Correlated Information and Direct Mechanism Design

David Rahman*
University of Minnesota

December 17, 2010

Abstract

In this paper I study the possibility of full surplus extraction on arbitrary type spaces using direct mechanisms, with three main results: (i) I characterize full surplus extraction, (ii) I characterize full surplus extraction assuming incentive compatibility, and (iii) I show that virtually full surplus extraction implies full surplus extraction. The characterizing condition in (ii) above is equivalent to McAfee and Reny's (1992a) condition in their restricted environment. Since they only characterized virtually full surplus extraction, it follows that their proposed mechanisms incur a loss relative to the direct mechanisms studied here, with which full surplus extraction is possible.

JEL Classification: C62, D71, D82.
Keywords: surplus extraction, direct mechanisms, convex independence.

*I owe many thanks to Christoph Müller for helpful conversations and encouragement. Please send any comments to dmr@umn.edu.

1 Introduction

In this paper I study surplus extraction in arbitrary type spaces with direct mechanisms. Although there is already notable work on the topic of surplus extraction, most prominently by Cremer and McLean (1988) and McAfee and Reny (1992a), there is not yet a systematic characterization for arbitrary settings that employs general mechanisms. Moreover, there is growing interest in mechanism design when type spaces are general enough to cover the universal one. The work of Heifetz and Neeman (2006) is a notable example of this interest.

I contribute to this topic with three main results. In Theorem 1, I provide a necessary and sufficient condition on agents' beliefs such that there exists a direct mechanism that extracts all of an agent's surplus. In Theorem 2 I provide a necessary and sufficient condition on agents' beliefs such that there exists a direct mechanism that extracts all the surplus, assuming that the surplus-extracting allocation is implementable. In Theorem 3, I show that for any type space, virtually full surplus extraction implies full surplus extraction.

These results improve on the existing literature in several ways. Theorem 1 extends Cremer and McLean's characterization to arbitrary type spaces. Cremer and McLean showed that, for any (quasi-linear) utility function there exists a surplus-extracting mechanism if and only if agents' beliefs exhibit convex independence. Crucially, they assumed that the space of agents' types is finite. In Example 2 I show that convex independence is no longer enough to guarantee surplus extraction when the type space is infinite. I then supply the corresponding condition for arbitrary type spaces, which I call virtual convex independence. On the other hand, Example 2 shows that requiring full surplus extraction for every utility function is too restrictive because for some utility functions the surplus-extracting allocation is simply not implementable. This problem is emphasized in Proposition 1, which essentially argues that a finite type space is *necessary* to guarantee surplus extraction for every utility function.

This motivates Theorem 2, which generalizes McAfee and Reny's characterization. To see how, let me first briefly summarize their work. McAfee and Reny restricted attention to a compact-continuous environment and divided the surplus extraction problem into two stages. In the first stage, the agent is offered a menu consisting of finitely many "participation fee schedules." In the second stage, the agent is asked to reveal his type with the promise that his previous choice of participation fee will not be used by the principal in the second stage. McAfee and Reny leave the second stage implicit—i.e., they assume that providing incentives is possible in the second stage—and characterize beliefs such that virtually all the surplus can be extracted in the first stage. In other words, they asked for surplus extraction for every utility function *as long as* the surplus-extracting allocation is implementable, thereby avoiding the problems of Theorem 1 alluded to above. Because they focused on finite participation schedules, they were not able to extract all the surplus. Instead, they settled with extracting virtually all the surplus, i.e., they showed that there exists a sequence of schedules that uniformly extracts all but a vanishing amount of the surplus. However, since their argument is non-constructive, it leaves open the question of whether or not such a sequence of schedules might converge to one that extracts all the surplus.

Theorem 2 generalizes McAfee and Reny's characterization in the following ways. First, while keeping the second stage implicit as they do, I use direct revelation mechanisms in the first stage, rather than finite participation fee schedules. This allows me to use the same techniques as for Theorem 1 to obtain a characterization of full surplus extraction, not just virtually full, for arbitrary environments, not just compact-continuous ones. However, the condition I derive, called asymptotic convex independence, coincides with McAfee and Reny's in their restricted compact-continuous setting. In other words, in compact-continuous environments, the same condition on beliefs generates full surplus extraction in the first stage with direct mechanisms but only virtually full surplus extraction with finite participation fee schedules. This suggests that full surplus extraction generally fails with finite participation fee schedules. In Theorem 3 I confirm this intuition by showing that in arbitrary environments, with direct revelation mechanisms virtually full surplus extraction implies full surplus extraction. This result holds regardless of whether or not the second stage is left implicit.

Moreover, all of the results above are derived by extending the duality techniques that Cremer and McLean (1988) used in their setting with finitely many types to environments with arbitrary type spaces. This contrasts the work of McAfee and Reny (1992a,b). They emphasized that their approach was not just an application of duality, but rather somewhere "[. . .] between the Stone-Weierstrass Theorem and a corollary to the Hahn-Banach Theorem." (McAfee and Reny, 1992b, p. 61.) Contrariwise, in this paper I show that with direct mechanisms, surplus extraction is characterized in arbitrary environments using duality.

2 Model

Consider the following relatively standard mechanism design environment. There is an agent with private information and a principal who solicits this information from the agent. The agent sends a message to the principal which may or may not be truthful. The principal subsequently observes a signal possibly correlated with the agent's information, such as output or other agents' types.

Formally, let T be an arbitrary set with typical element t, interpreted as the collection of all *types* for the agent that the principal deems possible. Let X be a nonempty set of outcomes and $(Y, \mathscr{Y})$ a measurable space of possible signals that the principal may observe. For each type t, let $p(t)$ be a countably additive probability measure on $\mathscr{Y}$ describing the likelihood of signals given t. Let $u(t, x, y) \in \mathbb{R}$ be the agent's utility from choice x when his type is t and the realized signal is y. An *allocation* is a map $\mathbf{x} : T \times Y \to X$, where $\mathbf{x}(t, y)$ represents the choice made by the principal when the agent's report is t and the realized signal is y. An *incentive scheme* (or simply *scheme*) is a map $\xi : T \times Y \to \mathbb{R}$, where $\xi(t, y)$ represents the payment from the agent to the principal when his report is t and the realized signal is y. An incentive scheme is denominated in money, which enters the agent's utility linearly with unit marginal utility, as usual. A *mechanism* is any pair $(\mathbf{x}, \xi)$ as above.

