Can Reputation Ensure Efficiency in the Structured Finance Market?

Mahmoud Elamin*

Department of Economics, University of Minnesota (UMN)

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Abstract

In Elamin (2010a), I found that the CRA can not credibly fully reveal its information about the quality of the projects it rates when the ratings are unverifiable. In the model with unverifiable ratings an informationally-efficient equilibrium, where the investors are always fully informed about the quality of the project they face, did not exist. This paper analyses the infinitely repeated game with unverifiable ratings and asks: can fear to lose reputation discipline the CRA and induce it to behave truthfully? Reputation here is considered as incomplete information about the type of the CRA. With some probability, the CRA could be a truthful type that always truthfully reveals the information it has about the projects it rates. I analyze the game of Elamin (2010a) between a sequence of short-run investors and short-run firms and a strategic long-run CRA. Nature selects a type for the CRA in the beginning of time and only informs the CRA of it. At every period, the (updated) probability that the CRA is of the truthful type is its reputation. In the model with only two types of projects, if the CRA's reputation is high enough, then there is an informationally-efficient equilibrium. But if there are more than two types of projects, then no matter how high the CRA's patience level or its reputation are, there is no informationally-efficient equilibrium. I also find that in the two types of projects case, if the firms are informed of the type of the CRA then there is no informationally-efficient equilibrium. The many types of projects case is clearly the relevant case to consider, therefore I conclude that the fear to lose reputation is not enough deterrent for the CRA in the structured finance market.

*Address: 4-101 Hanson Hall 1925 Fourth Street South, Minneapolis, MN 55455 e-mail: *elami001@umn.edu*. The author is grateful to Christopher Phelan, Jan Werner, David Rahman, Aldo Rustichini for advice and suggestions, seminar participants at the Minnesota Math Econ Seminar, Minnesota Math Econ Workshop and the Minnesota Public Finance Workshop for helpful comments.

1 Introduction

This paper evaluates the claim that the Credit Rating Agency's (CRA's) concerns about its reputation are potent enough to deter it from misbehaving in the structured finance market. Despite the reputation concerns, the last financial crisis showed a tremendous failure in rating structured finance products. The CRAs themselves admit that they face a deep crisis following the financial crisis fiasco. They acknowledge the need to work on restoring the market's faith and confidence in their ratings' credibility.

My research focused on the role of the CRAs in the structured finance market. Elamin (2010a) identified a key feature of structured finance products: the rating of those products are less verifiable than bond ratings. It showed that if the ratings are less verifiable then the CRA can not fully fulfill its role in the structured finance market. Elamin (2010a) considered an infinitely repeated game between a sequence of short-run firms and short-run investors and a long-run CRA. The firms are cash constrained and need to borrow from the investors to finance their projects. Projects could be either good or bad. They only differ in their probability of default and hence are ex-post undistinguishable. Projects pay zero in default and the same amount when they do not default. A new project is selected every period for the firm playing in that period. The firm decides whether to access the CRA to signal the type of project it has. The CRA is perfectly informed of the type of project the firm has when accessed, and makes a public announcement (rating) about the project type. Investors Bertrand compete after seeing the move of that period's firm and the CRA's announcement if there is any. Elamin (2010a) showed that in this infinitely repeated game no matter how high the discount factor of the CRA is there is no infomationally-efficient equilibrium, where the investors are fully informed of the type of project they face every period on the equilibrium path.

This paper starts from where Elamin (2010a) ends and builds on it. Here I tackle the question: can we get an informationally-efficient equilibrium in the infinitely repeated game of Elamin (2010a) when we consider reputation as incomplete information about the type of the CRA?

In general, reputation is thought of in two ways. First, reputation is synonymous with an infinitely repeated game. The fact that interactions are repeated means that a player should care not only about his payoff today but also about his payoff from tomorrow on. The mere existence of continuation payoffs in infinitely repeated games is considered as a representation of reputation. Think of this in the following sense, if a player behaves properly today then the players will reward him from tomorrow on. If that player does not behave properly then he is punished from tomorrow on. When punishment starts it is considered that the player lost his reputation. The second concept of reputation is represented as incomplete information about the type of the player. Reputation as incomplete information can be represented in one of the following two practically equivalent ways: the first is that with some probability the player is a committed robot who always plays one particular action. The second is that the incomplete information is about the payoff structure of the player. With some probability, the payoffs of the player are such that it is a dominant strategy for him to always play a certain action.

The infinitely repeated game concept of reputation was tackled already by Elamin (2010a), it was shown there that it does not make the CRA fully fulfill its role in the structured finance market. In this paper, I will consider reputation as incomplete information about the type of the CRA. With some prior probability the CRA might be a robot that when accessed always truthfully reveals its information about the projects it rates.

