially sprayed with a coating of oil. Next, a ‘chip spreader’ distributes a uniform layer of sand and aggregate on the road. Finally, rollers are used to bind the oil, sand and aggregate. Bidding is conducted using sealed

bids. While there are a large number of fringe firms in the industry, the market is dominated by a few large bidders that regularly compete against each other. Since all of the bids are publicly available shortly after they are submitted, collusion has occurred in seal coating in many markets. Bajari and Ye note that three of the largest bidders in their data have been fined for previous attempts to rig bids. The owner of the largest firm in the data set served prison time for a bid rigging conviction.

Bajari and Ye consider a first-price auction model similar to the example discussed in Section 1. However, they drop the assumption that all bidders are *ex ante* identical. In the construction industry, they argue it is important to allow for asymmetric bidders for three reasons. First, transportation costs are substantial in this market so that firms located closest to the project will tend to have lower cost. Second, there is a skewed size distribution of firms in the industry. Therefore, it is important to allow for firm specific difference in productivity. Third, project backlog increases the opportunity cost of taking on additional work and is likely therefore to be an additional source of *ex ante* asymmetries.

In the model, N firms compete for a contract to build a single and indivisible public works project. Firm i 's cost to complete the project, c_i , is a random variable with cumulative distribution function $F_i(\cdot; z_i; \theta_i)$ and probability density function $f_i(\cdot; z_i; \theta_i)$. Here z_i reflects publicly observed cost shifters from firm i . For instance, in the application, these include distance to the project, a firm fixed effect to capture differences in productivity, backlog at the time bids are submitted and an engineering cost estimate. The term θ_i is a set of firm specific parameters. In the model, firm i is risk neutral and has profits of $b_i - c_i$ if it is the low bidder and zero otherwise.

Let $G_i(b; z)$ be the equilibrium distribution of bids submitted by firm i . Note that the distribution of the bids depends on $z = (z_1, \dots, z_N)$, the publicly observed information for all firms in the industry. Then i 's expected profits from submitting a bid of b_i when i 's costs are c_i is equal to

$$(b_i - c_i) \prod_{j \neq i} (1 - G_j(b_i; z)) \quad (8)$$

It can easily be shown that the first-order condition to the model must satisfy

$$c_i = b_i - \left[\sum_{j \neq i} \frac{g_j(b_i; z)}{1 - G_j(b_i; z)} \right]. \quad (9)$$

As in Section 1, if the economist has estimates of \hat{G}_i and \hat{g}_i , it is possible to generate an estimate of c_i by evaluating the empirical analogue of the above equation for all bidders in the sample.

Bajari and Ye (2003) propose three tests for collusive bidding. We next describe the basic spirit of these tests, referring the interested reader to the text for complete details. The first test for competitive bidding is that conditional on z , the bids of all firms $I = 1, \dots, N$ must be distributed independently. This is a fairly robust prediction of the theory of competitive bidding and is in fact more general than the particular model described above. Because bidders have private information which is independently distributed, their bids, which are a deterministic function of this private information, must also be independently distributed. Obviously, one limitation of such a test is if some component of z is observed by the firms, but not by the econometrician. Following Porter and Zona (1993; 1999), their estimation strategy allows for the inclusion of an auction-specific fixed effect. Thus, they control for project specific cost shifters which are common to all of the firms.

Second, they demonstrate that the equilibrium distribution of competitive bids must be exchangeable. Let π be a permutation of the bidder identities $\{1, \dots, N\}$, that is, a one-to-one map from $\{1, \dots, N\}$ to $\{1, \dots, N\}$. If the equilibrium bid function is unique, the bid distribution must be exchangeable: that is,

$G_i(b; z_1, z_2, z_3, \dots, z_N) = G_{\pi(i)}(b; z_{\pi(1)}, z_{\pi(2)}, z_{\pi(3)}, \dots, z_{\pi(N)})$. In words, exchangeability means that if you permute the cost shifters of all the bidders, then the equilibrium bids must also permute in a symmetric fashion. Conditional independence and exchangeability are necessary for equilibrium bidding. If other regularity conditions hold, conditional independence and exchangeability are also sufficient for competitive bidding: that is, the economist can reverse engineer a competitive bidding model that rationalizes the observed bids.

Porter and Zona (1993, 1999) study the bidding behaviour of known cartels in construction and in the supply of school milk. Many of the irregular patterns of bidding

that they describe can be characterized as failures of conditional independence and exchangeability. For instance, the bids of cartel members are more correlated with each other than with non-cartel members. Also, cartel members do not shift their bids aggressively in response to shifts in the z_i of other cartel members which is a failure of exchangeability.

