EC 489/Deniz Cicek
Summer 2007

Name Surname:

Final

90 Minutes

*Answer **all** of the four questions below.*

1. Let $\succeq$ be a reflexive, complete and transitive preference relation defined on $\mathbb{R}^L_+$. Suppose $\succeq$ is strictly convex and has the following property

$$x' \geq x \text{ implies that } x' \succeq x, \text{ for every } x, x' \in \mathbb{R}^L_+.$$

   (a) Define strict convexity. (6 Points)

   (b) Define local non-satiation. (6 Points)

   (c) Prove that $\succeq$ is locally non-satiated. (8 Points)

2. Consider a pure exchange economy with commodity space $\mathbb{R}^L$ (so that there are L perfectly divisible commodities) and N traders $i = 1, ..., N$, each having consumption set $\mathbb{R}^L_+$, initial endowment vector $e_i \in \mathbb{R}^L_+$, and preferences represented by a continuous utility function $u_i : \mathbb{R}^L_+ \to \mathbb{R}$.

   (a) Define competitive equilibrium in this pure exchange economy. (5 Points)

   (b) Does competitive equilibrium necessarily exist in this economy? Justify your answer. (5 Points)

   (c) Define Pareto Optimality. (5 Points)

   (d) State the First Welfare Theorem. (5 Points)

   (e) Briefly discuss the economic significance of the First Welfare Theorem. (5 Points)

3. Consider the exchange economy in question 2. Assume $L = 2$, $N = 2$, $e_1 = (6, 0)$ and $e_2 = (3, 9)$. Suppose utility functions are as follows:

$$U_1(x_1, y_1) = x_1 y_1$$
$$U_2(x_2, y_2) = \min\{x_2, 2y_2\}$$

(a) Draw the appropriate Edgeworth box, show the initial endowment point and initial indifference curves VERY CLEARLY. (7 Points)

(b) Find the Walrasian (competitive) equilibrium. (8 Points)

(c) Show the equilibrium allocation, budget line and the indifference curves on the same Edgeworth diagram VERY CLEARLY. (7 Points)

(d) Consider the allocation $\hat{\omega} = ((5, 7), (4, 2))$. Note that $\hat{\omega}$ is Pareto efficient. Calculate prices $\hat{p_x}$ and $\hat{p_y}$ and lump-sum cash transfers $\hat{T_1}$ and $\hat{T_2}$ that will support $\hat{\omega}$ as a competitive equilibrium allocation. (8 Points)

4. Consider the Chicken Game (CG):

	Swerve	*Straight*
Swerve	$4, 4$	$1, 6$
Straight	$6, 1$	$-3, -3$

(a) Find the Nash Equilibria of the CG. (10 Points)

(b) Consider the infinitely repeated game with discount factor $\delta \in (0, 1)$ given by the CG as the stage game.

i. Consider the following strategy for player i, $i = 1, 2$: "Play *Swerve* at all periods regardless of what was played in the history."
Prove that this is not a subgame perfect equilibrium for any $\delta \in (0, 1)$. (5 Points)

ii. Consider the following strategy for player i, $i = 1, 2$: "Play *Swerve* at $t = 1$. For $t > 1$, play *Swerve* if no one deviated from *Swerve* in the history; otherwise play whatever action the other player played in the last period $(t - 1)$".
Prove that this is not a subgame perfect equilibrium for any $\delta \in (0, 1)$. (5 Points)

iii. Find a subgame perfect equilibrium of the infinitely repeated CG. (Hint: Think simple!)(5 Points)
(If you find a subgame perfect equilibrium that can sustain the outcome $(Swerve, Swerve)$ at all periods, you will get 5 points bonus.)