

Golosov and Tsyvinski (2007) "*Optimal Taxation
with Endogenous Insurance Markets*", QJE

Wyatt Brooks and Alessandro Dovis

October 2, 2008

Introduction

- ▶ Should the government provide insurance against idiosyncratic income uncertainty?

Introduction

- ▶ Should the government provide insurance against idiosyncratic income uncertainty?
- ▶ In environment with informational friction competitive private markets can provide insurance

Introduction

- ▶ Should the government provide insurance against idiosyncratic income uncertainty?
- ▶ In environment with informational friction competitive private markets can provide insurance
- ▶ If individual *assets trades* are *publically observable* then the FWT applies: CE allocations are Constraint Pareto Efficient (CPE) [Prescott and Townsend (1984)]

Introduction

- ▶ Should the government provide insurance against idiosyncratic income uncertainty?
- ▶ In environment with informational friction competitive private markets can provide insurance
- ▶ If individual *assets trades* are *publically observable* then the FWT applies: CE allocations are Constraint Pareto Efficient (CPE) [Prescott and Townsend (1984)]
- ▶ Public provision of insurance only crowds out provision by private firms

This paper's contributions

Key ingredients:

- ▶ *Individual assets trade are not observable*

This paper's contributions

Key ingredients:

- ▶ *Individual assets trade are not observable*
- ▶ *Prices in the anonymous financial market are endogenous*

This paper's contributions

Key ingredients:

- ▶ *Individual assets trade are not observable*
- ▶ *Prices in the anonymous financial market are endogenous*

Questions:

- ▶ Is the FWT holding in such environment?
- ▶ What is the optimal policy when private markets respond endogenously to governmental actions?
- ▶ How much gain (if any) is there to government intervention?

Results

- ▶ With *hidden trading* individual consumption allocation satisfies an *intertemporal Euler Equation* in contrast to the *Inverse Euler Equation* without hidden trade

Results

- ▶ With *hidden trading* individual consumption allocation satisfies an *intertemporal Euler Equation* in contrast to the *Inverse Euler Equation* without hidden trade
- ▶ Intertemporal Marginal Rate of Substitutions (MRS) are equated across agents

Results

- ▶ With *hidden trading* individual consumption allocation satisfies an *intertemporal Euler Equation* in contrast to the *Inverse Euler Equation* without hidden trade
- ▶ Intertemporal Marginal Rate of Substitutions (MRS) are equated across agents
- ▶ Decentralized allocations are generally not Constrained Pareto Efficient

Results

- ▶ With *hidden trading* individual consumption allocation satisfies an *intertemporal Euler Equation* in contrast to the *Inverse Euler Equation* without hidden trade
- ▶ Intertemporal Marginal Rate of Substitutions (MRS) are equated across agents
- ▶ Decentralized allocations are generally not Constrained Pareto Efficient
- ▶ With iid shocks imposing a moderate linear tax on capital income is welfare improving

Results

- ▶ With *hidden trading* individual consumption allocation satisfies an *intertemporal Euler Equation* in contrast to the *Inverse Euler Equation* without hidden trade
- ▶ Intertemporal Marginal Rate of Substitutions (MRS) are equated across agents
- ▶ Decentralized allocations are generally not Constrained Pareto Efficient
- ▶ With iid shocks imposing a moderate linear tax on capital income is welfare improving
- ▶ The result is not general, it depends on the underlying shock process

Further Results

- ▶ Private providers of insurance *take prices as given*

Further Results

- ▶ Private providers of insurance *take prices as given*
- ▶ The governmental authority internalizes the fact that the offered contract has an effect on prices

Further Results

- ▶ Private providers of insurance *take prices as given*
- ▶ The governmental authority internalizes the fact that the offered contract has an effect on prices
- ▶ Role for the government: limited to *correcting such an externality*

Outline

Environment

Retrading Market

Incentive Compatibility

Constrained Pareto Efficient Allocation

Competitive Equilibrium

Failure of FWT

iid shock, $T = 2$

Absorbing state

Comparison with Previous Results

Environment

Standard dynamic Mirrlees economy, no aggregate uncertainty:

- ▶ $t = 0, 1, \dots, T \leq \infty$
- ▶ *Continuum* $[0, 1]$ of *ex-ante* identical households
- ▶ Agent's type: $\theta^T \equiv \{\theta_1, \dots, \theta_T\} \in \Theta^T$ for $\Theta \equiv \{\theta(1), \dots, \theta(n)\}$
- ▶ $\pi(\theta^T) : \Theta^T \rightarrow [0, 1]$ probability distribution
- ▶ θ^T is private information and is learned through time
- ▶ $y_t = \theta_t l_t$ where:
 - ▶ y_t is effective labor (publicly observable)
 - ▶ l_t is effort (private information)

Preferences:

▶ vNM utility: $\sum_{t=0}^T \beta^t U(c_t, l_t)$,

$$U(c, l) = u(c) + v(l),$$

$u : \mathbb{R}_+ \rightarrow \mathbb{R} \in C^1$ str. increasing, str. concave,

$v : \mathbb{R}_+ \rightarrow \mathbb{R} \in C^1$ str. decreasing

Technology:

▶ $F(K_t, Y_t)$, CRS, where Y_t is aggregate effective labor

▶ Capital is owned and accumulated by the firms

Before realization of agents' type:

- ▶ Competitive firms and/or the government commit to a contract $(c, y) \equiv \{c_t, y_t\}$.

By the Revelation Principle we can restrict to (c, y) s.t. $\forall t$

$$c_t, y_t : \Theta^t \rightarrow \mathbb{R}_+$$

Let the space of possible agents' reports be:

$$\Sigma \equiv \{\sigma : \Theta^T \rightarrow \Theta^T \text{ s.t. } \forall t \sigma_t \text{ is } \theta^t \text{-measurable}\}$$

Before realization of agents' type:

- ▶ Competitive firms and/or the government commit to a contract $(c, y) \equiv \{c_t, y_t\}$.

By the Revelation Principle we can restrict to (c, y) s.t. $\forall t$
 $c_t, y_t : \Theta^t \rightarrow \mathbb{R}_+$

Let the space of possible agents' reports be:

$$\Sigma \equiv \{\sigma : \Theta^T \rightarrow \Theta^T \text{ s.t. } \forall t \sigma_t \text{ is } \theta^t \text{-measurable}\}$$

In any t in the anonymous retrading market:

- ▶ Agents trade one-period risk-free assets at a price Q_t [perfect enforcement]

Let $(x, s) \equiv \{x_t, s_t\}$ s.t. $\forall t x_t, s_t : \Theta^t \rightarrow \mathbb{R}$, be actual consumption and saving

Proposition

The restriction to only risk free asset trading is wlog as it is the only security that will be traded in equilibrium.

Proof $s^i(\theta^t, \tilde{\theta}_{t+1})$ pays 1 in $t + 1$ if agent i reports $\tilde{\theta}^{t+1}$

Case (i):

- ▶ Suppose one agent sells $s^i(\theta^t, \tilde{\theta}_{t+1})$ s.t. $q(s^i((\theta^t, \tilde{\theta}_{t+1}))) > 0$
- ▶ Then set $s^i(\tilde{\theta}^t) = -\infty$ and $\sigma_{t+1}(\theta^t, \theta_{t+1}) \neq \tilde{\theta}_{t+1} \rightarrow \leftarrow$

Case (ii)

- ▶ Suppose one agent buys $s^i(\theta^t, \tilde{\theta}_{t+1})$ and $q(s^i((\theta^t, \tilde{\theta}_{t+1}))) < Q_t$
- ▶ Then sell short risk-free and $s^i(\tilde{\theta}^{t+1}) = \infty \rightarrow \leftarrow$
- ▶ If $q(s^i(\theta^t, \tilde{\theta}_{t+1})) \geq Q$ then risk free bond is preferable



Household Problem

Given a contract (c, y) a household chooses (σ, x, s) as follows:

$$V((c, y), Q) = \max_{\sigma, x, s} \sum_{t=0}^T \sum_{\theta^t} \beta^t \pi(\theta^t) U \left(x(\theta^t), \frac{y(\sigma(\theta^t))}{\theta_t} \right)$$

s.t.