Bajari and Ye (2003) test for conditional independence and exchangeability in their data set. Given the limited number of observations available to them, they test these conditions in a regression framework. Essentially, they run a regression of b_i on z_i and z_{-i} , including auction fixed effects and bidder fixed effects. Conditional independence is tested by asking whether the fitted residuals from bidder i 's bid function is correlated with the fitted residuals from $j \neq i$'s bid function. Exchangeability is formulated as a test of the equality of certain regression coefficients. In total, 46 separate hypothesis tests are conducted. Forty-one of these tests are consistent with the implications of competitive bidding (that is, conditional independence and exchangeability). Therefore, they argue that most of the bids in the market appear to be competitive. However, reduced form tests suggest that bidding by two coalitions of firms appear to be suspicious. They label these coalitions 'candidate cartels'. Interestingly, all of the members of the candidate cartels were previously been convicted for bid rigging.

The third and final test for bid rigging uses structural estimates based on eq. (9). Bajari and Ye consider a non-nested hypothesis test between three models. Model M1 is that the data-generating process is the no collusion model. Model M2 is that the first candidate cartel engaged in efficient collusion, but that other firms in the industry are competitive. Model M3 is that the second candidate cartel engages in bid rigging. The costs c_i can be estimated under each of these three alternatives using the empirical analogue of (9). The different models generate different first-order conditions and hence, different estimated costs, c_i .

Bajari and Ye then ask which set of markups is 'most reasonable'. To answer this question, they consulted with two managers at one of the biggest firms in this market (which was not in a candidate cartel). From each manager, they elicited their beliefs about the distribution of markups in this industry. Bajari and Ye argue that is reasonable to suppose that these managers have informative priors about markups for two reasons.

First, all bidders in this industry must be bonded. The bonding companies are contractually liable to complete the project if the contractors go bankrupt. Contractors are typically required to give weekly profit and loss statements to the bonding companies. The bonding companies are therefore well informed about profit margins for firms in the industry. Profit margins in the industry are a common topic of conversation between contractors and bonding companies and are one source of information.

Second, the contractors in this industry compete against each other quite frequently and over many years. The contractors have access to similar cost information and study the bids of competing contractors in detail after the bids are publicly opened. Given that contractors closely follow cost conditions and bids in the industry, they will have a lot of information about their competitors' markups. There is an issue, of course, about whether the contractors would lie about their beliefs. However, Bajari and Ye shared their estimates with the contractor, which included empirical analysis of the behaviour of competing firms. Lying about the industry would reduce the value of these estimates. Also, the information from the contractor that was verifiable from external sources about the industry did seem to be accurately reported.

The stated beliefs of the experts were quite close. Below, we average the elicited beliefs from the contractors:

$$\begin{aligned}
 &25\text{th percentile} = 3\% \\
 &50\text{th percentile} = 5\% \\
 &75\text{th percentile} = 7\% \\
 &99\text{th percentile} = 15\%.
 \end{aligned}
 \tag{10}$$

For example, the 25th percentile of the bids has a markup of three per cent and the median bid has a markup of five per cent.

Table 2 shows the estimated distribution of markups from the three alternative structural models, M1, M2 and M3.

Table 2 Distribution of markups under alternative models

Percentile	M1	M2	M3
10	0.01229	0.01273	0.0114
20	0.01597	0.01818	0.0182

30	0.02077	0.02422	0.0256
40	0.02536	0.03201	0.0343
50	0.03329	0.04126	0.0447
60	0.04227	0.05434	0.0584
70	0.05692	0.0754	0.0930
80	0.1000	0.1621	0.1756
90	0.2381	0.3354	0.5826

Bajari and Ye note that the markups under M1 (competitive bidding) correspond most closely to the elicited prior beliefs. The markups under models M2 and M3 seem to be too large, particularly on the tails. They argue that this is evidence against the collusive models since they generate markups that seem implausibly large compared to the beliefs of an informed party. Bajari and Ye formalize this intuition by posing the selection of M1, M2 or M3 as a problem in statistical decision theory. As the table above suggests, the competitive model M1 is most favoured. Therefore, they cautiously interpret the data as being consistent overall with non-collusive behaviour.

3. Conclusion

In this short survey, I have attempted to provide an overview of recent empirical papers concerning auctions. Many recent papers build on the pioneering work of Guerre, Perrigne and Vuong (2000). A key insight of this paper was that a first-price sealed-bid auction model can be simply estimated using a two-step procedure. In the first step, the economist flexibly estimates the empirical distribution of the bids. In the second step, the economist evaluates the empirical analogue of the first-order condition for utility maximization. The method of Guerre, Perrigne and Vuong estimates the structural primitives of the model without imposing ad hoc parametric restrictions. We also discussed two applications of these recently developed estimators. Hortaçsu (2002) studied bidding in Treasury auctions in Turkey. His model predicted that discriminatory auctions generate higher revenue than uniform price auctions. Bajari and Ye (2003) applied these methods to test for collusion in sealed-bid auctions. They applied these

methods to searching for suspicious bidding patterns in a market where the largest firms had recently been sanctioned for collusion.