The expected utility to the agent from an allocation $\mathbf{x}$ when his type is t—assuming that he tells the truth—is given by

$$v(t) \;=\; \int_Y u(t, \mathbf{x}(t, y), y) p(dy|t),$$

and the expected utility gain from reporting s when his type is actually t is given by

$$\Delta v(t, s) \;=\; \int_Y [u(t, \mathbf{x}(s, y), y) - u(t, \mathbf{x}(t, y), y)] p(dy|t).$$

For the functions above to be well-defined, we must impose some restrictions on v and $\mathbf{x}$. Otherwise, the integrals above may not exist.

Assumption 1. Both $v(t)$ and $\Delta v(t, s)$ are well-defined and real-valued for all (t, s).

I shall maintain this assumption throughout. One way to guarantee that it holds is to assume that $v(t, \mathbf{x}(s, y), y)$ is a bounded, $\mathscr{Y}$-measurable function of y for all (t, s). But this is certainly not the only way.

Definition 1. The mechanism $(\mathbf{x}, \xi)$ is called *incentive compatible* if

$$\Delta v(t,s) \;\leq\; \int_Y [\xi(s,y) - \xi(t,y)] p(dy|t) \qquad \forall (t,s).$$

It is called *individually rational* if

$$v(t) \;\geq\; \int_Y \xi(t,y) p(dy|t) \qquad \forall t.$$

An allocation $\mathbf{x}$ is called *implementable* if there exists a scheme ξ such that $(\mathbf{x}, \xi)$ is incentive compatible and individually rational. In this case, we say ξ *implements* $\mathbf{x}$.

Just as before, for the inequalities above to be well-defined, we must impose some restrictions on ξ. Otherwise, the integrals above may not exist.

Assumption 2. Every scheme ξ satisfies the following property: $\xi(t,y)$ is a bounded $\mathscr{Y}$-measurable function of y for all t, i.e., $\xi \in B(Y)^T$.

I shall also maintain this assumption throughout. Whereas Assumption 1 is relatively uncontroversial, Assumption 2 has a little more content. It reflects a trade-off between restrictions on p versus ξ. For the inequalities defining incentive compatibility above to be well-defined, $\xi(s)$ must be $p(t)$-integrable for all (t,s). Bounded measurability guarantees this without imposing restrictions on p. (McAfee and Reny assume this and much more.) On the other hand, if we assumed that the set of all $p(t)$-integrable functions $L(t)$ was the same set L for all t then we could relax Assumption 2 by requiring only that $\xi \in L^T$. Alternatively, we might also assume that $\xi \geq 0$ and $\xi(t) \in L(t)$ for all t, but this requires that payments be bounded below and would require imposing restrictions on the function u for individual rationality to be feasible.

Definition 2. Say that *all the surplus can be extracted* from $(v, \mathbf{x})$ if there exists a scheme ξ that implements $\mathbf{x}$ and

$$v(t) \;=\; \int_Y \xi(t,y) p(dy|t) \qquad \forall t.$$

Such a ξ is called a *surplus-extracting scheme*. Say that *virtually all the surplus can be extracted* from $(v, \mathbf{x})$ if for every $\varepsilon > 0$ there is a scheme ξ that implements $\mathbf{x}$ and

$$0 \;\leq\; v(t) - \int_Y \xi(t,y) p(dy|t) \;\leq\; \varepsilon \qquad \forall t.$$

Extracting all the surplus means that every individual rationality constraint binds. As will be seen, the fact that we normalized every type's outside option to zero is without loss of generality for our results. Finally, virtually full surplus extraction is defined uniformly across all types, as in McAfee and Reny (1992a).

3 Convex Independence

Before presenting the main results of the paper, in this section I briefly describe the notion of convex independence, which will be useful in the sequel. First I define it, and then, to gain intuition, I present an example where it fails, offer an equivalent formulation, and suggest an interpretation.

To motivate convex independence, recall Cremer and McLean's (1988) contribution. In a setting with finitely many types, they showed that agents' conditional probability vectors exhibit convex independence (defined below) if and only if for any profile of utility functions, every allocation is implementable with a scheme that makes every individual rationality constraint bind. Hence, the scheme extracts all the surplus.

Definition 3. p exhibits *convex independence* if $p(t) \notin \operatorname{conv}\{p(s) : s \neq t\}$ for all t.[1]

Consider the following simple example where convex independence fails.

Example 1. Let $T = \{0, \frac{1}{2}, 1\}$, $Y = \{a, b\}$, and $p(t) = t\delta_a + (1-t)\delta_b$, where δ stands for Dirac measure. Convex independence clearly fails, since $p(\frac{1}{2}) = \frac{1}{2}p(0) + \frac{1}{2}p(1)$, and hence $p(\frac{1}{2})$ lies in the convex hull of $\{p(0), p(1)\}$.

Cremer and McLean's result is often summarized by the slogan "if types are correlated then you can extract the surplus." Of course, this is just a slogan and it is easy to see, as Example 1 shows, that "correlated types" is not enough for surplus extraction. Indeed, types *are* correlated in Example 1 (think of y as others' types), yet convex independence fails. Therefore the surplus cannot always be extracted.

I now derive an easy equivalent formulation of convex independence that will be useful in the sequel. Let $U = \{u \in \mathbb{R}^{T \times T} : u(t,t) = 0\}$ be the vector of possible utility gains, where $u(t,s)$ stands for the gains from deviating to s when the agent's type is t.

Lemma 1. *p exhibits convex independence if and only if given any $\lambda \in \mathbb{R}_+^{(T\times T)}$,*[2]

$$\sum_{s \in T} \lambda(t,s)[p(t) - p(s)] \;=\; 0 \quad \forall t \in T \qquad \Rightarrow \qquad \sum_{(t,s)} \lambda(t,s)u(t,s) = 0 \quad \forall u \in U.$$

A proof of this and all other results in this paper can be found in Appendix A.

For intuition, by Lemma 1 convex independence may be written as follows.

[1] Here, "conv" stands for convex hull.

[2] As a matter of notation, $\mathbb{R}_+^{(T\times T)}$ is the set of non-negative $T \times T$ matrices with finite support.

Notation. Let $\Delta(T|T)$ be the set of non-negative $T \times T$ matrices π such that (i) $\pi(\cdot|t) = \delta_t(\cdot)$ for all but finitely many t (where $\delta_t(s) = 1$ if $t = s$ and zero otherwise),[3] and whenever $\pi(\cdot|t) \neq \delta_t(\cdot)$, (ii) $\pi(\cdot|t)$ has finite support and (iii) $\sum_s \pi(s|t) = 1$.