$$x(\sigma(\theta^t)) + Q_t s(\sigma(\theta^t)) \leq$$
$$\leq c(\sigma(\theta^t)) + s(\sigma(\theta^{t-1}))$$
$$s(\theta^0) = 0$$

Equilibrium in the Retrading Market

Definition

An equilibrium on the retrading market given (c, y) is a tuple

$$((\sigma, x, s), Q)$$

s.t

- ▶ (σ, x, s) solves the household problem given $(c, y), Q$
- ▶ $\sum_{\theta^t} \pi(\theta^t) s(\sigma(\theta^t)) = 0 \quad \forall t$

Assumption: For any (c, y) there exists a unique equilibrium $((\hat{\sigma}, \hat{x}, \hat{s}), \hat{Q})$ in the retrading market. We will use the following notation:

$$\hat{V}(c, y) = V((c, y), \hat{Q})$$

and denote

$$\hat{Q} \equiv \hat{Q}(c, y)$$

Assumption: For any (c, y) there exists a unique equilibrium $((\hat{\sigma}, \hat{x}, \hat{s}), \hat{Q})$ in the retrading market. We will use the following notation:

$$\hat{V}(c, y) = V((c, y), \hat{Q})$$

and denote

$$\hat{Q} \equiv \hat{Q}(c, y)$$

Moreover we define:

$$\begin{aligned}
 V((c, y), Q)(\sigma) &= \max_{x(\sigma), s(\sigma)} \sum_{t=0}^T \sum_{\theta^t} \beta^t \pi(\theta^t) U \left(x(\sigma(\theta^t)), \frac{y(\sigma(\theta^t))}{\theta_t} \right) \\
 \text{s.t.} \quad &x(\sigma(\theta^t)) + Q_t s(\sigma(\theta^t)) \leq c(\sigma(\theta^t)) + s(\sigma(\theta^{t-1})) \\
 &s(\theta^0) = 0 \text{ given.}
 \end{aligned}$$

Constrained Pareto Efficiency

- ▶ An allocation $(c, y, K), (x, s)$ is *incentive-compatible* if:

$$\begin{aligned} V((c, y), \hat{Q})(\sigma^*) &= \sum_{t=0}^T \sum_{\theta^t} \beta^t \pi(\theta^t) U \left(x(\theta^t), \frac{y(\theta^t)}{\theta^t} \right) \\ &\geq V((c, y), \hat{Q})(\sigma) \quad \forall \sigma \in \Sigma \end{aligned}$$

where σ^* is the *truth-telling* reporting strategy.

- ▶ Possibility of *double deviation*:
 - ▶ Misreporting their type
 - ▶ Adjust trading strategy while deviating
- ▶ $(c, y, K), (x, s)$ is feasible if:

$$\sum_{\theta^t} \pi(\theta^t) x(\theta^t) + K_{t+1} \leq F \left(K_t, \sum_{\theta^t} \pi(\theta^t) y(\theta^t) \right) \quad \forall t$$

Proposition

If $(c, y, K), (x, s)$ is incentive-compatible and feasible then there exists a $(c', y, K), (x, 0)$ that is incentive-compatible and feasible.

Proof

- ▶ $c'(\theta^t) \equiv x(\theta^t) = c(\theta^t) - Q_t s(\theta^t) + s(\theta^{t-1})$.
- ▶ Any (σ', x', s') that improves upon $(c', y, K), (x, 0)$ would also improve upon $(c, y, K), (x, s)$
- ▶ if $(c, y, K), (x, s)$ is incentive compatible, then so is $(c', y, K), (x, 0)$
- ▶ Also, $(c', y, K), (x, 0)$ clearly satisfies feasibility.