In reality, the CRAs insist adamantly that their business is built on market participants' trust and confidence in their ratings. The much-repeated claim goes something along the following lines: CRAs can not afford not to do a good job rating products because if they do their reputation will collapse. Reputation as the story goes is the main and only capital in the credit rating business. If the CRA's reputation collapses, the ratings then have no credibility and market participants would ignore them. This would deal a fatal blow to the credit rating business. But does this claim really hold? We have seen that the first concept of reputation is not enough to get things right in the structured finance market. Will using the second concept of reputation save the day?

But what realistic concerns of market participants justify using reputation as incomplete information about the type of the CRA? In general, market participants are not sure about the exact mechanics that govern the CRA's internal structure. A lot of safeguards has been put in place by the CRAs to deal with possible conflicts of interests inherent in the rating business. Market participants might not be fully confident of the potency of these safeguards. Moreover market participants might be under the impression that if the CRA does not do the right thing, that would be considered as fraud. Intentionally falsifying or misrepresenting information might be punished by the government. These considerations among others create incomplete information about the type of the CRA in the mind of market participants leading it to have a reputation as modeled here.

This paper builds on the infinitely repeated game presented in Elamin (2010a). Reputation here is considered as incomplete information about the type of the CRA. With some probability, the CRA could be a truthful type that always reveals truthfully. I build on the infinitely repeated game of Elamin (2010a) with unverifiable ratings between a sequence of short-run investors and short-run firms and a strategic long-run CRA. Nature selects a type for the CRA in the beginning of time and only informs the CRA of it. At every period, the (updated) probability that the CRA is the truthful type is considered to be its reputation. The CRA is fully informed of the project type when accessed. Its role is to mitigate the information asymmetry between the investors and the firms. The question that motivates us here is: will the CRA fully fulfills its role in the game? This leads to a definition of an equilibrium of interest which I call the informationally-efficient equilibrium. In an informationally-efficient equilibrium the investors are fully informed of the type of project they face in every period along the equilibrium path and the CRA's role is fully fulfilled. In the model with only two types of projects, I derive a cutoff reputation level and show that if the CRA's initial reputation is above that cutoff then there is an informationally-efficient equilibrium. If the CRA's initial reputation is below that cutoff level then, independent of how high the discount factor is, there is no informationally-efficient equilibrium. This existence result seems to be encouraging at a first glance. It seems reputation does ensure efficiency in the structured finance market. To show that this encouragement is not in its place, I next test the limits of the existence result I found in the two project types case. Is this existence result an anomaly? Or is it a particular instance of a more general result? I find that the existence result does not hold in the two directions I extended the model. The first extension I considered is the case where there are still only two project types but the firms in the game are informed of the type of the CRA. The second extension is where the CRA is the only player informed of its type but there are more than two projects. If there are only two types of projects but the firms are informed of the type of the CRA, then no matter how high the CRA's patience level or its initial reputation are, there is no informationally-efficient equilibrium. In the case where the CRA is the only player informed of its type but there are more than two types of projects, then no matter how high the CRA's patience level or its initial reputation are, there is no informationally-efficient equilibrium. Hence we conclude that the existence result is indeed an anomaly. Reputation does not ensure efficiency in the infinitely repeated game with unverifiable ratings.

The rest of the paper is organized as follows: section 2 presents the infinitely repeated game with reputation, section 3 presents the case of two projects but with firms informed of the CRA's type, section 4 presents the case of more than two projects and section 5 concludes.

2 Infinitely Repeated Game with Reputation

In this section I consider an infinitely repeated game between short-run investors, short-run firms and a long-run CRA with reputation. At the beginning of time Nature selects a type for the CRA and only informs the CRA of it. The CRA could be a truthful type that when accessed always reveals its information truthfully or a strategic type that chooses a strategy to maximize its discounted payoffs. Every period the investors and the firm playing in that period see the public history that has transpired up to then. They use the public history to update the probability that the CRA is of the truthful type. This updated probability is considered to be the CRA's reputation.

I use the sequential equilibrium concept and in this section I define the concept of efficiency I will use. The efficiency concept I consider is motivated by the absence of information asymmetry between investors and firms. Hence the efficient equilibrium is called an informationally-efficient equilibrium. In an informationally-efficient equilibrium investors are informed of the type of project they face every period on the equilibrium path and the information asymmetry between investors and firms is mitigated. The role of the CRA is to mitigate this information asymmetry. I ask the following question here: can reputation help the CRA achieve its role fully?