$\Delta(T|T)$ is the set of matrices that differ from the identity on finitely many rows, where they are probability vectors with finite support. Dividing λ in Lemma 1 by $\sum_s \lambda(t, s)$ for each t in its support, convex independence means that given $\pi \in \Delta(T|T)$,

$$\sum_{s \in T} \pi(s|t)p(s) \;=\; p(t) \quad \forall t \quad \Rightarrow \quad \pi(s|t) = 0 \text{ if } s \neq t.$$

This may be interpreted as "conditional" detectability. If type t could become type s with probability $\pi(s|t)$ then convex independence would mean that there is no way of becoming other types that is indistinguishable from remaining the original type.

4 Guaranteeing Surplus Extraction

In this section I present Theorem 1, which generalizes Cremer and McLean's result to arbitrary type spaces. I then argue in Proposition 1 that guaranteeing surplus extraction for all utility functions is too restrictive with infinitely many types. But first, I present an example that shows how convex independence is generally not enough to guarantee surplus extraction. I will refer to this example repeatedly.

Example 2. Let $T = [0, 1]$, $Y = \{a, b, c\}$ and $p(t) = (1 - t)^2\delta_a + t^2\delta_b + 2t(1 - t)\delta_c$. As Figure 1 below illustrates, clearly p exhibits convex independence.

However, implementability may fail, making surplus extraction impossible. To see this, by a result in Rahman (2010), given the function Δv there is a scheme ξ that delivers incentive compatibility if and only if $\sup_\pi \{w \cdot \pi / \|D\pi\| : \pi \in \Delta(T|T)\} < \infty$, where $\|D\pi\| = \sum_s \|\sum_t \pi(s|t)[p(t) - p(s)]\|$ and $\|\sum_t \pi(s|t)[p(t) - p(s)]\|$ is calculated using the total variation norm. Given $k \in \mathbb{N}$, let $t_k = 1/k$. Define π_k by $\pi_k(0|t_k) = (1 - t_k^2)$, $\pi_k(1|t_k) = t_k^2$ and $\pi_k(\cdot|t) = \delta_t(\cdot)$ for all other t. By routine calculations, $\|D\pi_k\| = O(\frac{1}{k})$. Letting $\Delta v(t, 0) = \sqrt{t}$, it follows that $\Delta v \cdot \pi_k = (1 - \frac{1}{k^2})/\sqrt{k}$. Hence, $\lim \Delta v \cdot \pi_k / \|D\pi_k\| = +\infty$, so no scheme can deliver incentive compatibility. As a result, no fraction of the surplus may be extracted incentive compatibly because no payment scheme can implement the allocation induced by w.

[3] I am slightly abusing notation by using δ to denote sometimes Dirac measure and other times a canonical basis vector. There should be no associated confusion, though.

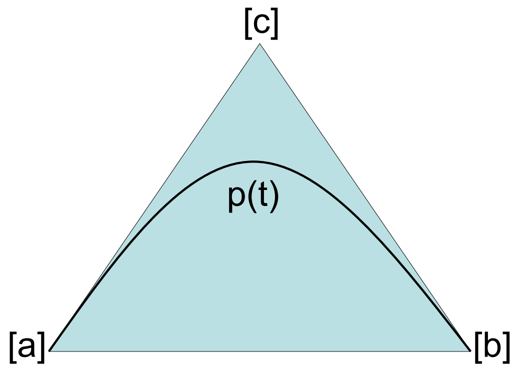

Figure 1: Convex independence holds but virtual convex independence fails.

I now generalize Cremer and McLean's result to arbitrary type and signal spaces.

Definition 4. p exhibits *virtual convex independence* if for any net $\{\lambda_\delta\} \subset \mathbb{R}_+^{(T\times T)}$,

$$\sum_{s\in T} \lambda_\delta(\cdot, s)[p(\cdot) - p(s)] \to 0 \quad \text{weakly}^{4} \quad \Rightarrow \quad \sum_{(t,s)} \lambda_\delta(t,s)u(t,s) \to 0 \quad \forall u \in U.$$

Clearly, virtual convex independence implies convex independence. To see this, just restrict attention to constant nets and use Lemma 1. Moreover, Example 2 shows that the converse implication fails, since convex independence holds there but virtual convex independence fails. As it turns out, virtual convex independence is just the condition that extends Cremer and McLean's result to arbitrary type spaces.

Theorem 1. *All the surplus can be extracted from any given* $(v, \mathbf{x})$ *if and only if* p *exhibits virtual convex independence.*

Arguably, Example 2 and Theorem 1 show that guaranteeing full surplus extraction for every utility function imposes too strong a restriction on the information structure, p. To see this, notice that the reason why surplus extraction cannot be guaranteed for every utility function in Example 2 is that for some utility functions there is no payment scheme that makes truth-telling incentive compatible. Unfortunately, a simple version of this problem can be transplanted into any environment with convex independence and infinitely many types. The next result makes such an argument.

Proposition 1. *p exhibits virtual convex independence if and only if it exhibits convex independence and* $|T| < \infty$.

[4]Henceforth, *convergence is weak unless otherwise stated.* Thus, $\sum_s \lambda_\delta(\cdot, s) \to 0$ means that $\sum_{(t,s)} \lambda_\delta(t,s)f(t) \to 0$ in $\mathbb{R}$ for every $f \in \mathbb{R}^T$.

5 Assuming Incentive Compatibility

Proposition 1 says that there is no condition on beliefs that guarantees full surplus extraction for every utility function when there are infinitely many types. Example 2 suggests two ways around this problem. One is to restrict the domain of utility functions to one where a given allocation can be made incentive compatible. This is problematic because this domain will generally depend on the information structure, making comparisons across environments difficult. Another approach is to settle for guaranteeing surplus extraction only when implementability is possible. This is the approach that we will follow in this section.

Even though they never explicitly mentioned the above problem, McAfee and Reny implicitly followed this approach by dividing the surplus extraction problem into the two stages described in the introduction. As a result they were able to obtain richer characterizations of surplus extraction. One way to interpret their results is that they answered the following question. Assuming a given allocation is implementable, when can (almost) all the surplus be extracted?

I follow a similar approach below, except that—instead of restricting attention to their finite participation schedules—I allow for general surplus-extracting mechanisms. I begin by defining formally what I mean by surplus extraction assuming incentive compatibility and characterizing it in Theorem 2. I then show in Proposition 2 how my characterization generalizes McAfee and Reny's condition.

