Midterm 1—February 21, 2008

Instructions: You have 1 hour and 15 minutes to answer the questions below. Provide answers as complete as possible. Good luck!

1. (a) Solve the following linear program, i.e., find an optimal solution and compute the value of the problem.

$$\begin{aligned} \min_{\pi \in \mathbb{R}^3} \ \pi_1 + \pi_2 + \pi_3 \ \text{ subject to} & \\ \pi_1 + \pi_2 &\geq 1, \\ \pi_1 + \pi_3 &\geq 1, \\ \pi_2 + \pi_3 &\geq 1, \\ \pi_1, \pi_2, \pi_3 &\geq 0. \end{aligned}$$

   (b) Solve the following linear program for every possible value of α:

$$\begin{aligned} \min_{\pi \in \mathbb{R}^3} \ \pi_1 + \pi_2 + \pi_3 \ \text{ subject to} & \\ \pi_1 + \pi_2 &\geq 1, \\ \pi_1 + \pi_3 &\geq 1, \\ \pi_2 + \pi_3 &\geq 1, \\ \pi_1 + \pi_2 + \pi_3 &\geq \alpha, \\ \pi_1, \pi_2, \pi_3 &\geq 0. \end{aligned}$$

2. Consider the following linear program:

$$\begin{aligned} \max_{x_1, x_2} \ x_1 + 2x_2 \ \text{ subject to} & \\ x_1 + \tfrac{8}{3}x_2 &\leq 2\alpha, \\ x_1 + x_2 &\leq \alpha + \beta, \\ 2x_1 \quad &\leq 3 + \beta \\ x_1, x_2 &\geq 0, \end{aligned}$$

   where α and β are real numbers.

   (a) Suppose that $\beta = 0$ is fixed. Completely classify the optimal solutions x^* of this linear program as well as the value of the problem in terms of the possible range of values that α could take. Also, completely classify the optimal solutions y^* of its dual problem as a function of α. Plot the graphs of y_1^* as a function of α, y_2^* as a function of α, y_3^* as a function of α, and $2y_1^* + y_2^*$ as a function of α.

   (b) Now suppose $\alpha = 1$ is fixed, and redo the previous exercise with β changing. Draw the graph of $y_2^* + y_3^*$ as a function of β. Comment.