□

Definition

(c, y, K) is CPE if it solves [SPP1]

$$\begin{aligned} \max_{c, y, K} \quad & \sum_{t=0}^T \sum_{\theta^t} \beta^t \pi(\theta^t) U \left(c(\theta^t), \frac{y(\theta^t)}{\theta^t} \right) \\ \text{s.t.} \quad & \sum_{\theta^t} \pi(\theta^t) c(\theta^t) + K_{t+1} \leq F \left(K_t, \sum_{\theta^t} \pi(\theta^t) y(\theta^t) \right) \quad \forall t \\ & \sum_{t=1}^T \sum_{\theta^t} \beta^t \pi(\theta^t) U \left(c(\theta^t), \frac{y(\theta^t)}{\theta^t} \right) \geq \hat{V}(c, y) \end{aligned}$$

Proposition

(c, y, K) solves [SPP1] iff solves [SPP2] defined as:

$$\begin{aligned} \max_{c, y, K, Q} \quad & \sum_{t=0}^T \sum_{\theta^t} \beta^t \pi(\theta^t) U \left(c(\theta^t), \frac{y(\theta^t)}{\theta^t} \right) \\ \text{s.t.} \quad & \sum_{\theta^t} \pi(\theta^t) c(\theta^t) + K_{t+1} \leq F \left(K_t, \sum_{\theta^t} \pi(\theta^t) y(\theta^t) \right) \quad \forall t \\ & \sum_{t=1}^T \sum_{\theta^t} \beta^t \pi(\theta^t) U \left(c(\theta^t), \frac{y(\theta^t)}{\theta^t} \right) \geq V(c, y, Q)(\sigma) \quad \forall \sigma \\ & Q_t u'(c(\theta^t)) = \beta \sum_{\theta^{t+1}} \pi(\theta^{t+1} | \theta^t) u'(c(\theta^{t+1})) \quad \forall t \quad \forall \theta^t \end{aligned}$$

Proof The constraint sets are equivalent.



Characterization of CPE allocation

- ▶ $\forall \theta^t \in \Theta^t$

$$MRS(\theta^t) \equiv \frac{u'(c(\theta^t))}{\beta \sum_{\theta^{t+1}} \pi(\theta^{t+1}|\theta^t) u'(c(\theta^{t+1}))} = 1/Q_t$$

then: MRS are equalized across histories.

- ▶ This result does not hold generally with observable asset trading

Competitive Equilibrium

Representative Firm Problem:

$$\begin{aligned} \max_{c,y,d,K,b} \quad & \sum_{t=0}^T \left(\prod_{\tau=1}^{t-1} q_{\tau} \right) d_t \\ \text{s.t.} \quad & \sum_{\theta^t} \pi(\theta^t) c(\theta^t) + K_{t+1} + d_t + q_t b_{t+1} \\ & \leq F \left(K_t, \sum_{\theta^t} \pi(\theta^t) y(\theta^t) \right) \quad \forall t \\ & V((c, y), Q)(\sigma^*) \geq V((c, y), Q)(\sigma) \quad \forall \sigma \in \Sigma \\ & V((c, y), Q)(\sigma^*) \geq \underline{U} \end{aligned}$$

Note: firm takes Q as given in the IC

Competitive Equilibrium

Definition

A CE is $(\{c_t, y_t, K_t, q_t, d_t, b_t, Q_t\}, (\underline{U}))$ s.t.

- ▶ (c, y, K, d, b) solves the firm's problem given (q, Q, \underline{U})
- ▶ The household chooses the contract (c, y) that offers the highest ex-ante utility
- ▶ Aggregate feasibility:

$$\sum_{\theta^t} \pi(\theta^t) c(\theta^t) + K_{t+1} \leq F \left(K_t, \sum_{\theta^t} \pi(\theta^t) y(\theta^t) \right) \quad \forall t$$

- ▶ Q_t clears the retrading market for given (c, y) , and agents choose their savings and reporting strategies optimally.