Definition 5. *All the surplus can be extracted from* $(v, \mathbf{x})$ *assuming incentive compatibility* if there is a scheme ξ that (i) extracts all the surplus, so $\int_Y \xi(t, y)p(dy|t) = v(t)$ for all t, and (ii) satisfies the following system of inequalities:

$$0 \;\leq\; \int_Y [\xi(s, y) - \xi(t, y)]p(dy|t) \qquad \forall(t, s).$$

Intuitively, the system of inequalities above says that ξ does not disrupt any incentive compatibility constraints. Hence, if $\mathbf{x}$ is implementable, say with scheme ζ, then it is possible to find another scheme ξ such that $\zeta + \xi$ still implements $\mathbf{x}$ and extracts the surplus. Our next goal is to characterize information structures that guarantee this to be the case for every $(v, \mathbf{x})$. Then we can find a ξ such that $\xi + \zeta$ extracts the surplus—i.e., all the surplus can be extracted conditional on $\mathbf{x}$ being implementable—regardless of v. This captures McAfee and Reny's idea of an implicit second stage.

Definition 6. p exhibits *asymptotic convex independence* if for every net $\{\pi_\delta\}$ in $\Delta(T|T)$,

$$\sum_{s\in T}\pi_\delta(s|\cdot)p(s)\to p(\cdot) \qquad \Rightarrow \qquad \sum_{s\in T}\pi_\delta(\cdot|s)\to 1.$$

Asymptotic convex independence is strictly weaker than virtual convex independence and strictly stronger than convex independence. To see this, asymptotic convex independence follows from virtual convex independence by restricting λ_δ to add up to one for every t, and Example 2 presents an information structure where virtual convex independence fails yet asymptotic convex independence holds (convex independence holds there, too). Moreover, convex independence follows from asymptotic convex independence by restricting attention to constant nets. We now present a simple example where convex independence holds but asymptotic convex independence fails.[5]

Example 3. Let $T = Y = \mathbb{N}$, $p(t+1) = \delta_t$ for each $t \in \mathbb{N}$, and $p(1) = \sum_k 2^{-k}\delta_k$.

This example shows that the crucial difference between convex independence and asymptotic convex independence is that it allows for infinite combinations. It turns out that this characterizes surplus extraction assuming incentive compatibility.

Theorem 2. *All the surplus can be extracted from any* $(v, \mathbf{x})$ *assuming incentive compatibility if and only if* p *exhibits asymptotic convex independence.*

Theorem 2 is proved similarly to Theorem 1, using duality to characterize surplus extraction given incentive compatibility. Let us now discuss how Theorem 2 and asymptotic convex independence generalize the work of McAfee and Reny.

Assuming T is a compact metric space, McAfee and Reny (1992a, p. 404) characterize *virtually full* surplus extraction assuming incentive compatibility (I explain Section 6 what this means exactly) with the following condition: for every $t \in T$ and $\mu \in \Delta(T)$,[6]

$$p(t) = \int_T p(s)\mu(ds) \qquad \Rightarrow \qquad \mu = \delta_t.^{7} \tag{$*$}$$

First of all, Theorem 2 does not require continuity and compactness. Secondly, it can be shown that asymptotic convex independence is equivalent to condition $(*)$ in their restricted setting. Intuitively, it is well known that—when T is a compact metric space—the set of Borel probability measures with finite support is (weak$*$)

[5] I apologize for such a tongue-twisting paragraph.

[6] As a matter of notation, $\Delta(T)$ is the set of Borel probability measures on T.

[7] McAfee and Reny also assume that p is continuous and $p(t)$ has a continuous density for all t.

dense in $\Delta(T)$ (e.g., Aliprantis and Border, 2006, Theorem 15.10). Therefore, any $\mu(t) \in \Delta(T)$ is the limit of a sequence[8] of probability measures $\{\pi_m(t)\}$ with finite support. In other words, the key difference between asymptotic convex independence and condition $(*)$ is between weak and pointwise convergence. In McAfee and Reny's restricted setting, this difference vanishes.

Proposition 2. *Suppose that T is a compact metric space and both v and p are continuous. All the surplus can be extracted from any given $(v, \mathbf{x})$ assuming incentive compatibility with a continuous scheme if and only if condition $(*)$ holds.*

Thirdly, Theorem 2 considers general, direct revelation mechanisms, whereas McAfee and Reny restrict attention to finite "participation fee schedules" which, as described in the introduction, are clearly not general mechanisms. Therefore, in principle they may incur some loss of generality. In fact, there is a loss associated with their finite participation schedules. This is reflected in the fact that McAfee and Reny are only able to characterize *virtually* full surplus extraction, rather than full extraction of the surplus. Theorem 2 asserts that in their restricted environment, their condition $(*)$ characterizes full surplus extraction with direct mechanisms.

This observation does not follow from their work—at least not easily. McAfee and Reny show that there is a sequence of participation schedules such that each one uniformly extracts more and more of the surplus until the amount of surplus left for the agent vanishes asymptotically. However, it is unclear from their argument whether or not this sequence of schedules might meaningfully converge.

Furthermore, since the description above of full surplus extraction assuming incentive compatibility is given by a system of linear inequalities, the proof of Theorem 2 relies squarely on duality. In particular, it does not require any approximation results in the spirit of the Stone-Weierstrass Theorem, say. This is useful because the characterizing condition in Theorem 2 (and in Theorem 1, too) may be simply understood as the dual characterization of surplus extraction.

This is different from the results of McAfee and Reny (1992b). In infinite dimensions, duality involves some primal linear inequalities and the closure of the associated dual linear inequalities (see, e.g., Clark, 2006). However, virtually full surplus extraction is not the closure of dual linear inequalities because the closure is not taken with respect to the assumed incentive compatibility.

[8]Since T is a compact metric space, it is separable, hence $\Delta(T)$ is, too, so its topology is first countable. Therefore, without loss of generality we may focus on sequences rather than nets.

6 Virtually Full Surplus Extraction

The previous discussion motivates understanding the general relationship between full surplus extraction and virtually full surplus extraction. This is our next task.

Virtually full surplus extraction is defined in Definition 2. The following remains to be defined. *Virtually all the surplus can be extracted* from $(v, \mathbf{x})$ if for every $\varepsilon > 0$ there exists a scheme ξ such that $0 \leq \int_Y [\xi(s,y) - \xi(t,y)]p(dy|t)$ for all (t,s) and

$$0 \ \leq \ \int_Y [v(t, \mathbf{x}(t,y), y) - \xi(t,y)]p(dy|t) \ \leq \ \varepsilon \qquad \forall t.$$

Since virtually full surplus extraction bounds the surplus uniformly in t, it was natural for McAfee and Reny (1992a) to restrict v to be bounded. Furthermore, given their finite menus, it was also natural to restrict v to be continuous and T to be compact.

