Political Economy of Mechanisms
Acemouglu, Golosov, Tsyvinski

Jahiz Barlas Sewon Hur Nathalie Pouokam

University of Minnesota

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Overview

Model of political economy

- Citizens make economic decisions
- Self interested politicians make allocative decisions (without restrictions)

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Main Question:

- Does providing incentives to politicians create distortions?
- If so, when can these distortions be eliminated?

Answers and Contributions

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- Incentives create distortions in the political economy
- Given sufficient restrictions on the patience parameters, these distortions disappear

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Contributions

- No restriction on class of taxes
- Allow for nonstationary voting rules
- Incorporating a political economy model into a neoclassical growth model.

Environment

- t=0,1,...

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- Identical citizens: i $\in \mathrm{I}$ where $\mathrm{I} = [0,1]$

Assumption (1)

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- Identical politicians: $\iota \in \mathfrak{I}$ where $\mathfrak{I}$ = [0,1]

Assumption (2)

v: $\mathbb{R}_+ \rightarrow \mathbb{R}$ *is a* C^2 *and concave function with* $v'(x) > 0 \; \forall x \in \mathbb{R}_+$ *and* $\delta \in (0,1)$. *Politicians do not have access to capital markets.*

Assumption (3)

$Y_t = F(K_t, L_t) = \widetilde{F}(K_t, L_t) + (1-\delta)K_t$ is strictly increasing, C^1 in K and L, CRS and satisfies:

1. $\lim_{L \to 0} F_L(K, L) = \infty \qquad \forall\ K \geq 0$
2. $\lim_{K \to 0} F_K(K, L) < 1 \qquad \forall\ L \in [0, \overline{L}]$

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Define:

$$L_t = \int_{i \in \mathrm{I}} l_{i,t}.\mathrm{di}$$

$$C_t = \int_{i \in \mathrm{I}} c_{i,t}.\mathrm{di}$$

Game Structure

At t, economy starts with politician $\iota \in \mathfrak{I}$ in power and a stock of capital K_t. Then:

1. Citizens choose labor $[l_{i,t}]_{i \in \mathrm{I}}$ and L_t is used for output $F(K_t, L_t)$

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2. Politician chooses:
    - rents $x_t \in \mathbb{R}_+$
    - citizen consumption function $c_t : \mathbb{R}_+ \to \mathbb{R}_+$
    - $K_{t+1} \in \mathbb{R}_+$ subject to constraint $K_{t+1} \leq F(K_t, L_t) - C_t - x_t$

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   - $K_{t+1} \in \mathbb{R}_+$ subject to constraint $K_{t+1} \leq F(K_t, L_t) - C_t - x_t$
3. Elections are held and citizens jointly decide whether to keep the politician in power or replace him: $\rho_t \in \{0, 1\}$ for $\rho_t = 1$ denoting replacement

Remark:
Citizens make economic decisions individually, but political decisions collectively. We assume the political decision ρ_t is made by a random agent
Therefore, history h^t is:

$$h^t \equiv (K_O, \iota_0, [l_{i,0}]_{i\in\mathrm{I}}, x_0, c_0, \rho_0, K_1, ..., K_t, \iota_t, [l_{i,t}]_{i\in\mathrm{I}}, x_t, c_t, \rho_t, K_{t+1})$$

Define:

$$h_1^{t-1} = (h^{t-1}, [l_{i,t}]_{i\in\mathrm{I}})$$
$$h_2^{t-1} = (h_1^t, [c_{i,t}]_{i\in\mathrm{I}}, K_{t+1}, x_t,)$$

Definition

Given h^{t-1}, a sub-game perfect equilibrium (SPE) is vector:

$$([l^*_{i,t}(h^{t-1})]_{i\in\mathrm{I}}, x^*_t(h^{t-1}_1), [c^*_{i,t}(h^{t-1}_1)]_{i\in\mathrm{I}}, K^*_{t+1}(h^{t-1}_1), \rho^*_t(h^{t-1}_2))$$

such that the set of allocations are best responses to each other.

Definition

The best SPE is the SPE that maximizes utility of the citizens.