For purposes of comparison, the firm's problem can be rewritten as [F2]:

$$\begin{aligned}
 \max_{c,y,K} \quad & \sum_{t=0}^T \sum_{\theta^t} \beta^t \pi(\theta^t) U \left(c(\theta^t), \frac{y(\theta^t)}{\theta^t} \right) \\
 \text{s.t.} \quad & \sum_{\theta^t} \pi(\theta^t) c(\theta^t) + K_{t+1} \leq F \left(K_t, \sum_{\theta^t} \pi(\theta^t) y(\theta^t) \right) \forall t \\
 & \sum_{t=1}^T \sum_{\theta^t} \beta^t \pi(\theta^t) U \left(c(\theta^t), \frac{y(\theta^t)}{\theta^t} \right) \geq V(c, y, Q)(\sigma) \forall \sigma \in \Sigma \\
 & \{Q_t\} \text{ given}
 \end{aligned}$$

using the fact that $\forall (c, y, K), (x, s)$ is incentive-feasible then it exists a $(c', y, K), (x, 0)$ that is incentive-feasible.

Characterization of CE

Any CE allocation must satisfy:

$$q_t = Q_t = 1/F_k(K_{t+1}, Y_{t+1})$$

$$u'(c(\theta^t)) = \beta(1/Q_t) \sum_{\theta^{t+1}} \pi(\theta^{t+1}|\theta^t) u'(c(\theta^{t+1}))$$

That is:

$$MRS(\theta^t) = MRT_t \quad \forall t, \forall \theta^t$$

Failure of the First Welfare Theorem

- ▶ In CE: $MRS(\theta^t) = MRS(\tilde{\theta}^t) = MRT_t$
- ▶ In CPE: $MRS(\theta^t) = MRS(\tilde{\theta}^t)$ not necessarily equal to MRT_t

The FWT does not apply generally

- ▶ Comparing SPP2 and F2: same objective function, different constraint set.
- ▶ If (c, y, K) are attainable in F2 given Q , then (c, y, K, Q) are attainable in SPP2
- ▶ *Vice-versa* not generally true
- ▶ *Externality* problem

iid shock, two states

We'll show a case when government intervention through
distorting taxation improves CE allocation.

Let $T = 2$, $\Theta \equiv \{\theta_l, \theta_h\}$, $\theta_t \sim iid \forall t = 1, 2$.

Let $v : \mathbb{R}_+ \rightarrow \mathbb{R}$ be concave.

IC requires that c is monotonic.

Proposition

The constrained efficient allocation requires

$$1/Q < F_k(K_2, Y_2).$$

Therefore the CE allocation is not constrained Pareto efficient.

Proof Outline

We want to characterize the CPE allocation.

Claim: The only binding ICs involve $\sigma_t(\theta^t) = \theta_l$ where $\theta_t = \theta_h$.

Claim: For any possibly binding IC the deviation strategy is s.t. $s(\sigma(\theta^t)) \geq 0$, and for least one history $s(\sigma(\theta^t)) > 0$.

Case (i): $\sigma(\theta_1, \theta_2) = \theta_1 \quad \forall \theta_2 \in \Theta$, then

$$Qu'(c(\theta_1)) = \beta \sum_{\theta_2} \pi(\theta_2) u'(c(\theta_1, \theta_2))$$

$$Qu'(c(\theta_1) - Qs(\sigma)) = \beta u'(c(\sigma(\theta_1, \theta_2)) + s(\sigma))$$

both hold only if $s(\sigma) > 0$.

Case (ii): $\sigma(\theta_1) = \theta_1$ and truth-telling at $t = 2$, then:

$$Qu'(c(\theta_1)) = \beta \sum_{\theta_2} \pi(\theta_2) u'(c(\theta_1, \theta_2))$$

$$Qu'(c(\theta_1) - Qs(\sigma)) = \beta \sum_{\theta_2} \pi(\theta_2) u'(c(\theta_1, \theta_2))$$

both hold only if $s(\sigma) = 0$.

Claim: If an IC is binding then for the alternative σ

$$\frac{\partial V(\sigma)}{\partial Q} \leq 0$$

with strict inequality in at least one case.