By using direct revelation mechanisms, Theorem 2 attains full surplus extraction (assuming incentive compatibility) in more general settings that McAfee and Reny did with a condition that in their restricted setting is equivalent. On the other hand, they only characterized virtually full surplus extraction with their participation fee schedules. This begs the question, what characterizes virtually full surplus extraction with direct revelation mechanisms? I answer this question in the next result.

Theorem 3. *With direct revelation mechanisms, given any* $(v, \mathbf{x})$*, virtually all the surplus can be extracted from* $(v, \mathbf{x})$ *if and only if all the surplus can be extracted from* $(v, \mathbf{x})$*. This result still holds "assuming incentive compatibility."*

Theorem 3 says that with direct mechanisms, any condition that captures full surplus extraction also captures virtually full surplus extraction, and vice versa. Therefore, McAfee and Reny's characterization of virtually full surplus extraction (assuming incentive compatibility)—rather than full extraction—relies squarely on their restriction to finite participation fee schedules. Allowing for general, direct revelation mechanisms, their condition leads not just to virtually full surplus extraction, but to full extraction of the surplus.

As a final comment, notice that Theorem 3 above does not require continuity or compactness to obtain equivalence between virtually full and full surplus extraction. Therefore the loss of generality from McAfee and Reny's use of finite participation fee schedules transcends their restricted environment.

7 Conclusion

In this paper, I revisited the question surplus extraction by looking at direct mechanisms and arbitrary type spaces. I characterized beliefs that guarantee full surplus extraction in terms of virtual convex independence (Theorem 1), and argued that the addition of infinitely many types did not add much to the case of finitely many types (Proposition 1). Following McAfee and Reny (1992a), I relaxed the guarantee of surplus extraction to one that assumes incentive compatibility, and obtained a generalization of their condition $(*)$ to arbitrary type spaces (Theorem 2, Proposition 2). By using direct mechanisms I was able to obtain full surplus extraction rather than just virtually full. Although McAfee and Reny's use of "finite participation fees" may have appeal by conceivably appearing realistic, they do incur some loss of generality. Moreover, I also showed that the distinction between full and virtually full surplus extraction breaks down with direct mechanisms (Theorem 3).

A Proofs

Before proving the results in the main text, I present some notation and an ancillary result that will be used repeatedly in the proofs. The result is Clark's (2006) extension of The Theorem of the Alternative.

Let X and Y be ordered, locally convex real vector spaces, with positive cones X_+ and Y_+ and topological dual spaces X^* and Y^* such that $X^{**} = X$ and $Y^{**} = Y$. Let $A : X \to Y$ be a continuous linear operator with adjoint operator $A^* : Y^* \to X^*$ and fix any $b \in Y$. Finally, for any set S let $\overline{S}$ denote its closure.

Lemma A.1 (Clark, 2006, page 479)**.** *For any $b \in Y$, there exists $x \in X_+$ such that $A(x) = b$ if and only if $A^*(y_0^*) \in \overline{X_+^* - \{A^*(y^*) : y^*(b) = 0\}}$ implies that $y_0^*(b) \geq 0$.*

For any set X, $\mathbb{R}^X$ stands for the space of functions $X \to \mathbb{R}$ with the product topology. The dual space of $\mathbb{R}^X$ is the subspace of functions with finite support. (See, e.g., Conway, 1990, p. 115.) This subspace is denoted by $\mathbb{R}^{(X)}$. When endowed with the weak$*$ topology, its dual space is the primal space again, i.e., $\mathbb{R}^X$. (See, e.g., Conway, 1990, p. 125.)

I will use this duality between the space of all vectors with the product topology and the space of vectors with finite support to characterize the system of linear inequalities that describes surplus extraction.

A.1 Proof of Lemma 1, Theorem 1 and Proposition 1

I begin with the easy proof of Lemma 1.

Proof of Lemma 1. For sufficiency, convex independence implies that whenever $\pi(t) \geq 0$ has finite support and $\sum_s \pi(s|t) = 1$ for all t, if $p(t) = \sum_s \pi(s|t)p(s)$ then $\pi(s|t) = 1$ exactly when $s = t$. Since $\pi(t)$ is a probability measure, rearranging yields that $\sum_s \pi(s|t)[p(t) - p(s)] = 0$ implies $\sum_s \pi(s|t)u(t, s) = 0$ for all $u \in U$. Multiplying by any positive constant delivers sufficiency.

For necessity, suppose that convex independence fails, i.e., there is a type t such that $p(t) \in \operatorname{conv}\{p(s) : s \neq t\}$. Hence, there is a vector $\pi(t)$ with finite support such that $\sum_s \pi(s|t)[p(s) - p(t)] = 0$ yet $\pi(s|t) > 0$ for some $s \neq t$. Let $u(t, s) = 1$ and zero elsewhere, and define $\lambda(t, \cdot) = \pi(\cdot|t)$ and zero elsewhere. The result now follows. $\square$

Next, I prove Theorem 1. The proof proceeds in three steps. First, I describe full surplus extraction with a family of linear inequalities. In the second step, we apply Lemma A.1 to obtain a necessary and sufficient condition for full surplus extraction. Finally, in the last step we equate this dual condition to virtual convex independence.

Recall that, by definition, all the surplus can be extracted if there exists a scheme ξ—called a *surplus-extracting scheme*—such that

$$\begin{aligned} v(t) &= \int_Y \xi(t,y)p(dy|t) \qquad \forall t, \quad \text{and} \\ \Delta v(t,s) &\leq \int_Y [\xi(s,y)-\xi(t,y)]p(dy|t) \qquad \forall (t,s), \end{aligned}$$

Clearly, this is a system of linear inequalities with respect to ξ. Appealing to duality, we obtain the following characterization of existence of solutions to this linear system.

Lemma 2. *There exists a surplus-extracting scheme ξ if and only if for every net $\{(\lambda_\delta,\eta_\delta)\}$ such that $\lambda_\delta \in \mathbb{R}_+^{(T\times T)}$ and $\eta_\delta \in \mathbb{R}^{(T)}$,*

$$\eta_\delta(\cdot)p(\cdot)+\sum_{s\in T}\lambda_\delta(s,\cdot)p(s)-\lambda_\delta(\cdot,s)p(\cdot) \;\to\; 0 \qquad \Rightarrow \qquad \lim \lambda_\delta\cdot(\Delta v+w) \;\leq\; 0,$$

where $w(t,s)=v(s)-v(t)$.

