

### Assignment 1

1. Exercises 2.1, 2.2 and 3.1 through 3.8 in SLP.
2. Consider the following “Edgeworth-box” economy with two commodities (say apples and bananas). There are two (types of) households (labeled 1 and 2). The preferences of both households are the same and are given by

$$u(c_A, c_B) = \log c_A + \mathbf{b} \log c_B$$

Household 1 is endowed with  $w_A$  units of apples and 0 units of bananas and household 2 is endowed with 0 units of apples and  $w_B$  units of bananas.

- a. Define a competitive equilibrium.
  - b. Suppose  $w_A = w_B = 1$  and  $\mathbf{b} = 1$ . Compute the competitive equilibrium (or equilibria, if there is more than one).
  - c. Define a Pareto-optimum.
  - d. What conditions do the set of Pareto-optima have to satisfy?
  - e. Suppose  $w_A = w_B = 1$  and  $\mathbf{b} = 1$  as in part (b). What prices and transfers support the following allocation:  $(c_A^1, c_B^1) = \left(\frac{3}{4}, \frac{3}{4}\right)$  and  $(c_A^2, c_B^2) = \left(\frac{1}{4}, \frac{1}{4}\right)$ .
3. Consider the following 2-period economy. There is a single consumption good in each period. There are 2 households who have identical preferences over consumption given by:

$$u(c_1, c_2) = \log c_1 + \mathbf{b} \log c_2.$$

Household 1 has endowments given by  $(w_1, 0)$  and household two has endowments given by  $(0, w_1)$ .

- a. Define an Arrow-Debreu competitive equilibrium.
  - b. Suppose households attempt to smooth their consumption over time by borrowing and lending. Define a sequential markets competitive equilibrium. Show that a sequential market's equilibrium and an Arrow-Debreu equilibrium give identical allocations.
  - c. Suppose  $w_1 = w_2 = 1$  and  $\mathbf{b} = 1$ . Calculate the competitive equilibrium.
  - d. Interpret the price of second period consumption as an interest rate. How does the interest rate vary as the distribution of endowments across households varies?
  - e. Suppose households cannot borrow or lend from each other. Try to define a competitive equilibrium in this case. Show that a policy which transfers goods from the rich to the poor in each period is Pareto-improving.
4. Consider the Solow growth model. The economy lasts forever. There is a single good in each period which can be consumed or invested. The production function for producing this good is  $f(k_t)$  where  $f$  is strictly increasing, differentiable and satisfies  $\lim_{k \rightarrow 0} f'(k) = \infty$  and  $\lim_{k \rightarrow \infty} f'(k) = 0$   $f(0) = 0$ .
- Consumption decisions are summarized by  $c_t = (1 - s)f(k_t)$  where  $s$  (the savings rate) is a fixed number. The law of motion for capital accumulation is  $k_{t+1} = f(k)_t + (1 - \mathbf{d})k_t - c_t$ . A steady-state is a capital stock such that  $k_{t+1} = k_t = k^*$ .
- a. Show that a steady state exists. Give a formula for the steady state.
  - b. Suppose that the economy starts at some arbitrary  $k_0 \neq k^*$ . Show that it converges monotonically to the steady state.
5. Obtain national income account data from the bureau of Economic Analysis (BEA) website .
- a. Real GDP against time from 1952 to 2005 (annual data).
  - b. Ratio of the components of the product side of real GDP (such as personal consumption expenditures) to GDP. What trends do you see?
  - c. Plot the ratio of the components of national income such as wages and salaries to national income?
6. How would you make sense of the findings in Question 5 in terms of the growth model?