I show that in the two types of projects case we consider in this section, if the initial reputation of the CRA for truthfulness is above a certain cutoff reputation level then there exists an informationally-efficient equilibrium. If the initial reputation is below that cutoff then an informationally-efficient equilibrium does not exist.

A word is in order about why the investors and firms are short-run players. The fact that investors are short-run players prevents collusion in the infinitely repeated game. To give an idea, assume there are two investors who are long-lived and consider the following scheme: investor 1 gives an offer that wipes out the firm 1 at period 1, while investor 2 gives an unacceptable offer that gives negative profits. In period 2, investor 2 gives an offer that wipes out firm 2 and investor 1 gives an unacceptable offer. Investors competing to provide funding to firms do not collude. So I model investors as short-run players to prevent collusion. On the other hand, modeling firms with projects as short-run firms is in line with the structured finance market I want to model. As explained in the introduction of Elamin (2010a), structured finance securities' payment depends only on cash flows specific to the particular pool of loans that back these securities. Issuers sell the pool of loans to a special purpose vehicle constructed for the purpose of dealing with that pool. An issuer then receives a cash payment for the pool and his role ends. The special purpose vehicle itself issues the structured finance securities backed by the pool. The original issuer is not liable if the special purpose vehicle he constructed defaults. This quick background motivates viewing firms as short-run players (or as one-shot projects).

2.1 Stage Game and Setup

The stage game of this infinitely repeated game with reputation is similar to the infinitely repeated game in Elamin (2010a). Time is discrete and infinite. A sequence of cash constrained firms need the investor's funding for a project. The firm either has a good project or a bad project. Firms know the type of project they have. They could ask the CRA to rate their project signaling the type of project they have to investors.

At every period, there is a short-run firm, called firm t, and two short-run investors $t1$ and $t2$ who play in that period only and exit the game. Firm t is cash constrained and needs the investors' funding. A CRA is the only long-run player in the game. At the beginning of time, Nature selects a type for the CRA and only informs the CRA of it. The CRA can be one of two possible types: either a strategic type that selects a strategy to maximize its infinite sum of discounted payoffs, call this type ζ_0 or a truthful type that acts as a robot always truthfully revealing all the information it has about the project's type once accessed, call this type ζ_T. Nature selects the truthful type with a prior probability $0 < \alpha_1 < 1$.

Nature selects a new project every period (iid), it picks a project with a default probability from the set of possible default probabilities $\{p_H, p_L\}$ with prior η on p_L where $0 < p_H < p_L < 1$ and p_L is the low quality project. Firm t playing in period t sees the move of nature and decides whether to access the CRA (A) or not (NA). Denote that action by a_t and let $a_t = 0$ if firm t did not access the CRA at time t, and $a_t = 1$ if it decides to access. At period t, if accessed by Firm t, the CRA sees the true default probability of the project selected for time t and makes a public announcement from the set $\{H, L\}$. Investors $t1$ and $t2$ moving in that period are each endowed with 1 unit of a good. They simultaneously make offers (R_{ti}, b_{ti}), where $R_{ti} \in \Re$ is the required rate of return by investor ti and $0 \leq b_{ti} \leq 1$ is the issuance size demanded by investor ti. Investors $t1$ and $t2$ move in the following three possible contingencies (occurring in period t): firm t did not access the CRA, it accessed the CRA and H was announced, or it accessed the CRA and L was announced. In all contingencies, firm t then observes the offers $\{(R_{t1}, b_{t1}), (R_{t2}, b_{t2})\}$ set by the investors of time t. If the offers give the firm the same profits then Nature moves again picking each offer with probability $\frac{1}{2}$. If the offers give firm t different profits, then firm t decides which offer to pick. If the firm t has decided to access the CRA and b is the issuance size of the picked offer, only the amount $(1-\epsilon)b$ is invested in the time t project, and the amount ϵb is paid to the CRA upfront. If the CRA was not accessed, b is invested in the project. Then the time t project realizes as a public outcome, it either defaults or pays. And the game in period t ends. Firm t exits the game. Investors $t1$ and $t2$ consume and exit. Then we start period $t+1$ with the same CRA, different firm with a new project (firm $t+1$) and two new investors.