Proof. By Lemma A.1, there is a surplus-extracting scheme if and only if for any net $\{(\lambda_\delta,\lambda_\delta^+,\eta_\delta)\}$ with $\lambda_\delta^+\geq 0$ and $(\lambda_\delta,\eta_\delta)\cdot(\Delta v,v)=0$ for all δ, it follows that $(\lambda_0,\eta_0)\cdot(\Delta v,v)\geq 0$ whenever (a) $-\lambda_0=\lim \lambda_\delta^+ +\lambda_\delta$ and (b) the following holds:

$$-\eta_\delta(\cdot)p(\cdot)-\sum_s[\lambda_\delta(s,\cdot)p(s)-\lambda_\delta(\cdot,s)p(\cdot)] \to \eta_0(\cdot)p(\cdot)+\sum_s \lambda_0(s,\cdot)p(s)-\lambda_0(\cdot,s)p(\cdot).$$

But by (a), $(\lambda_0,\eta_0)\cdot(\Delta v,v)\geq 0$ is equivalent to $\lim -\lambda_\delta^+\cdot\Delta v-\lambda_\delta\cdot\Delta v+\eta_0\cdot v\geq 0$, and by (b), this is equivalent to

$$\lim(\lambda_\delta^+ +\lambda_\delta)\cdot\Delta v+\sum_t\Big[\sum_s[\lambda_0(s,t)-\lambda_0(t,s)]+\eta_\delta(t)+\sum_s[\lambda_\delta(s,t)-\lambda_\delta(t,s)]\Big]\cdot v(t)\leq 0.$$

By construction of $\{(\lambda_\delta,\eta_\delta)\}$ and (a), this is finally equivalent to

$$\lim\sum_{(s,t)}\lambda_\delta^+(t,s)[\Delta v(t,s)+w(t,s)] \;\leq\; 0,$$

and the claimed result follows. ∎

Lemma 3. *For every net $\{(\lambda_\delta, \eta_\delta)\}$ such that $\lambda_\delta \in \mathbb{R}_+^{(T\times T)}$ and $\eta_\delta \in \mathbb{R}^{(T)}$,*

$$\eta_\delta(\cdot)p(\cdot) + \sum_{s\in T} \lambda_\delta(s,\cdot)p(s) - \lambda_\delta(\cdot,s)p(\cdot) \;\to\; 0$$

implies $\lim \lambda_\delta \cdot u = 0$ *for all* $u \in U$ *if and only if* p *exhibits virtual convex independence.*

Proof. Sufficiency is immediate by letting $\eta_\delta(t) = \sum_s \lambda_\delta(t,s) - \lambda_\delta(s,t)$ for all (δ, t). For necessity, suppose that p exhibits virtual convex independence and that the above limiting condition holds for $\{(\lambda_\delta, \eta_\delta)\}$. We will show that $\lim \lambda_\delta \cdot u = 0$ for all $u \in U$. By integrating with respect to y, notice that $\eta_\delta(\cdot) - \sum_s \lambda_\delta(\cdot, s) - \lambda_\delta(s,\cdot) \to 0$ is necessary. Substituting, we obtain $\sum_s \lambda_\delta(s,\cdot)[p(s) - p(\cdot)] \to 0$. Hence, by virtual convex independence, $\lambda_\delta \cdot u = 0$ for all $u \in U$, as required. □

Proof of Theorem 1. That virtual convex independence implies full surplus extraction now follows from Lemmata 2 and 3. Conversely, suppose that virtual convex independence fails, so there is a net $\{\lambda_\delta\}$ with $\lambda_\delta \geq 0$ such that $\sum_s \lambda_\delta(s,\cdot)[p(s) - p(\cdot)] \to 0$ yet $\lim \lambda_\delta \cdot u > 0$ for some $u \in U$. Now define $w(t,s) = u(t,s)$ for all (t,s). By Lemmata 2 and 3, there is no surplus-extracting scheme. □

Proof of Proposition 1. Necessity is immediate because convex independence implies virtual convex independence. For sufficiency, if convex independence fails then virtual convex independence must fail, so assume convex independence and T is infinite. Let $\{t_k\}_{k=0}^\infty$ be an infinite sequence of distinct types. Define $\pi_k(t_0|t_k) = 1/k$ and $\pi_k(\cdot|t) = \delta_t(\cdot)$ for all other t, and $\Delta v(t_k, t_0) = \sqrt{\|D\pi_k\|}$ for all $k \in \mathbb{N}$ ($\Delta v(t,s)$ equals zero elsewhere). Now we have the same problem as in Example 2, so by Theorem 1 virtual convex independence fails. □

A.2 Proof of Theorem 2

We will broadly follow the same steps as for the previous proof, but discuss in some detail the dual condition to surplus extraction assuming incentive compatibility before equating it to asymptotic convex independence.

Recall that by definition all the surplus can be extracted from $(v, \mathbf{x})$ assuming incentive compatibility if there is a scheme ξ such that

$$v(t,t) \;=\; \int_Y \xi(t,y)p(dy|t) \qquad \forall t, \qquad \text{and}$$

$$0 \leq \int_Y [\xi(s,y) - \xi(t,y)]p(dy|t) \qquad \forall (t,s).$$

Lemma 4. *All the surplus can be extracted from $(v, \mathbf{x})$ assuming implementability if and only if for every net $\{(\lambda_\delta, \eta_\delta)\}$ with $\lambda_\delta \in \mathbb{R}_+^{(T\times T)}$ and $\eta_\delta \in \mathbb{R}^{(T)}$,*

$$\eta_\delta(\cdot)p(\cdot) + \sum_{s\in T} \lambda_\delta(s,\cdot)p(s) - \lambda_\delta(\cdot,s)p(\cdot) \;\to\; 0 \quad \Rightarrow \quad \lim \eta_\delta \cdot v \;\le\; 0. \qquad (\dagger)$$

The proof of this result is almost identical to that of Lemma 2, hence omitted. Our next step in the proof of Theorem 2 is to show that the dual condition (†) above is equivalent to asymptotic convex independence. But before we take this step, let us manipulate and interpret the dual condition, to help understand it. As a useful preliminary step, let us temporarily assume that both T and Y are finite sets.