Definition

An SPE is renegotiation proof if, after any h^t, there does not exist another SPE that can make all active players weakly better off and some strictly better off.

We now map the game structure à la Chari Kehoe (1990):

1. Citizens choose labor $f^i_{1,t}(h^{t-1}) = l_{i,t}(h^{t-1})$ and contingency rules.
2. Politician chooses $\sigma_t(h_1^{t-1}) = \{\{c_{i,t}(h_1^{t-1})\}_{i \in I}, K_{t+1}(h_1^{t-1}), x_t(h_1^{t-1})\}$ and a contingency plan for future actions for any possible future history.
3. Citizens choose $f^i_{2,t}(h_2^{t-1}) = \rho^i_t(h_2^{t-1})$ and contingency rules.

For any policy plan, $\sigma = (\sigma_0, \sigma_1, ...)$, $\sigma^t = (\sigma_t, \sigma_{t+1}, ...)$ denotes the continuation of σ. A similar notation holds for f.

Given some history h^{t-1}, $h^t = [h^{t-1}, \sigma_t(h^{t-1}), f_t(h^{t-1})]$ is the continuation of history as induced by (σ, f)

Citizens' Problem:
Given (h^{t-1}):

$$\max_{f^t} \sum_{s=0}^{\infty} \beta^s U[c_{t+s}(h^{t+s-1}), l_{t+s}(h^{t+s-1})] \qquad \forall t \geq 0$$

Given $(h^{t-1}, \sigma(h^{t-1}))$:

$$\max_{f^t} \sum_{s=0}^{\infty} \beta^s U[c_{t+1+s}(h^{t+s}), l_{t+1+s}(h^{t+s})] \qquad \forall t \geq 0$$

Politician's Problem:

$$\max_{\sigma} \sum_{s=0}^{\infty} \delta^s v(x_{t+s}(h^{t+s-1})) \qquad \forall t \geq 0$$

$$s.t. \quad C_{t+s}(h^{t+s-1}) + K_{t+s+1}(h^{t+s-1}) + x_{t+s}(h^{t+s-1}) = F(K_{t+s}(h^{t+s-1}), L_{t+s}(h^{t+s-1})) \qquad \forall t \geq 0, \forall h^{t+s-1}$$

Definition

A sustainable equilibrium is a pair (σ, f) that satisfies:

1. Given policy plan σ, the continuation of the allocation rule f solves the citizens' problem given h^{t-1} at the first stage and (h_2^{t-1}) at the third stage.
2. Given allocation rule f, the continuation of the policy plan σ solves the politician's problem given h_1^{t-1}

Social Planner's Problem

The Social Planner's Problem (SPP) is:

$$\max_{\{C_t, L_t, K_{t+1}, x_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(C_t, L_t)$$

$$s.t \quad C_t + K_{t+1} + x_t \leq F(K_t, L_t) \qquad \forall t \qquad (RC)$$

$$\omega_t \equiv \sum_{s=0}^{\infty} \delta^s v(x_{t+s}) \geq v(F(K_t, L_t)) \qquad \forall t \qquad (SC)$$

$$(c_t, l_t) \in \Lambda \qquad \forall t$$

where Λ contains $c_t \geq 0$, $l_t \in [0, \overline{L}]$ and $U(c_t, l_t) \geq 0$

Proposition (1)

The allocation obtained from a best SPE is identical to a solution of the SPP and involves no replacement of the initial politician along the equilibrium path. Moreover, this allocation can be supported as a renegotiation proof SPE.

Proof:
Let $\widetilde{A}$ be a sustainable equilibrium allocation and A^* be a solution to the SPP.

Step 1: $\widetilde{A}$ satisfies RCs and SCs.
This follows from the politician's problem. $\widetilde{A}$ satisfies RC trivially, and SC is satisfied as if it is not, there would be deviation of politician.

Thus, $\widetilde{A}$ can not yield strictly higher utility than an allocation A^* that solves the SPP problem.

To prove the theorem then, we show that A^* can be supported as an SPE allocation that is renegotiation proof.