I now turn to defining payoffs. Investors are identical and have log utility. In all cases, the unpicked investor consumes his endowment and gets zero utility. If the CRA was not accessed at time t, the picked investor ti consumes c_{ti} when the project selected at time t defaults, and $c_{ti} + R_{ti}b_{ti}$ when the project pays. If the CRA was accessed, the picked investor ti consumes $1 - b_{ti}$ when the project defaults, and $c_{ti} + R_{ti}(1-\epsilon)b_{ti}$ when the project pays where $c_{ti} = 1 - b_{ti}$. Assume the selected offer in period t is (R_{ti}, b_{ti}). Firm t earns zero profit when the project defaults (limited liability) and if the project pays it earns: $(\bar{R} - R_{ti})b_{ti}$ if the CRA was not accessed in period t and $(1-\epsilon)(\bar{R} - R_{ti})b_{ti}$ if the CRA was accessed in period t. I will not define payoffs for the truthful CRA since it acts as a robot. The payoff of the strategic CRA with discount factor $0 < \delta < 1$ in the infinitely repeated game is the discounted sum of its one period payoff. Let the selected issuance size in period t be b_t and let a_t be the indicator function of the access decision by firm t as explained before. The strategic CRA evaluates the sequence $\{b_t\}_{t=1}^{\infty}$ in the following way:

$$(1-\delta)\sum_{t=1}^{\infty} a_t \delta^{t-1} \epsilon b_t$$

Let $q = \eta p_L + (1-\eta) p_H$ be the posterior on default when the prior is retained. I will now present two conditions on the parameters of the game under which the game becomes interesting. Elamin (2010a) contains a discussion of these conditions:

Condition 1. $p_L < \frac{(1-\epsilon)\bar{R}-1}{(1-\epsilon)\bar{R}}$

Condition 2. $(1-\epsilon)\bar{R} - \frac{((1-\epsilon)\bar{R}-1)^q}{(1-q)^{1-q}q^q} > \bar{R} - \frac{(\bar{R}-1)^{p_L}}{(1-p_L)^{1-p_L}{p_L}^{p_L}}$

To make visualizing the game easier a detailed chronological timeline follows.

1. At the beginning of time, Nature picks a type for the CRA and only informs the CRA of it. The CRA is either a strategic type ζ_0 or a truthful type ζ_T.

2. Then at every period t, Nature moves and picks a time t project from the set of possible default probabilities $\{p_H, p_L\}$ with prior η on p_L where $0 < p_H < p_L < 1$ and p_L is the low quality project.

3. Firm t sees the move of nature and then decides to access (A) the CRA or not access (NA).

4. If accessed by firm t, the CRA sees the move of nature on project type in that period and makes a public announcement from the set $\{H, L\}$.

5. Two investors t1 and t2 each with an endowment of one unit of a good, simultaneously make offers (R_{ti}, b_{ti}) in the following three possible contingencies: firm t did not access the CRA, it accessed the CRA and H was announced, or it accessed the CRA and L was announced. Note that R_{ti} is the required rate of return required by investor ti, a real number in $\Re$ and b_{ti} is the issuance size ti is willing to buy where $0 \leq b_{ti} \leq 1$.

6. In all contingencies, firm t observes the offers $\{(R_{t1}, b_{t2}), (R_{t2}, b_{t2})\}$ set by the investors playing in period t. If the offers give the firm the same profits then Nature moves again picking each offer with probability $\frac{1}{2}$. If the offers give the firm different profits, then the firm decides which offer to pick.

7. Let b_t be the issuance size of the picked offer. If the firm has decided to access the CRA then the amount $(1 - \epsilon)b_t$ is invested in the project, and the amount ϵb_t is paid to the CRA upfront. If the CRA was not accessed then b_t is invested in the project.

8. The time t project realizes as a public outcome, it either defaults or pays. Investors $t1$ and $t2$ consume and exit the game.

9. The time t game ends. And we start period $t + 1$ with two new investors, a new firm with a new project and the same long-run CRA.

2.2 Histories and Strategies

I now define the relevant histories of the infinitely repeated game necessary to define the strategies of the players. At any time period t, a player's strategy might possibly depend on anything he knows at that period of what has transpired in the past, be it private (like the true type of the project for a CRA that was accessed in some previous period) or public (like the decision to access the CRA or not, or if the project defaulted or paid). But here I impose the standard restriction that the players use the past in one way only: the players only use the public events from the past in their strategies. The players can still make their strategy today depend on what they see privately today, but it can depend on the past only through what is publicly known. There is an exception to this of course and that is that the CRA knows what type it is when it moves. At any time period, the CRA knows the move of nature on its type and uses the public history from the past and what it sees privately today when it picks a strategy. Hence, I will first define the public history, and then for each player add to it what he observes privately today and for the CRA its type. Note here that for simplicity, I will not mention the firm t's second move because as we saw in the Lemmas in Elamin (2010a) in any equilibrium the firm will not move again, and these nodes will be off-path. Tracking behavior in these nodes will be merely cumbersome with no real benefit. The Lemmas derived in Elamin (2010a) still apply to this environment with almost no change in the formulation or the proofs. The only difference is that the beliefs themselves might be different, but behavior *given beliefs* is still the same. These Lemmas are listed in this paper as Lemmas 1, 2, and 3.