Claim 1. *Suppose that both T and Y are finite sets. All the surplus can be extracted from any given $(v, \mathbf{x})$ assuming interim implementability if and only if p satisfies the following condition, called* convex dependence implies undetectability*: For any strategy π, if $p(t) = \sum_s \pi(s|t)p(s)$ for all t then $\sum_s \pi(t|s) = 1$ for all t.*

Proof. By the Theorem of the Alternative (see, e.g., Rockafellar, 1970, p. 198), for every $(v, \mathbf{x})$ there exists a scheme ξ such that $v(t) = \sum_y \xi(t,y)p(y|t)$ for all t and $0 \le \sum_y[\xi(s,y)-\xi(t,y)]p(y|t)$ for all (t,s) if and only if for every v and every pair (η,λ) with $\lambda \ge 0$, if $\eta(t)p(t) = \sum_s \lambda(s,t)p(s) - \lambda(t,s)p(t)$ for all t then $\sum_t \eta(t)v(t) \le 0$. This latter condition is equivalent to the following: if $\eta(t)p(t) = \sum_s \lambda(s,t)p(s) - \lambda(t,s)p(t)$ for all t then $\eta \equiv 0$. Rearranging terms, the antecedent may be written equivalently as $[\eta(t) + \sum_s \lambda(t,s)]p(t) = \sum_s \lambda(s,t)p(s)$. Integrating out y, notice that $\eta(t) + \sum_s \lambda(t,s) = \sum_s \lambda(s,t)$ for every t. Without any loss of generality, we may assume that $\lambda(t,t) > 0$, and since $\lambda \ge 0$, it follows that $\eta(t) + \sum_s \lambda(t,s) = \sum_s \lambda(s,t) > 0$. By choosing $\lambda(t,t)$ appropriately, we may assume without loss that $\sum_s \lambda(s,t) = \Lambda$ does not depend on t. Dividing both sides of the previous system of equations by Λ, we finally obtain that if $p(t) = \sum_s \pi(s|t)p(s)$ then $\sum_s \pi(t|s) = 1$. □

To see how this condition works, consider the following example.

Example 4. Let $T = \{a,b,c\}$ and $Y = \{0,1\}$. Define $p(a) = p(b) = \delta_0$ and $p(c) = \delta_1$. Here convex independence *does not* imply undetectability. To see this, consider the following strategy: $\pi(b|a) = \pi(b|b) = \pi(c|c) = 1$ and $\pi(s|t) = 0$ for all other (t,s). Clearly, $p(t) = \sum_s \pi(s|t)p(s)$ for all t, yet $\sum_s \pi(a|s) = 0$.

This example suggests that "convex dependence implies undetectability" is intimately related to convex independence. This intuition is correct, as the next result shows.

Claim 2. *The information structure p exhibits convex independence if and only if convex dependence implies undetectability.*

Proof. If convex independence fails then $p(\hat{t}) \in \operatorname{conv}\{p(s) : s \neq \hat{t}\}$ for some type $\hat{t}$. Let $\hat{\pi}(t) = \delta_t$ if $t \neq \hat{t}$ and $\hat{\pi}(\hat{t})$ be any strategy that solves $p(\hat{t}) = \sum_s \pi(s|t)p(s)$. Now $\sum_s \hat{\pi}(\hat{t}|s) \neq 1$, so convex dependence does not imply undetectability. Conversely, assuming convex independence, if $p(t) = \sum_s \pi(s|t)p(s)$ for all t then $\pi(t) = \delta_t$ for all t, hence $p(t) = \sum_s \pi(t|s)p(s)$ for all t. Integrating out y, $\sum_s \pi(t|s) = 1$ for all t. □

Notice that Claim 2 holds regardless of whether or not T and Y are finite sets. It follows from this last claim that when both T and Y are finite, all the surplus can be extracted assuming incentive compatibility if and only if p exhibits convex independence. By Cremer and McLean's result, it follows that convex independence characterizes both full surplus extraction and surplus extraction assuming incentive compatibility.

Let us now extend Claim 1 to the case where both T and Y may be infinite sets.

Lemma 5. *All the surplus can be extracted from any given $(v, \mathbf{x})$ assuming interim implementability if and only if p exhibits asymptotic convex independence.*

Proof. By Lemma 4, all the surplus can be extracted from any given $(v, \mathbf{x})$ assuming incentive compatibility if and only if condition (†) holds for all v. This is equivalent to requiring that for every net $\{(\lambda_\delta, \eta_\delta)\}$ with $\lambda_\delta \in \mathbb{R}_+^{(T\times T)}$ and $\eta_\delta \in \mathbb{R}^{(T)}$,

$$\eta_\delta(\cdot)p(\cdot) - \sum_{s\in T} \lambda_\delta(s,\cdot)p(s) - \lambda_\delta(\cdot,s)p(\cdot) \;\to\; 0 \quad\Rightarrow\quad \lim \eta_\delta \cdot v \;=\; 0 \quad \forall v.$$

In other words, $\eta_\delta \to 0$ weakly. Without loss, we may assume that $\lambda_\delta(t,t) \geq 1$ for all (t,δ) and that $\sum_s \lambda_\delta(s,t) = \Lambda_\delta \geq 1$ for all (t,δ), so $\sum_s \lambda_\delta(s,t)$ does not depend on t. Integrating out y yields $\eta_\delta(\cdot) - \sum_s \lambda_\delta(s,\cdot) - \lambda_\delta(\cdot,s) \to 0$. Now define $\pi_\delta(s|t) = \lambda_\delta(s,t)/\Lambda_\delta$. Dividing the antecedent above by Λ_δ and rearranging, we obtain the following equivalent condition: $[\eta_\delta(\cdot)/\Lambda_\delta + \sum_s \pi_\delta(\cdot,s)]p(\cdot) - \sum_s \pi_\delta(s,\cdot)p(s) \to 0$. Since Λ_δ is bounded below, $\eta_\delta(\cdot)/\Lambda_\delta \to 0$, too. Therefore, $\sum_s \pi_\delta(\cdot,s) \to 1$. This finally shows that (†) above is equivalent to the following: $\sum_s \pi_\delta(s|\cdot)p(s) \to p(\cdot)$ implies that $\sum_s \pi_\delta(\cdot|s) \to 1$. □

Theorem 2 now follows from Lemma 5.

A.3 Proof of Proposition 2

We now prove Proposition 2, which explains the relationship between asymptotic convex independence and condition $(*)$.

Lemma 6. *Asymptotic convex independence implies condition* $(*)$.