Step 2: A^* can be supported as an SPE

$f^t(K_t)$ is defined by:

$$\left\{ \begin{array}{ll} L_t(h^{t-1}) = L_t & \\ \rho_t(h_2^{t-1}) = 1 & \text{if } (x_t, K_{t+1})(h_1^{t-1}) \neq (x_t, K_{t+1}) \\ & \text{and zero otherwise} \end{array} \right.$$

$\sigma^t(K_t)$ is defined by:

$$\left\{ \begin{array}{ll} x_t(h_1^{t-1}) = x_t & \\ K_{t+1}(h_1^{t-1}) = K_{t+1} & \\ C_t(h_1^{t-1}) = C_t & \text{if } L_t(h^{t-1}) = L_t \text{ and 0 otherwise} \end{array} \right.$$

where:

$\{L_t, x_t, K_{t+1}, C_t\}$ is solution to the continuation of the SPP for capital stock K_t.

Note that if $K_t = K_t^*$, then the allocations will correspond to the A^* allocation.

Claim 1: $f \in BR(\sigma)$
Reason: Given σ, no other choice of labor supply can yield strictly higher utility to citizens. Also, σ assumes that if a politician is voted out then the continuation of σ for the new politician is still defined as in our construction.
Thus, given σ, there is no other voting rule that would make the citizens strictly better off hence $f \in BR(\sigma)$.

Claim 2: $\sigma \in BR(f)$
Reason: Given A^* solves the SPP problem, we have SC satisfied. If the politician is voted out for deviating, he would get $v(F(K_t, L_t))$ from deviation which is not optimal.

Hence, (σ, f) supports A^* as a SPE.

Step 3:
Idea: (σ, f) is renegotiation proof because it involves continuation of the best SPE.
Citizens and politicians cannot be made better off simultaneously.

Step 4:
For contradiction: Assume SPP solution can be supported with replacement of initial politician.
Consider $\{C_t^*, L_t^*, K_{t+1}^*, x_t^{'}\}$ where politician kept in power with new politician's payoff.
Since $x^{'} > 0$, we have:

$$\underbrace{\sum_{t=0}^{s} \delta^t v(x_t^*) + \sum_{t=s+1}^{\infty} \delta^t v(x_t^{'})}_{\text{Politician kept in power}} > \underbrace{\sum_{t=0}^{\infty} \delta^t v(x_t^*)}_{\text{Politician fired}}$$

Decreasing x_0 and increasing c_0 would give contradiction $\Rightarrow\Leftarrow$

Definition

An undistorted allocation is a sequence $\{C_t, L_t, K_{t+1}, x_t\}_{t=0}^{\infty}$ that, given sequence $\{x_t\}_{t=0}^{\infty}$, maximizes the SPP without the sustainability constraint.

Thus, an undistorted allocation where $(C_t, L_t) \in Int\Lambda$ satisfies:

$$F_L(K_t, L_t)U_C(C_t, L_t) = -U_L(C_t, L_t)$$

$$U_C(C_t, L_t) = \beta F_K(K_{t+1}, L_{t+1})U_C(C_{t+1}, L_{t+1})$$

Definition

An allocation $\{C_t, L_t, K_{t+1}, x_t\}_{t=0}^{\infty}$ features downward labor distortions at time t if the following holds:

$$F_L(K_t, L_t)U_C(C_t, L_t) > -U_L(C_t, L_t)$$

Definition

An allocation $\{C_t, L_t, K_{t+1}, x_t\}_{t=0}^{\infty}$ features downward intertemporal distortions at time t if the following holds:

$$U_C(C_t, L_t) < \beta F_K(K_{t+1}, L_{t+1})U_C(C_{t+1}, L_{t+1})$$

Cases

Best SPE in two separate environments considered:

- No Capital
  - Special Case: Stationary equilibria
- Capital

No Capital Case

Assumption (4)

Let:

$$(\widetilde{C}, \widetilde{L}) \in arg \max_{(C,L)\in\Lambda} L - C$$

Then:

$$\frac{v(\widetilde{L} - \widetilde{C})}{1 - \delta} > v(\widetilde{L})$$

Also:
Production Function is $Y_t = L_t$

Recursive Formulation

$$
\begin{aligned}
V(\omega) = \max_{C,L,x,\omega^+} \quad & U(C,L) + \beta V(\omega^+) \\
s.t. \quad & C + x \leq L \\
& v(x) + \delta\omega^+ \geq v(L) \\
& \omega = v(x) + \delta\omega^+ \\
& (C,L) \in \Lambda \\
& \omega \in [0, \overline{\omega}]
\end{aligned}
$$

where $\overline{\omega} = \frac{v(\widetilde{L} - \widetilde{C})}{1-\delta}$ is maximum attainable rent.