Recall:

$$\begin{aligned} V((c, y), Q)(\sigma) &= \max_{x, s} \sum_{t=0}^2 \sum_{\theta^t} \beta^t \pi(\theta^t) U \left(x(\sigma(\theta^t)), \frac{y(\sigma(\theta^t))}{\theta_t} \right) \\ &\text{s.t. } x(\sigma_1(\theta_1)) + Q_t s(\sigma_1(\theta_1)) \leq c(\sigma_1(\theta_1)) \quad \forall \theta_1 \\ &\quad x(\sigma_2(\theta_1, \theta_2)) \leq s(\sigma_1(\theta_1)) + c(\sigma_2(\theta_1, \theta_2)) \quad \forall \theta_1, \forall \theta_2 \end{aligned}$$

Envelope Thm implies:

$$\frac{\partial V(\sigma)}{\partial Q} = - \sum_{\theta_1 \in \Theta} \lambda(\theta_1) s(\sigma_1(\theta_1)) \leq 0$$

Considering the FONC of the SPP, after some heroic algebraic manipulation:

$$\begin{aligned}\eta(\theta)[Q^2 u''(c(\theta_h)) + \beta \sum_{\theta_i} \pi(\theta_i) u''(c(\theta_h, \theta_i))] &= \\ &= \pi(\theta_h) \lambda_2 [Q F_k(2) - 1], \\ - \sum_{\sigma} \mu(\sigma) \frac{\partial V(\sigma)}{\partial Q} + \sum_{\theta} \eta(\theta) u'(c(\theta_i)) &= 0\end{aligned}$$

$\frac{\partial V(\sigma)}{\partial Q} \leq 0$ for any binding deviating strategy (strict in one)

\Rightarrow at least one $\eta(\theta^2) < 0$

$\Rightarrow 1/Q < F_k(K_2, Y_2)$ as wanted.

□

Intuition

- ▶ In anticipation of future misreporting households save
- ▶ Increasing Q disincentivizes such a deviation strategy
- ▶ Relaxes IC constraints
- ▶ SP does not need to equate $1/Q$ to F_k

Intuition

- ▶ In anticipation of future misreporting households save
- ▶ Increasing Q disincentivizes such a deviation strategy
- ▶ Relaxes IC constraints
- ▶ SP does not need to equate $1/Q$ to F_k

Can distorting capital taxes be welfare improving in a CE?

Trade-off:

- ▶ Increasing $Q \Rightarrow$ relaxes IC
- ▶ Distorts investment decision

Proposition

A strictly positive linear capital tax with lump-sum rebate is welfare improving when θ^t is iid.

Proof

- ▶ Let R denote the return on capital.
- ▶ Govt Budget Constraint: $T = \tau Rk(\tau, T)$
- ▶ Using the IFT, we can find $T'(0) = Rk$.

- ▶ Then from [F2] we can define $W(\tau, T(\tau))$

$$\max_{c,y,K} \sum_{t=1}^2 \sum_{\theta^t} \beta^t \pi(\theta^t) U \left(c(\theta^t), \frac{y(\theta^t)}{\theta^t} \right)$$

$$s.t. \quad \sum_{\theta} \pi(\theta) c(\theta) + K_2 \leq F_{k,1} K_1 + F_{y,1} Y_1$$

$$\sum_{\theta^2} \pi(\theta^2) c(\theta^2) \leq (1 - \tau) F_{k,2} K_2 + F_{y,2} Y_2 + T$$

$$\sum_{t=1}^T \sum_{\theta^t} \beta^t \pi(\theta^t) U \left(c(\theta^t), \frac{y(\theta^t)}{\theta^t} \right) \geq V(c, y, Q(\tau))(\sigma)$$

- ▶ Let γ_t be the Lagrange multiplier on feasibility and $\lambda(\sigma)$ be the Lagrange multiplier on the IC.

- ▶ $\frac{\partial W(\tau, T(\tau))}{\partial \tau} \Big|_0 = W_{\tau}(0, 0) + W_T(0, 0) T'(0)$

- ▶ Then $W_T = \gamma_2$ and $W_{\tau} = -\gamma_2 Rk - \sum_{\sigma} \lambda(\sigma) \frac{\partial V(\sigma)}{\partial Q} \frac{\partial Q_t}{\partial \tau}$.

