ANSWER KEY

Due Monday, Sep 28

Before you begin writing your answers make sure that you read and fully understand the assignment rules.

Assignment rules:

1. Homework assignments must be submitted at the beginning of the lecture, in class, on the listed dates.
2. Late homework assignments will not be accepted under any circumstances.
3. Submitted assignments must be stapled. Non-stapled assignments will receive a maximum grade of 75%. A non-stapled homework will be graded as normal and then the grade will be multiplied by 75%.
4. Assignments must be typed, except graphs and formulas or they will receive a maximum grade of 75%. Otherwise, the homework will be graded as normal and then the grade will be multiplied by 75%.
5. In all the questions you are required to show the way you got to the solution. Presenting only the final answer, without showing your work will result in zero points for that question, even if the answer is correct.
6. In order to receive full credit, all diagrams must be fully labeled. That is, you are required to give a title to each diagram and label the axis.
7. You are encouraged to collaborate on the homework assignments, but submitted work must be your own. This means that you should write your answers independently. On each assignment you are required to note the names of your collaborators. NOTE: Turning in identical homeworks will result in a zero for both parties.
8. No emailed homeworks will be accepted unless prior authorization is acquired from the instructor
9. We all make mistakes (yes, even the instructors). For each substantive mistake found in this homework, a student first to report it will be rewarded with 5 points on top of his/her score for the homework (out of 100).

Total 100 possible pts

Data Analysis [30 pts]

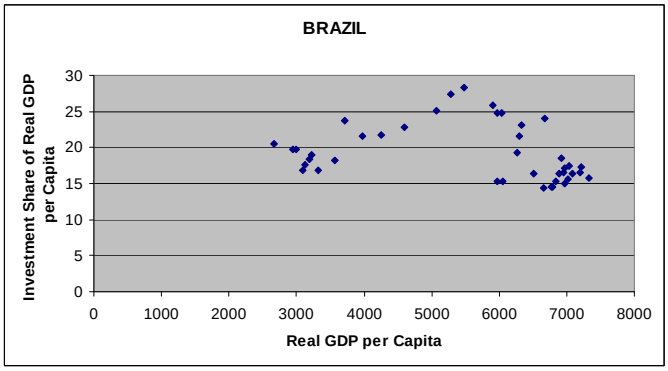
1. (15 pts) Go to the Penn World Table (PWT) website. Consider the following countries: France, Japan, Ecuador, Brazil, Chad, and Kenya. For each country determine the following between 1960 and 2000 (you do *not* need to submit your data):

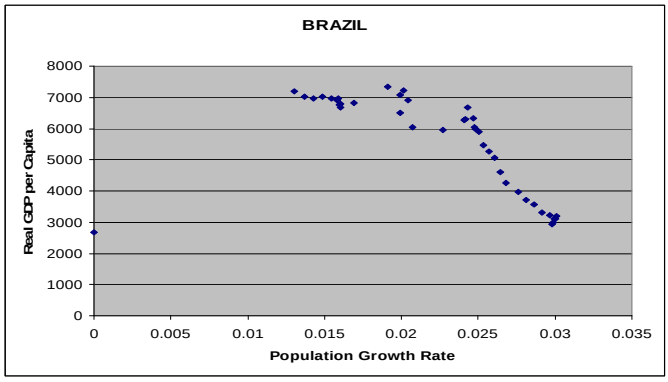
   - real GDP per capita (using "Constant Prices: Laspeyres" series)
   - investment share of real GDP per capita
   - population growth rate (you will need to calculate this yourself)

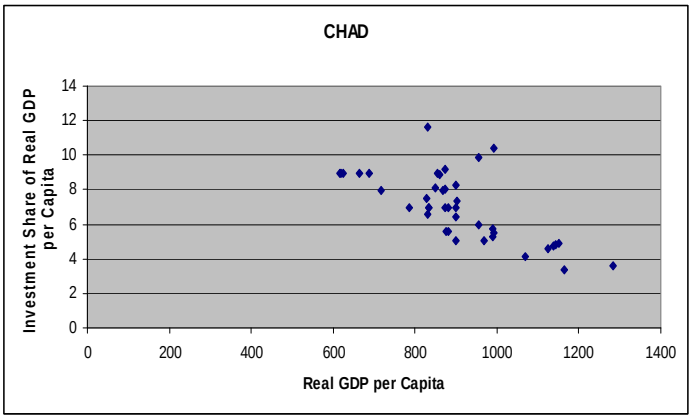
   a. (12 pts) For each country, plot the following graphs:

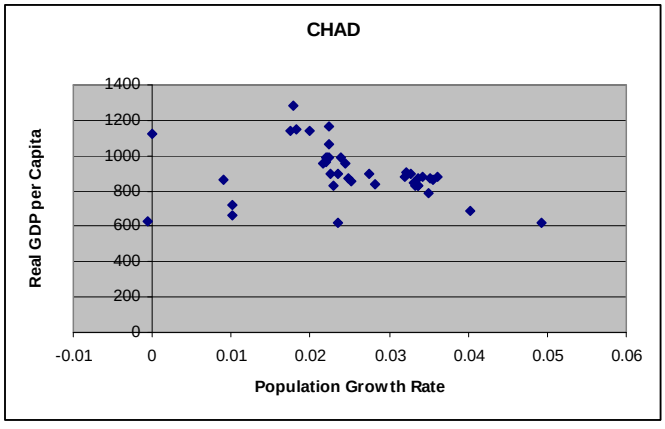
   - Real GDP per capita against investment share
   - Real GDP per capita against population growth rate

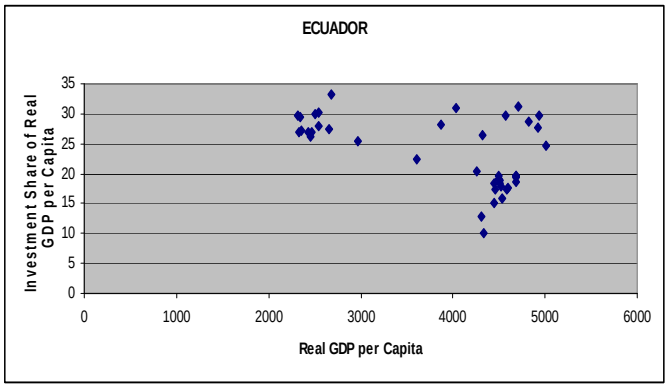
   Note: You should have 12 graphs in total. Submit your graphs (be sure to label).