Theorem (1)

Suppose that $Y_t = L_t$, $U_c(0,0) > -U_L(0,0)$ and assumptions 1,2 and 4 hold. Then the best SPE has the following characteristics:

1. *There are downward labor distortions at t=0.*
2. *When $\beta \leq \delta$, then*

$$\{\omega_t\}_{t=0}^{\infty} \nearrow \omega^* \qquad \text{and} \qquad \{C_t, L_t, x_t\}_{t=0}^{\infty} \to \{C^*, L^*, x^*\}$$

   where $U_C(C^*, L^*) = -U_L(C^*, L^*)$

3. *When $\beta > \delta$, then there are downward labor distortions even asymptotically.*

The allocations described above can be supported as a renegotiation proof SPE.

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Proof:

Lemma:

$$\sum_{s=0}^{\infty} \delta^s v(x_{0+s}) > v(L_0) \quad then \quad \sum_{s=0}^{\infty} \delta^s v(x_{t+s}) > v(L_{t+s}) \quad \forall t \geq 1$$

It suffices to show $\sum_{s=0}^{\infty} \delta^s v(x_{0+s}) = v(L_0)$. (as this $\Rightarrow \psi_0 > 0$)

Suppose for contradiction that SC_0 is slack.
Then by lemma, SC_t is slack $\forall t \geq 1$.
$\Rightarrow x_t = 0 \ \forall t$
$\Rightarrow \omega_0 = 0$
By assumption, $U_c(0,0) > U_L(0,0)$
So in absence of distortions $L_0 > 0$
Note that:

$$0 = \frac{v(0)}{1-\delta} < v(L_0)$$

Thus, deviation for politician is optimal which is a contradiction $\Rightarrow\Leftarrow$.
So there exist labor distortions at time $t = 0$.

■

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Suppose that $Y_t = L_t$, $U_c(0,0) > -U_L(0,0)$ and assumptions 1,2 and 4 hold. Then the best SPE has the following characteristics:

2. *When $\beta \leq \delta$, then*

$$\{\omega_t\}_{t=0}^{\infty} \nearrow \omega^* \qquad \text{and} \qquad \{C_t, L_t, x_t\}_{t=0}^{\infty} \rightarrow \{C^*, L^*, x^*\}$$

where $U_C(C^*, L^*) = -U_L(C^*, L^*)$

Exercise: Consider t=0,1 where $\beta < \delta$

Claim: $\omega_1 > v(L_1)$

Proof:
Suppose not, $\omega_0 \geq v(L_0)$ and $\omega_1 = v(L_1)$
Then consider the following variation:

$$\Delta x_1 > 0$$

$$\Delta x_0 < 0$$

s.t. $\Delta x_0 v^{'}(x_0) + \delta \Delta x_1 v^{'}(x_1) = 0$
Asymptotically, $v^{'}(x_0) = v^{'}(x_1)$
Hence, $\Delta x_1 = -\frac{1}{\delta}\Delta x_0$

Now, let

$$\Delta c_0 = -\Delta x_0$$

$$\Delta c_1 = -\Delta x_1$$

Change in citizen's utility: $-u_{c0}\Delta x_0 - \beta u_{c1}\Delta x_1$
$= \Delta x_0[\frac{\beta}{\delta} - 1] > 0$
This contradicts that the initial allocation solves the SPP.
Hence, $\omega_1 > v(L_1)$

■

Theorem (1)

Suppose that $Y_t = L_t$, $U_c(0,0) > -U_L(0,0)$ and assumptions 1,2 and 4 hold. Then the best SPE has the following characteristics:

3. *When $\beta > \delta$, then there are downward labor distortions even asymptotically.*

We present an exercise again with t=0,1

Claim: $\omega_1 = v(L_1)$
Suppose not, i.e. suppose $\omega_1 > v(L_1)$
Then consider

$$\Delta x_1 < 0$$
$$\Delta x_0 > 0$$

s.t. $\Delta x_0 v(x_0) + \delta \Delta x_1 v(x_1) = 0$
Hence, $\Delta x_1 = -\frac{1}{\delta}\Delta x_0$

Now, let

$$\Delta c_0 = -\Delta x_0$$
$$\Delta c_1 = -\Delta x_1$$

Change in citizen's utility:

$$\begin{aligned} & -u_{c0}\Delta x_0 - \beta u_{c1}\Delta x_1 \\ = \quad & \Delta x_0[\tfrac{\beta}{\delta} - 1] > 0 \end{aligned}$$

This contradicts that the initial allocation solves the SPP.
Hence, $\omega_1 > v(L_1)$

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Stationary Equilibria

We focus on stationary strategies.

Proposition (2)

Consider the environment without capital in Theorem 1, and suppose that Assumptions 1,3 and 4 hold and that $U_C(0,0) > -U_L(0,0)$*. Then, in the best stationary SPE, distortions never disappear.*

Proof:

Note along a stationary equilibrium path, $x_t = x$, $L_t = L$ then:
$\frac{v(x)}{1-\delta} \geq v(L)$ replaces the SC.
Suppose for contradiction that along this path, $\psi = 0$.
Then x=0 is solution to SPP and there are no distortions.

As $U_C(0,0) > -U_L(0,0)$, $L > 0$ implying:

$$0 = \frac{v(0)}{1-\delta} < v(L)$$

Thus, to have the politician not deviating, we need $x > 0$ which is a contradiction as x=0 is the solution to SPP. $\Rightarrow\Leftarrow$

$\blacksquare$

Hence, we see that in stationarity, there will always be distortions.

- Discount factors do not matter
- Gives importance to nonstationary SPE

Capital Case

Define:

$$\overline{C} = \min\{C : (C, \overline{L} \in \Lambda)\}$$
$$\overline{K} = arg \max_{K \geq 0}\{F(K, \overline{L}) - K - \overline{C}\}$$

Assumption (4')

1. $\frac{\delta v(F(\overline{K},\overline{L}) - \overline{C} - \overline{K})}{1-\delta} > v(F(\overline{K}, \overline{L}))$
2. $\overline{C} + \overline{K} \leq F(0, \overline{L})$

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Assumption (4')

1. $\frac{\delta v(F(\overline{K},\overline{L}) - \overline{C} - \overline{K})}{1-\delta} > v(F(\overline{K}, \overline{L}))$
2. $\overline{C} + \overline{K} \leq F(0, \overline{L})$

Allows for maximum utility for politician to be obtained.
Used in part 2 of theorem 2.

Theorem (2)

Suppose assumptions 1-3 and 4' hold. Then the best SPE has the following characteristics:

1. *There are downward labor distortions at some $t < \infty$ and downward intertemporal distortions at t-1 (provided that $t \geq 1$)*
2. *When $\beta \leq \delta$, then $\{C_t, K_t, L_t, x_t\}_{t=0}^{\infty} \rightarrow \{C^*, K^*, L^*, x^*\}$. At this allocation, the labor and intertemporal distortions disappear asymptotically.*
3. *When $\beta > \delta$, then there are downward labor distortions and intertemporal distortions, even asymptotically.*

The allocations described above can be supported as a renegotiation proof SPE.

Interesting things to note:

- Might not have labor distortions in time 0
- Existence of distortions in some other time
- Distortions of both kinds occur
- Politician payoffs no longer nondecreasing
- Wedge created provides rationale for long-run capital taxation

Further Extensions

- Endogenize politicians' discount factors
- Alternate environments
  - Mirleesian (AGT 2008)
- Different voting structures

APPENDIX : This appendix contains a formal proof of Theorem 1, parts b and c

Proof of (b):

Claim: $\{V_\omega(\omega_t)\}$ is nonincreasing.

