**Macroeconomic Theory**
**Econ. 8106, Fall 2008**
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# Assignment 2

**Due Thursday, November 13**.

1. Assume a Cobb-Douglas production function in a deterministic growth model. Assume leisure is inelastically supplied. Show that the capital-output ratio and the rental rate on capital in a steady state is invariant to changes in
    a. a proportional tax on labor income.
    b. changes in a proportional tax on consumption.
    c. invariant to permanent TFP changes.
    d. not invariant to changes in a tax on capital income.

    Who bears the burden of a tax on labor income?

2. Use the method of undetermined coefficients to solve for the linearized decision rules in a one-sector growth model with $b = 0.5$, $d = 0.1$, $F(k_t, \ell_t) = k_t^{0.25} \ell_t^{0.75}$, and preferences $\sum_{t=0}^{\infty} b^t \log c_t$. Assume labor is inelastically supplied.
3. Exercises 6.2, 6.3, 6.5, 6.6, 6.8, 6.9 from SLP.
4. **Uncertainty in a finite, pure endowment economy**

    Consider an economy with 2 consumers (denoted by 1 and 2) and 2 periods (1 and 2) and a single non-storable consumption good. Preferences of both consumers are ordered by $Eu(c_1, c_2)$, where *E* denotes the expectations operator and $u(c_1, c_2) = \log c_1 + \log c_2$. The endowments for the 2 consumers are:

    - In period 1: $w_1 = (1,1)$,
    - In period 2: $w_{21} = (1,0)$, with probability *p* and (0,1) with probability $(1 - p)$.

    In the following questions, define an equilibrium for this economy and compute it.

**(Question 4 – continued)**

a) Agents are allowed to trade the consumption good among themselves within a given period and state but not across different periods and states, i.e., contracts of the type: I will give you something if state 1 occurs and you will give me something if state 2 occurs are ruled out.

In what follows assume agents can trade within period and state of the world as in (a).

b) Agents are also allowed to trade a riskless bond at period 1. This is an asset that costs $q$ at period 1 and gives payoff 1 in terms of the consumption good in period 2.

c) Assume now that there is also a storage technology that the agents can use with return $R > 0$, i.e., if they save 1 unit of consumption in period 1, they will get $R$ units of consumption in period 2 no matter what the state is. Compare the equilibrium allocations to those of (a) and (b).

d) Agents are able to trade state contingent bond in period 1.

e) Arrow-Debreu markets are opened. Compare the equilibrium allocation of consumption to that of (d). Define *perfect insurance*. Show that we will get it with $p = 0.5$.

f) In addition to (e) assume there is a storage technology as in (c). Compare the results you get to those of (e).

5. Consider a growth model with an equal number of two types of agents. Their utility functions are the same and are $\sum b^t U(c_t)$. The underlying state $s_t$ can take on one of two values $\{H, L\}$. If $s_t = H$, type 1 agents have an endowment of 2 units of labor and type 2 agents have an endowment of 0 units of labor. Assume initial endowments of capital are the same.

a) Define a competitive equilibrium with complete markets. What social planning problem does this equilibrium solve? Show that this planning problem is identical to an economy with just one type of agents.

b) Define a competitive equilibrium in which the two types of agents can only use capital to save across time. Is an equilibrium allocation with complete markets an equilibrium allocation here? Why not? (Hint: Conjecture that it is in the case where

the complete markets economy is at a steady state. Then describe the evolution of capital stocks assuming a sample path that consists of a very long string of $s_t = H$.

**Extra Credit**: Suppose now that the initial endowments of capital are not the same. Describe how you would compute the weights on the social planner's problem that supports a competitive equilibrium. (You do not have to compute these weights.)