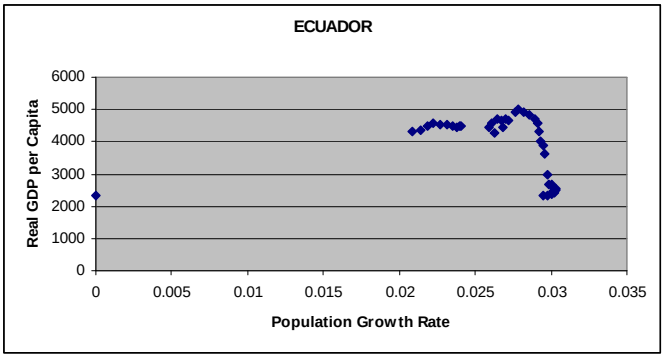


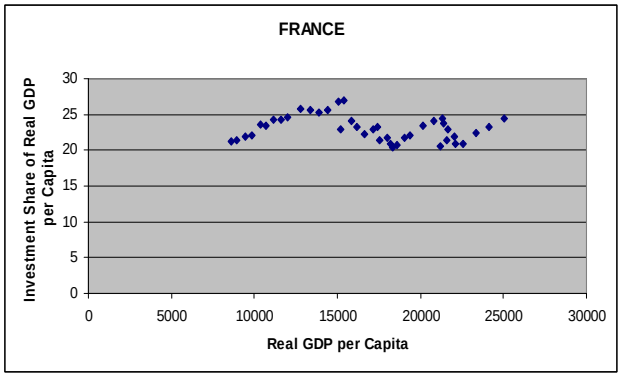


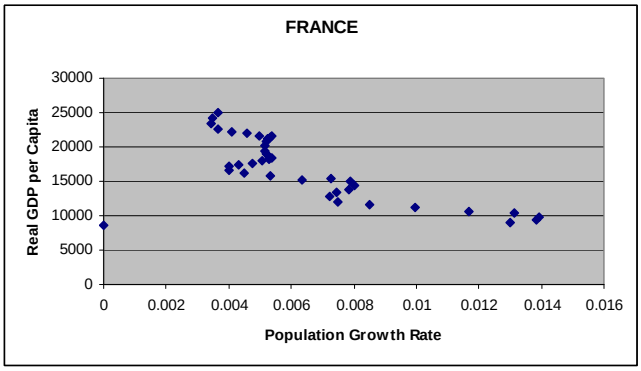


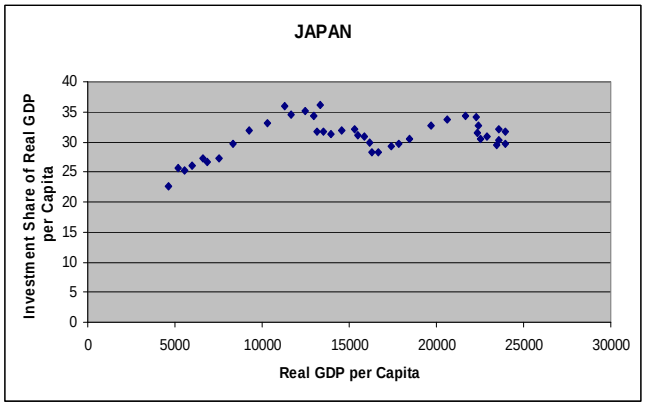


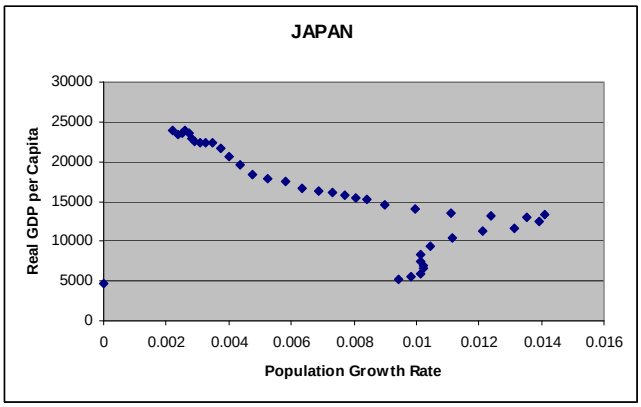

ECUADOR
Investment Share of Real GDP per Capita
0
5
10
15
20
25
30
35
0
1000
2000
3000
4000
5000
6000
Real GDP per Capita

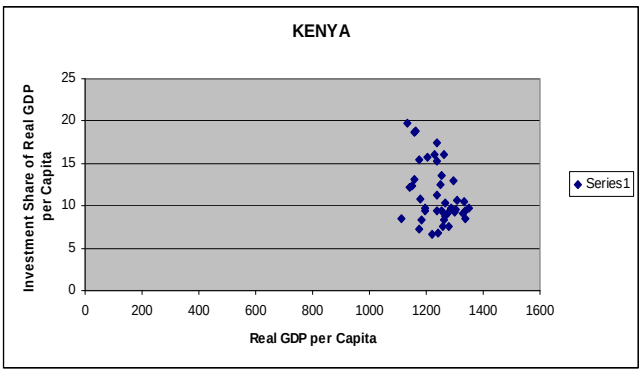

ECUADOR
Real GDP per Capita
0
1000
2000
3000
4000
5000
6000
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
Population Growth Rate

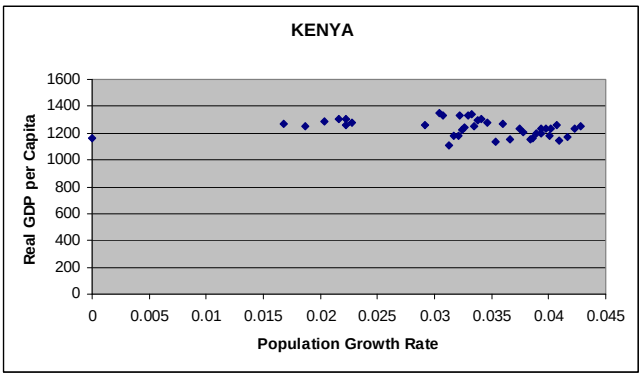

FRANCE
Investment Share of Real GDP per Capita
0
5
10
15
20
25
30
0
5000
10000
15000
20000
25000
30000
Real GDP per Capita


FRANCE
Real GDP per Capita
0
5000
10000
15000
20000
25000
30000
0
0.002
0.004
0.006
0.008
0.01
0.012
0.014
0.016
Population Growth Rate


