Economics 4113, Spring 2009. Instructor: David Rahman, University of Minnesota.

Homework 2—Due February 19, 2009

1. Write down the duals of the following LPs: (a) $\max\{cx : Ax = b, x \geq 0\}$, (b) $\min\{cx : Ax = b, x \geq 0\}$, (c) $\max\{cx : Ax \leq b\}$ (d) $\min\{cx : Ax \geq b\}$.

2. Find the optimal primal and dual solutions to the following LP:

$$\max_{x \geq \mathbf{0}} \; x_1 + x_2 - 3x_3 \quad \text{s.t. } x_1 + 2x_2 - 3x_3 = 4, \; 4x_1 + 5x_2 - 9x_3 = 13.$$

3. Convert the following optimization into a linear program.

$$\min_{x,y,z} \; |x| + |y| + |z| \quad \text{s.t. } x + y \leq 1, \; 2x + z = 3.$$

4. Let $V = \max\{\sum_{j=1}^{n} c_j x_j : \sum_{j=1}^{n} a_j x_j \leq b, \; x \geq \mathbf{0}\}$. Assume that all c_j and a_j are positive. Show that $V = b \max_j c_j/a_j$.

5. Use strong duality to prove the Theorem of the Alternative.

6. Consider the following linear programming problem:

$$\begin{aligned} \max_{x_1,x_2} \; \alpha x_1 + \beta x_2 \; & \text{ subject to} \\ x_1 + 2x_2 &\leq 4, \\ 2x_1 + x_2 &\leq 5, \\ x_1, x_2 &\geq 0, \end{aligned}$$

   where α and β are real numbers. Completely classify the optimal solutions x^* of this linear program as well as the value of the problem in terms of the possible range of values that α and β could take.