Economics 8106 - Macroeconomic Theory Fall 2008 **V.V. Chari**

Problem Set 3

Due Thursday, November 19, 2008

1. Consider the decision problem of a single worker who has preferences given by $\sum_{t=0}^{\infty} \beta^t u(c_t)$ where c_t denotes consumption, β is a discount factor and u is strictly concave. At the beginning of each period the worker is either unemployed or employed. If unemployed, the worker receives a wage offer from a distribution $G(w)$ and if employed, gets a new offer from a distribution $F(w)$. These offers are constant for the duration of the jobs. These draws are i.i.d. Jobs disappear with probability δ at the end of each period. The worker cannot save or borrow and if unemployed receives a benefit b.

(a) Set up the decision problem as a dynamic program.

(b) Show that the solution to the dynamic program is of the reservation wage form.

(c) Assume that $b = 0$, $u(0) = 0$ and that G and F are both the same with the supports of both distributions given by $[0, \bar{w}]$. What jobs if any does the worker reject?

2. Consider the problem faced by each of a large number of inventors. An inventor can either choose to invent a new product at a cost of b or not. If the inventor chooses to invent, the quality of the invention is random. Let z denote the quality of an invention where z is drawn independently across inventors and time from a distribution $F(z)$. The quality of the invention is also the per period profits from the invention. Once an invention is produced, the inventor must manage the product if he wants to sell it. While managing, an inventor has no time to invent. With probability p, an invention becomes worthless, and the inventor can go back to his first love, inventing new products. Assume investors are risk neutral and discount future profits.

(a) Set up a typical inventor's problem.

(b) Define a stationary equilibrium for the economy.

(c) Suppose the costs b fall. What happens to the fraction of those engaged in invention in a stationary equilibrium?

(d) Suppose the distribution F changes in a mean-preserving fashion. What happens to the fraction of those engaged in invention in a stationary equilibrium?

3. Consider an economy in which workers accumulate human capital while working. A worker currently working at a job produces a single nonstorable good according to h_t^α where h_t is human capital and $0 < \alpha < 1$. Jobs disappear at the end of the period with probability δ. if the job does not disappear, the worker's human capital next period is $h_{t+1} = \gamma h_t$ where $\gamma > 1$. If the job disappears, the worker becomes unemployed. Unemployed workers receive one new job offer in each period. The offers are parameterized by a random variable z which is uniformly distributed between 0 and 1 and is i.i.d. over time. A worker who accepts a job at t has human capital $h_t = zh_{t-1}$. That is, part of the human capital $(1-z)h_{t-1}$ disappears forever. Workers are risk-neutral and maximize $\sum \beta^t c_t$. Assume no borrowing or lending is possible.

(a) Set up the worker's problem as a dynamic program.

(b) Prove that unemployed workers have a reservation strategy of the form $z(h_{t-1})$ where they accept all offers greater than $z(h_{t-1})$ and reject all others.

(c) What can you say about $z(h_{t-1})$? Is it linear in h_{t-1}?

4. Problems 6.2, 6.4, 6.6, 6.8. 6.9, 6.11, 6.12, 6.13, 6.17 in Sargent & Ljungqvist.

5. Problems in Sections 10.4 and 10.5 in SLP.