Envelope Conditions: $V_\omega = \frac{\beta}{\delta} V_{\omega^+} + \psi$

Since $\frac{\beta}{\delta} \leq 1$, $\frac{\beta}{\delta} V_{\omega^+} \geq V_{\omega^+}$ from FOCs as $V_\omega < 0$

$V_\omega = \frac{\beta}{\delta} V_{\omega^+} + \psi \geq V_{\omega^+}$

Thus, $V_\omega(\omega_t)$ is nonincreasing.

As V is concave $\Rightarrow \{\omega_t\}_{t=0}^{\infty}$ is nondecreasing and thus $\{\omega_t\}_{t=0}^{\infty} \rightarrow \omega^* \in [0, \infty]$Case 2:

$\{V_\omega(\omega_t)\}$ converges to some $V_\omega > -\infty$ and $\{\omega_t\}_{t=0}^{\infty}$ has a subsequence converging to $\omega_\infty \in bd[0, \overline{\omega}]$.

Lemma: if $\omega_t = \overline{\omega}$ for some t, then $\omega_{t+j} > v(L_{t+j}) \forall j \geq 0$.

Following immediately from Lemma, SC becomes slack eventually hence $\psi_t \rightarrow \psi_\infty = 0$ and so we are done.

Then we have $x_t \to x^*$ and so $C_t \to C^*$ and $L_t \to L^*$.
Now, we need to show that the no-distortions condition $U_C(C^*, L^*) = -U_L(C^*, L^*)$ holds.
As $\{V_\omega(\omega_t)\}$ converges in the extended real line, there are 3 cases to consider:
Case 1:
$\{V_\omega(\omega_t)\}$ converges to some $V_\omega > -\infty$, and for all convergent subsequences $\{\omega_{t^n}\}$, $\{\omega_{t^n}\} \to \omega_\infty \in int[0, \overline{\omega}]$.
Then FOCs:

$$\begin{aligned} \frac{\beta}{\delta} V'(\omega^+) & = -\gamma - \psi \\ \Rightarrow V'(w^+) & \leq \frac{\beta}{\delta} V'(\omega^+) & = -\gamma - \psi \end{aligned}$$

Taking limits $\Rightarrow -\gamma_\infty \leq -\gamma_\infty - \psi_\infty$ which implies $\psi_\infty = 0$ and the result holds.

Case 3:
$\{V_\omega(\omega_t)\} \to -\infty$ and there is a subsequence of $\{\omega_t\}_{t=0}^{\infty}$ converging to some $\omega_\infty \in int[0, \overline{\omega}]$
Recall $\frac{\beta}{\delta} V^{'}(\omega^{+}) = -\gamma - \psi$, so $V_\omega \to -\infty \Rightarrow$ either $\psi_t \to \infty$ or $\gamma_t \to \infty$.
But $v^{'}(x_t) = \frac{\lambda_t}{\psi_t + \gamma_t}$, so either $\lambda_t = +\infty$ or $v^{'}(x_t) = 0$.
If $\lambda_t = +\infty$ then $U_C = +\infty$, so by concavity of U, we have $c_t \to 0$. But $(c_t, l_t) \in \Lambda$, $c_t \to 0 \Rightarrow l_t \to 0$ and hence $x_t \to 0$ so that $\omega_t \to 0$ which contradict $\omega_\infty \in (0, \overline{\omega})$. This rules out this possibility.
If $v^{'}(x_t) = 0$ then $x_t = +\infty$, since $v^{'}$ is positive and decreasing, then by assumption, now $x_t = +\infty$ is NOT possible as $Y_t \leq \overline{Y} < \infty$

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Proof of (c):
The main idea is to show that when $\beta > \delta$, ω_t will never converge to the boundary.
Now, if ω never goes to the boundary, then:

$$\begin{aligned} \frac{\beta}{\delta} V_\omega &= V_\omega - \psi \\ \Rightarrow V_\omega[\tfrac{\beta}{\delta} - 1] &= -\psi \end{aligned}$$

Thus, $\psi > 0$ in the limit.

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