

Economics 4113: Midterm 2—April 3, 2008

INSTRUCTIONS: You have 1 hour and 15 minutes to answer the questions below. Provide answers as complete as possible. Good luck!

An individual lives for two years. He earns income I_1 during the first year and I_2 during the second year. Each year, the individual may consume two goods, x and y . Let x_1 denote the amount of good x consumed in the first year, and x_2 the amount of good x consumed in the second year; similarly for y_1 and y_2 . Let p_1 and q_1 be the prices of goods x and y , respectively in the first year, and p_2 and q_2 those in the second year. Assume for now that the individual cannot borrow or save. The individual's budget problem is given below:

$$\max_{x,y \geq 0} \alpha \ln(x_1) + \beta \ln(y_1) + \delta(\alpha \ln(x_2) + \beta \ln(y_2)) \quad \text{subject to}$$
$$p_1 x_1 + q_1 y_1 \leq I_1 \quad \text{and} \quad p_2 x_2 + q_2 y_2 \leq I_2.$$

where α and β are positive, $0 < \delta < 1$, I_1, I_2 are also positive, and $\alpha + \beta = 1$.

1. Solve the optimization problem, i.e., calculate the optimal consumption of each good at each date. Also calculate the marginal utility of money income at both dates 1 and 2. Find an expression for the indirect utility

$$V(p_1, q_1, p_2, q_2, I_1, I_2),$$

i.e., the value function of this budget problem.

2. Now suppose that the individual can borrow and save at the rate of interest $r > 0$. Let s be the amount of money that the individual saves in the first year to have available in the second year. (If $s < 0$ then we interpret this as borrowing.)

Thus, if an individual saves an amount s then the income has available to purchase goods in date 1 equals $I_1 - s$, whereas his income in date 2 equals $I_2 + (1 + r)s$. In date 2, after saving an amount s , the individual receives principal (s) plus interest (rs). Implicitly we are assuming that the borrowing rate equals the lending rate.

Let $W(s) = V(p_1, q_1, p_2, q_2, I_1 - s, I_2 + (1 + r)s)$ be the indirect utility after saving an amount s from date 1 for date 2. The individual's saving problem is

$$\max_s W(s).$$

Decide if W is a concave function or a convex function (or neither) of s . Find the optimal saving decision s^* . Under what conditions is $s^* = 0$? What is the individual's optimal decision if $\delta = 0$? Now, suppose that $\delta = 1/(1 + r) > 0$. How much money does the individual end up spending in date 1 and date 2? What is the intertemporal expenditure ratio $(I_1 - s^*)/[I_2 + (1 + r)s^*]$? Find the intertemporal consumption ratios x_1^*/x_2^* and y_1^*/y_2^* after saving s^* .