- ▶ Then $\frac{\partial W(\tau, T(\tau))}{\partial \tau} \Big|_0 = - \sum_{\sigma} \lambda(\sigma) \frac{\partial V(\sigma)}{\partial Q} \frac{\partial Q_t}{\partial \tau}$
- ▶ Recall: $\frac{V(\sigma)}{\partial Q} \leq 0 \forall \sigma$ s.t. the IC is binding and strictly negative for at least one of such σ .
- ▶ $Q = \frac{1}{R_2(1-\tau)}$.
- ▶ Using the Envelope Theorem we get $\frac{\partial Q}{\partial \tau} = \frac{1}{R_2(1-\tau)^2} > 0$.

Therefore

$$\frac{\partial W(\tau, T(\tau))}{\partial \tau} \Big|_0 > 0$$

so there is some positive capital tax that improves welfare.



Absorbing Shocks

Taxing capital is not a general result. Consider the following process $\Theta \equiv \{0, 1\}$:

$$\pi(\theta_1 = 0) = 1/2, \pi(\theta_1 = 1) = 1/2, \quad \Pi = \begin{pmatrix} 1 & 0 \\ 1 - \rho & \rho \end{pmatrix}$$

The constrained efficient allocation solves

$$\begin{aligned} \max_{c, y, K} \quad & 0.5[u(c_1) + v(y_1) + \beta u(c_{11}) + \beta v(y_{11})] + \\ & 0.5[u(c_0) + \beta \rho u(c_{00}) + \beta(1 - \rho)(u(c_{01}) + v(y_{01}))] \\ \text{s.t.} \quad & \sum_{t=1}^T \sum_{\theta^t} \beta^t \pi(\theta^t) U(c(\theta^t), y(\theta^t)/\theta^t) \geq \hat{V}(c, y) \\ & \text{feasibility} \end{aligned}$$

Claim

At the binding ICs the optimal deviating strategy always involves either saving or borrowing.

Consider deviating strategy $\sigma_2(\theta_1 = 0, \theta_2 = 1) = 0$, then:

$$Qu'(c_0) = \beta[\rho u'(c_{00}) + (1 - \rho)u'(c_{01})]$$

$$Qu'(c_0 - Qs) = \beta u'(c_{00} + s)$$

Both hold only if $s > 0$.

However, consider deviating strategy $\sigma_1(\theta_1 = 1) = 0$,
 $\sigma_2(\theta_1 = 1, \theta_2 = 1) = 1$, it must be:

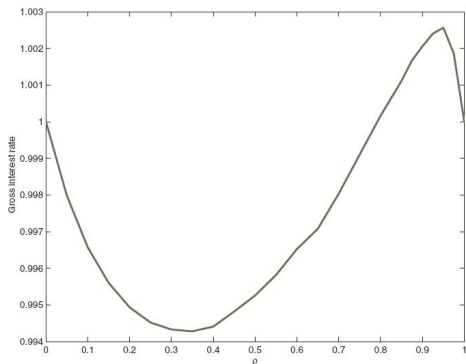
$$Qu'(c_0) = \beta(1 - \rho)u'(c_{01}) + \beta\rho u'(c_{00})$$

$$Qu'(c_0 - Qs) = \beta u'(c_{01} + s)$$

Both hold only if $s < 0$.

- ▶ Increasing $1/Q$ may be detrimental
- ▶ it will tighten the last IC
- ▶ If ρ is high this is the prevailing effect
- ▶ The welfare improving tax will be negative.

Computational Experiment



Suppose that $\beta = 1$ and $F(K, Y) = K + Y$, so that the equilibrium Q is 1. Then, find the optimal Q as a function of ρ , the probability of transitioning from state zero to state 1.

Comparison with Previous Results

Cole-Kocherlakota (ReStud, 2001), Abraham-Pavoni (JEEA, 2005)
among others

- ▶ Introduce hidden trading in an environment with informational frictions
- ▶ Fixed *exogenous interest rate* (backyard technology)
⇒ is no externality problem
- ▶ CE is efficient, so no role for government.
- ▶ In this paper: return on bonds is endogenous, so government action can improve welfare.

Further Developments

- ▶ Implications for consumption dynamics and asset pricing (introducing aggregate uncertainty)
- ▶ Imperfect *enforcement* in the retrading market and for the competitive firms
- ▶ Role of the retrading market as a constraint for a self-interested government (political economy)