

Homework 6—Due May 14, 2009

1. Consider a production function

$$y = A \prod_{j=1}^n x_j^{\alpha_j}$$

where y is output, the x_j 's are inputs, and A and the α_j 's are positive constants. Let $w = (w_1, \dots, w_n)$ be a vector of input prices. Show that the minimum cost of producing a given output level y is

$$C(w, y) = \beta(y/A)^{1/\beta} \prod_{j=1}^n (w_j/\alpha_j)^{\alpha_j/\beta},$$

where $\beta = \sum_j \alpha_j$. Assuming that $\beta < 1$, calculate the corresponding maximum profit function $\pi(p, w)$, where p is the output price. What goes wrong if $\beta \geq 1$?

2. Suppose that the direct utility function is $U(x, y) = (\alpha x^\rho + \beta y^\rho)^{1/\rho}$, with $\alpha, \beta > 0$ and $\rho < 1$.

- (a) Show that the expenditure function is of the form

$$E(p, q, u) = \rho u (ap^r + bq^r)^{1/r},$$

where p and q are the prices of each of the two goods, u is the utility level, and a, b and r are constants that can be expressed in terms of α, β and ρ .

- (b) Find the compensated demand functions and show that the ratio of the expenditure-minimizing quantities equals

$$x/y = (a/b)(q/p)^{1-r}.$$

- (c) The elasticity of (x/y) with respect to (q/p) is given by

$$\frac{d \ln(x/y)}{d \ln(q/p)}$$

and is called the elasticity of substitution. Show that in this example it is constant and equal to $1 - r$. What condition must be imposed on ρ to ensure a non-negative elasticity of substitution, i.e., $r < 1$?

3. Consider an economy with two agents, A and B . Agent A is endowed with a left glove and agent B with a right glove. Let $v_i > 0$ be the utility to agent i if he receives one left glove and one right glove. If an agent receives only one glove or no gloves then he gets a utility of 0.
- (a) Find the core of this economy.
 - (b) Calculate the set of equilibrium prices in this economy. (You may assume that even if an agent receives more than one pair of gloves his utility does not increase beyond v_i .)
 - (c) Now suppose that there are n identical copies of agent A and $n+1$ copies of agent B , for any natural number n . Find the core of this economy as well as the set of equilibrium prices. (Hint: You may assume equal treatment of equals.)

Acknowledgment: Questions 1 and 2 above were taken from Dixit's (1995) textbook.