JAPAN
Investment Share of Real GDP per Capita
0
5
10
15
20
25
30
35
40
0
5000
10000
15000
20000
25000
30000
Real GDP per Capita


JAPAN
Real GDP per Capita
0
5000
10000
15000
20000
25000
30000
0
0.002
0.004
0.006
0.008
0.01
0.012
0.014
0.016
Population Growth Rate


KENYA
Investment Share of Real GDP per Capita
0
5
10
15
20
25
0
200
400
600
800
1000
1200
1400
1600
Real GDP per Capita
Series1


KENYA
Real GDP per Capita
0
200
400
600
800
1000
1200
1400
1600
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
0.045
Population Growth Rate

b. (3 pts) What can you observe in the data and graphs about the relationship between real GDP per capita and investment? What about between real GDP per capita and population growth rate? Are the relationships you observe consistent with the Solow Model? Explain.

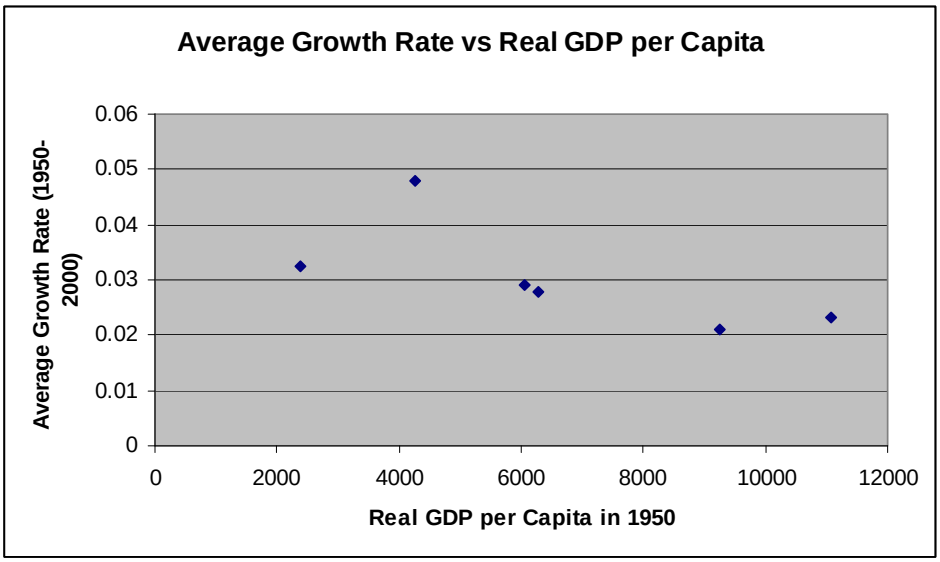
Ans: From the graphs, we can observe a roughly positive relationship between real GDP per capita and investment share, and a negative relationship between real GDP per capita and population growth rate. These observations are intuitive. With a higher investment share of output, one would expect to see more products being produced, and hence a higher GDP, and therefore, a higher real GDP per capita. On the other hand, with a higher population growth rate, more people have to share the GDP, so the real GDP per capita tends to fall.

2. (15 pts) Go to the Penn World Table (PWT) website. Consider the following countries: Japan, USA, Italy, Australia, France, and Belgium.

a. (6 pts) For each country calculate the growth rate of real GDP per capita from 1950 to 2000. Then, for each country, find the average growth rate for that period (you do *not* need to submit your data). Fill in the following table:

Country	**Real GDP per Capita in 1950**	**Average Growth Rate (1950-2000)**
Australia	9261.068	0.021
Belgium	6290.08	0.028
France	6042.439	0.029
Japan	4273.952	0.048
Italy	2391.685	0.032
USA	11086.6	0.023

b. (4 pts) On one graph, plot the average growth rates of these countries against the levels of real GDP per capita in 1950. Submit the graph. Clearly label your graph.



c. **(**5 pts**)** If the graph in part (b) were your only evidence, what would your conjecture be as to why some countries in the sample could grow faster than others? Explain. Is your observation consistent with the Solow Model?

Ans: From the graph, we can observe a roughly negative relationship between the growth rate of real GDP per capita and the per capita GDP in 1950. Notice that the countries we chose are mostly developed countries, i.e. they have similar patterns of investment, population growth, etc. Therefore, in terms of the Solow model, these countries should have roughly the same steady states.