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See also auctions (theory); auctions (empirics); auctions (experiments); cartels; game theory; games of incomplete information; industrial organization (empirical studies); nonparametric structural models

Bibliography

Athey, S. and Haile, P. 2002. Identification of standard auction models. *Econometrica* 70, 2107–40.

Athey, S. and Haile, P. 2007. Nonparametric approaches to auctions. In *Handbook of Econometrics*, vol. 6, ed. J. J. Heckman and E. E. Leamer. Amsterdam: North-Holland.

Bajari, P. and Hortacısu, A. 2003. The winner's curse, reserve prices, and endogenous entry: empirical insights from eBay auctions. *RAND Journal of Economics* 34, 329–55.

Bajari, P. and Ye, L. 2003. Deciding between competition and collusion. *Review of Economics and Statistics* 85, 971–89.

Brendstrup, B. and Paarsch, H. 2003. Nonparametric estimation of Dutch and first-price, sealed-bid auction models with asymmetric bidders. Working paper, University of Iowa.

Campo, S. E., Guerre, I., Perrigne and Vuong, Q. 2002. Semiparametric estimation of first-price auctions with risk averse bidders. Working paper, Pennsylvania State University.

Campo, S., Perrigne, I. and Vuong, Q. 2003. Asymmetry in first-price auctions with affiliated private values. *Journal of Applied Econometrics* 18, 197–207.

Donald, S. and Paarsch, H. 1993. Piecewise pseudo-maximum likelihood estimation in empirical models of auctions. *International Economic Review* 34, 121–48.

Flambard, V. and Perrigne, I. 2006. Asymmetry in procurement auctions: evidence from snow removal contracts. [*Economic Journal*](#) 116, 1014–36.

Guerre, E., Perrigne, I. and Vuong, Q. 2000. Optimal nonparametric estimation of first-price auctions. *Econometrica* 68, 525–74.

- Hendricks, K., Pinkse, J. and Porter, R. 2003. Empirical implications of equilibrium bidding in first-price, symmetric, common value auctions. *Review of Economic Studies* 70, 115–45.
- Hirano, K. and Porter, J. 2003. Asymptotic efficiency in parametric structural models with parameter dependent support. *Econometrica* 71, 1307–38.
- Hendricks, K. and Porter, R. 2007. Lectures on auctions: an empirical perspective. In *Handbook of Industrial Organization*, vol. 3, ed. M. Armstrong and R. Porter. Amsterdam: Elsevier.
- Hong, H. and Paarsch, H. J. 2006. *An Introduction to the Structural Econometrics of Auction Data*. Cambridge, MA: MIT Press.
- Hong, H. and Shum, M. 2002. Increasing competition and the winner’s curse: evidence from procurement. *Review of Economic Studies* 69, 871–98.
- Hortaçsu, A. 2002. Mechanism choice and strategic bidding in divisible good auctions: an empirical analysis of the Turkish treasury auction market. Working paper, University of Chicago.
- Li, T. and Perrigne, I. 2003. Timber sale auctions with a random reserve price. *Review of Economics and Statistics* 85, 189–200.
- Li, T., Perrigne, I. and Vuong, Q. 2002. Structural estimation of the affiliated private value auction model. *RAND Journal of Economics* 33, 171–93.
- Maas, P. 1997. *Underboss: Sammy the Bull Gravano’s Story of Life in the Mafia*. New York: HarperCollins.
- Paarsch, H. 1992. Deciding between common and private values paradigms in empirical models of auctions. *Journal of Econometrics* 51, 191–215.
- Pesendorfer, M. 2000. A study of collusion in first-price auctions. *Review of Economic Studies* 67, 381–411.
- Pesendorfer, M. and Jofre-Bonet, M. 2003. Estimation of a dynamic auction game. *Econometrica* 71, 1443–89.
- Porter, R. and Zona, J. D. 1993. Detection of bid rigging in procurement auctions. *Journal of Political Economy* 101, 518–38.
- Porter, R. and Zona, J. D. 1999. Ohio school milk markets: an analysis of bidding. *RAND Journal of Economics* 30, 263–88.

Wilson, R. 1979. Auctions of shares. *Quarterly Journal of Economics* 93, 675–89.

Index terms

auctions

bid rigging

cartels

collusion

conditional independence

discriminatory auctions

exchangeability

first-price auctions

sealed-bid auctions

statistical decision theory

structural estimation

uniform price auctions