Proof. If condition $(*)$ fails then a net of strategies $\{\pi_\delta\}$ exists with $\sum_s \pi_\delta(s|t)p(s) \to p(t)$ for all t yet $\pi_\delta(\hat{t}) \not\to \delta_{\hat{t}}$ for some $\hat{t}$. Let $\hat{\pi}_\delta(t) = \pi_\delta(t)$ if $t = \hat{t}$ and δ_t otherwise. Now, $\sum_s \hat{\pi}_\delta(\hat{t}|s) = \hat{\pi}_\delta(\hat{t}|\hat{t}) \not\to 1$, so asymptotic convex independence fails. □

To prove Proposition 2, consider the system of inequalities that describe surplus extraction assuming incentive compatibility. Given the restrictions of Proposition 2 and that ξ is continuous in t, by Lemma A.1 a feasible ξ exists for all v if and only if for every sequence $\{(\lambda_m, \eta_m)\}$ with $\lambda_m \in \mathbb{R}_+^{(T\times T)}$ and $\eta_m \in \mathbb{R}^{(T)}$,

$$\eta_m(\cdot)p(\cdot) - \sum_{s\in T} \lambda_m(s,\cdot)p(s) - \lambda_m(\cdot,s)p(\cdot) \;\to\; 0 \quad\Rightarrow\quad \lim \eta_m \cdot v \;=\; 0 \quad \forall v,$$

where now weak convergence in the above antecedent is with respect to all continuous functions on T, and by viewing $\eta_m(\cdot)p(\cdot) - \sum_s \lambda_m(s,\cdot)p(s) - \lambda_m(\cdot,s)p(\cdot)$ as a measure with finite support. Manipulating this implication as in the proof of Lemma 5, we obtain the following equivalent condition: $\sum_s \pi_m(s|\cdot)p(s) \to p(\cdot)$ implies that $\sum_s \pi_m(\cdot|s) \to 1$. Since T is a compact metric space, so is $\Delta(T)$, and the finite measures are dense.

Consider an arbitrary function $\mu : T \to \Delta(T)$ such that $\int_T p(s)\mu(ds|t) = p(t)$. Let $\{\pi_m\}$ be any sequence[9] such that $\pi(\cdot|t)$ has finite support and $\pi_m(\cdot|t) \to \mu(\cdot|t)$ for all t. Therefore, $\sum_s \pi_m(s|t)p(s) - p(t) \to 0$ for all t. Since $\sum_s \pi_m(s|t)p(s) - p(t)$ has finite support as a function of t, it also converges in the weak$*$ topology, i.e., $\sum_t \xi(t)[\sum_s \pi_m(s|t)p(s) - p(t)]$ converges to the same limit for all continuous ξ, hence, this limit is zero.

By asymptotic convex independence, it follows that $\sum_s \pi_m(\cdot|s) \to 1$. But since pointwise convergence is implied by weak convergence, it follows that $\sum_s \pi_m(t|s) \to 1$ for all t, i.e., $\mu(\{t\}|t) = 1$. We have now established that condition $(*)$ is implied by asymptotic convex independence, proving Proposition 2.

[9]By McAfee and Reny's assumptions, without loss we may focus on sequences rather than nets. See Footnote 8.

A.4 Proof of Theorem 3

Finally, we turn to prove Theorem 3. We will just prove the first statement here, as the same argument establishes the second one (the second one being "assuming incentive compatibility"). First of all, one direction is immediate, since full surplus extraction implies virtually full surplus extraction. For the converse, recall that by definition virtually all the surplus can be extracted from $(v, \mathbf{x})$ if for every $\varepsilon > 0$ there is a scheme ξ such that

$$\begin{aligned} \Delta v(t,s) \ &\leq\ \int_Y [\xi(s,y) - \xi(t,y)]p(dy|t) \qquad \forall (t,s), \quad \text{and} \\ 0 \ &\leq\ \int_Y [v(t,\mathbf{x}(t,y),y) - \xi(t,y)]p(dy|t) \ \leq\ \varepsilon \qquad \forall t. \end{aligned}$$

Our usual duality argument yields the following equivalence, whose proof is omitted since it follows the same lines as previous ones.

Lemma 7. *Virtually all the surplus can be extracted from $(v, \mathbf{x})$ if and only if for every $\varepsilon > 0$ and every net $\{\lambda_\delta\}$ such that $\lambda_\delta \geq 0$,*

$$\begin{aligned} [\lambda_\delta(1,\cdot) - \lambda_\delta(0,\cdot)]p(\cdot) + \sum_{s\in T} \lambda_\delta(s,\cdot)p(s) - \lambda_\delta(\cdot,s)p(\cdot) \ &\to\ 0 \\ \Rightarrow \quad \lim \lambda_\delta \cdot (\Delta v + w) - \varepsilon \sum_{(t,s)} \lambda_\delta(t,s) \ &\leq\ 0, \end{aligned}$$

where $w(t,s) = v(s) - v(t)$.

By Lemma 7, virtual surplus extraction requires that for every net $\{\lambda_\delta\}$ satisfying the antecedent above and every $\varepsilon > 0$, $\lim \lambda_\delta \cdot (\Delta v + w) - \varepsilon \sum_{(t,s)} \lambda_\delta(t,s) \leq 0$. But since $\varepsilon > 0$ is arbitrary, this implies that $\lim \lambda_\delta \cdot (\Delta v + w) \leq 0$. However, this is precisely the requirement for full surplus extraction. This establishes the first part of Theorem 3. A proof of the second part is omitted on the grounds that it is very close to this argument.

References

Aliprantis, C. and K. Border (2006): *Infinite dimensional analysis: a hitchhiker's guide*, Springer Verlag. 10

Clark, S. A. (2006): "Necessary and Sufficient Conditions for Infinite-Dimensional Linear Inequalities," *Positivity*, 10, 475–489. 10, 13

Conway, J. B. (1990): *A Course in Functional Analysis*, Graduate Texts in Mathematics, New York: Springer, second ed. 13

Cremer, J. and R. McLean (1988): "Full extraction of the surplus in Bayesian and dominant strategy auctions," *Econometrica*, 56, 1247–1257. 1, 2, 5, 6, 7, 17

Heifetz, A. and Z. Neeman (2006): "On the Generic (Im)Possibility of Full Surplus Extraction in Mechanism Design," *Econometrica*, 74, 213–233. 1

McAfee, R. and P. Reny (1992a): "Correlated Information and Mecanism Design," *Econometrica*, 60, 395–421. 0, 1, 2, 4, 8, 9, 10, 11, 12, 18

——— (1992b): "A Stone-Weierstrass theorem without closure under suprema," *Proceedings of the American Mathematical Society*, 114, 61–67. 2, 10

Rahman, D. (2010): "Detecting Profitable Deviations," Mimeo. 6

Rockafellar, R. T. (1970): *Convex Analysis*, Princeton University Press. 16