Why do some countries grow faster than the other? Not all countries start at the same capital per capita level. Those countries that start with a lower capital per capita would grow faster, according to the Solow model, at least in the sample of developed countries.

Basic Solow Model [70 pts]

3. (20 pts) Consider a country where the savings rate is 16%, while the capital income share is 1/3. Take the depreciation rate to be 1% and the population growth rate to be 3%. Answer the following questions:

a. (10 pts) Derive the equation for the steady state level of per capita GDP in this economy. Calculate it with the parameters given.

Ans: For detailed derivation, refer to your notes. The equation for the steady state level of per capita GDP is given by:

$$y_{ss} = \left[s / (\delta + n) \right]^{\alpha/(1-\alpha)}$$

For the given parameters, note first that $\alpha/(1-\alpha) = 1/2$. Therefore, the steady state per capita output $y_{ss} = \text{sqrt}(0.16/0.04) = 2$

b. (5 pts) According to the basic Solow Model, what would be the savings rate needed for the country to double its steady state level of per capita income?

Ans: With population and depreciation rates unchanged, this economy would have to increase its savings rate to 64 percent to double its standards of living. This tells you that to account for the wide variation in per capita incomes across countries, savings rates cannot be the sole explanation (at least in the basic Solow Model). To formally derive the result, doubling per capita income would mean setting $y_{ss} = 4 = \text{sqrt}(s^*/0.04)$. Squaring both sides, $16 = s^*/0.04$ or $s^* = 0.64$

c. (5 pts) According to the basic Solow Model, what would be the population growth rate required to double the country's steady state level of per capita income?

Ans: Holding the other parameters fixed, Solow Model suggests that having zero population growth would help the country double its standards of living. Formally, $y_{ss} = 4 = \text{sqrt}[0.16/(0.01+n^*)]$. Squaring both sides, $16 = 0.16/(0.01+n^*)$ or $n^* = 0$.

4. (30 pts) Comparative statics: Consider two countries with the same population growth rate, the same savings rate, the same factor income shares, and the same rate of depreciation.

a. (10 pts) If country A is currently richer (i.e. has a higher level of capital per worker) than country B, which country has a higher growth rate of GDP per capita? Explain.

Ans: First, recall that the growth rate in GDP per capita is given by:

$$g_y = \alpha\, g_k$$

Solow Model tells us that the further away a country is from its steady state, the higher will be the growth rate in capital per worker since:

$$g_k = sk^{\alpha-1} - (n+\delta)$$

is a decreasing function of k. When a country is poorer (has lower k), the growth rate in capital per worker (hence in output per worker) is higher, i.e. country A will grow slower than country B. Note that this assumes that both countries are below their steady state levels of capital per worker.

On the other hand, if both countries are currently above their steady state levels, the richer country's output per worker will *shrink* faster than the poorer country's.

So, in absolute terms, the rate of change in output per worker will be higher in the richer country A, even if that rate of change is negative.

Refer to class notes for graphs depicting this.

b. (10 pts) Suppose country A has a higher income tax rate than the country B. If all the tax revenues are spent on wasteful spending by the government, which country will have a higher standard of living in the long run? Explain.

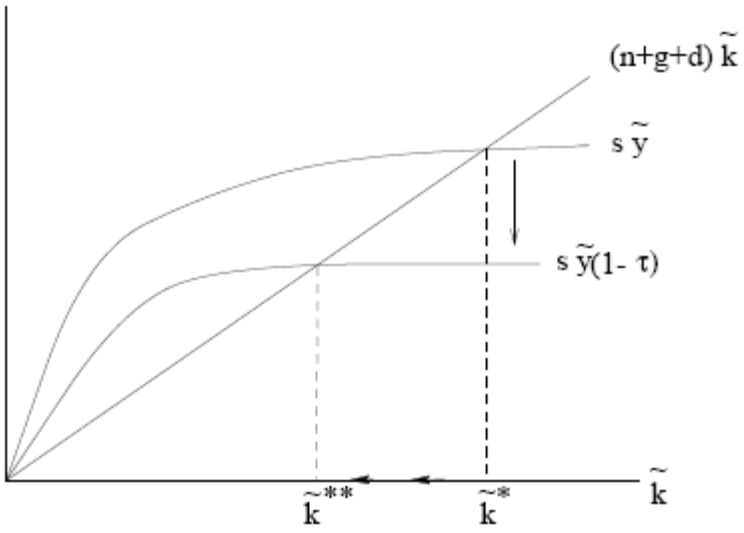
Ans: Recall that before any tax is imposed, the two countries will have identical steady state levels of capital per worker, determined from the following equation:

$$0 = sy_{ss} - (\delta+n)k_{ss} = sk_{ss}^{\alpha} - (\delta+n)k_{ss}$$

When a country imposes a tax rate τ on income y, the after tax income faced by the population is $(1-\tau)y$. So the new steady state will be determined by:

$$0 = s(1-\tau)y_{ss} - (\delta+n)k_{ss} = s(1-\tau)k_{ss}^{\alpha} - (\delta+n)k_{ss}$$

This shifts the investment curve down in the Solow diagram, lowering the steady state level of capital per worker k_{ss} (refer to class notes for the diagram). The higher the tax τ, the more pronounced will the downward shift in the investment curve will be, hence the lower will the resulting steady state level of per capita output.



We can think of the tax is lowering the effective savings rate in the country since the fraction of income saved would effectively fall from s to $s(1-\tau)$.

Thus, with the tax revenue τY wasted on non-productive government spending, country B with lower income tax will enjoy a higher standard of living in the long run.

c. (10 pts) Same as part (b), except that now all the tax revenues are spent on investment. Which country will have a higher standard of living in the long run now? Explain.

Ans: With government using tax revenues to finance investment, total investment in the country will change to:

$$I = \text{private savings} + \text{public savings} = s(1-\tau)Y + \tau Y$$

Therefore, the effective investment rate I/Y would equal to:

$$I/Y = s(1-\tau) + \tau = s + \tau(1-s) > s$$

which is greater than would be the case without public investment (and tax).

Note that the effective savings/investment rate increases with the size of the tax τ; therefore, country A with higher income tax will have a higher steady state level of output per worker (enjoy a higher standard of living). This, of course, assumes that government investment is as productive as private investment and that the taxes are levied in a non-distortionary way (two very big assumptions).

5. (20 pts) Simulating Transition Dynamics: Suppose that $Y = K^{1/3}L^{2/3}$, $s = 0.3$, $n = 0.03$, and $\delta = 0.01$. Let $k_0 = 100$.

a. (10 pts) Derive the discrete time version of the equation for the law of motion of capital in the Solow Model.

Ans: In the discrete time version, the law of motion for capital is as follows:

$$K_{t+1} = (1-\delta)K_t + I_t = (1-\delta)K_t + sY_t$$

Dividing both sides by population L_t, we can rewrite:

$$K_{t+1}/L_t = (1-\delta)\,K_t/L_t + s\,Y_t/L_t$$
$$K_{t+1}/L_{t+1} \cdot L_{t+1}/L_t = (1-\delta)\,K_t/L_t + s\,Y_t/L_t$$
$$k_{t+1} \cdot L_{t+1}/L_t = (1-\delta)\,k_t + s\,y_t$$

Note that in discrete time, L_{t+1}/L_t = population growth rate $(1+n)$. Hence,

$$k_{t+1}(1+n) = (1-\delta)\,k_t + s\,y_t$$

Substituting $y_t = k_t^{\alpha}$ and dividing by $(1+n)$, we finally get:

$$k_{t+1} = [(1-\delta)\, k_t + s\, k_t^{\alpha}] / (1+n)$$

Thus, if we know the parameters and the initial capital per worker level k_0, then we can find current output per worker y_0, and determine what will be the next period's capital per worker k_1. If we know k_1, then we can find y_1 and k_2. Following this logic, we can determine the entire path of endogenous variables k and y given some initial level of capital worker k_0 and the parameters.

Note that this implies steady state levels:

$$k_{ss} = \left[s / (\delta+n)\right]^{1/(1-\alpha)}$$
$$y_{ss} = \left[s / (\delta+n)\right]^{\alpha/(1-\alpha)}$$

b. (10 pts) Using part (a), calculate what would be the values of per capita y and k five and ten periods forward (you may want to do this in Excel).

Ans: Refer to the Excel file for details.

$k_5 = 88.18$, $k_{10} = 78.23$, $y_5 = 4.45$, $y_{10} = 4.28$