2.2.1 Public History

At any time period t, the entries recorded in the public history are the following elements: if the CRA was accessed or not by firm t, the public announcement if accessed, the offers of the investors, the chosen investor, and the realization of the project at the end of the period. A time t public history is an element:

$h_t^p \in \left[\{NA, (A, H), (A, L)\} \times \Re^2 \times [0, 1]^2 \times \{1, 2\} \times \{D, P\}\right]^{t-1}$

where:

- NA stands for CRA not accessed.
- $\{(A, H), (A, L)\}$ denotes the decision to access the CRA and the subsequent announcement.
- $\Re^2 \times [0, 1]^2$ denotes the set of possible required rates and possible issuance sizes of investors $t1$ and $t2$.
- $\{1, 2\}$ denotes the set of possible chosen investor in each period.
- $\{D, P\}$ denotes the set of what could happen to the project. It either defaults (D) or pays (P).

Let $h_1^p = \emptyset$, H_t^p be the space of all possible time t public histories h_t^p and $H^p = \bigcup_t H_t^p$.

2.2.2 Strategies

Firm t, sees the public history from the past and the true project type at period t and randomizes on access and not access. Firm t's strategy is a function $\sigma_{Firm\ t} : H_t^p \times \{p_H, p_L\} \rightarrow \triangle\{A, NA\}$. The CRA sees a private history in the time periods it was accessed. The CRA, knowing its type, only uses the public history from the past for its strategy. It specifies what it will do after access by a p_H firm and a p_L firm. The CRA's strategy is $\sigma_{CRA} : \zeta_0 \times H^p \times A \times \{p_H, p_L\} \rightarrow \triangle\{H, L\}$. Investor ti, sees the public history from the past and picks his offer given the three contingencies he finds himself in. Time t investor i's strategy is $\sigma_{ti} : H_t^p \times \{NA, (A, H), (A, L)\} \rightarrow \Re \times [0, 1]$. Investor ti picks his required rate of return and the issuance size given what he knows up to then.

2.3 Sequential Equilibrium

I now define the sequential equilibrium concept I will use. I note here that there are two "kinds" of incomplete information in this game: transient and permanent. The incomplete information about the type of project selected for investment at each period is transient in nature. A new project is selected every period. So there is no incomplete information carrying over from one period to the other concerning project types. The incomplete information about the type of the CRA playing in the game on the other hand is more permanent in nature. This incomplete information may carry over from one period to the other. Only the CRA is informed of its type. The firms and investors have to use the public information from the past, and whatever information they see in the period they play in to update their belief about the CRA being the truthful type. This posterior belief is the reputation of the CRA.

Definition 1. *A sequential equilibrium (SE) is a tuple* (σ, μ, α)*:*

1. $\sigma = \left(\{\sigma_{Firm\ t}\}_{t=1}^{\infty}, \sigma_{CRA}, \{\sigma_{t1}\}_{t=1}^{\infty}, \{\sigma_{t2}\}_{t=1}^{\infty}\right)$

    - $\sigma_{Firm\ t}$ *is the strategy of the time* t *firm.*

- $\sigma_{CRA} = \{\sigma_{CRA\ t}\}_{t=0}^{\infty}$ *is the strategy of the CRA in the infinitely repeated game, a collection of time* t *strategy components.*

- σ_{ti} *is the strategy of investor* ti.

2. $\mu = \{\mu_t\}_{t=1}^{\infty}$ *where:*

   $\mu_t = \big(\mu(H_t^p \times NA), \mu(H_t^p \times (A, L)), \mu(H_t^p \times (A, H))\big)$

   At each period t *given the public history,* μ_t *is a collection of three numbers between* 0 *and* 1. *These probabilities denote the beliefs of the investors in each of their information sets about the probability of facing a* p_L *project in each possible contingency.*

3. $\alpha = \{\alpha_t(H_t^p)\}_{t=1}^{\infty}$ *where:*

   $\alpha_t(H_t^p)$ *is the updated probability that the CRA is of the truthful type that firm* t *and investors* $t1$ *and* $t2$ *start the period with.*

Satisfying the following requirements:

i. *Given* μ, α, $\{\sigma_{Firm\ t}\}_{t=1}^{\infty}$, $\{\sigma_{t1}\}_{t=1}^{\infty}$, *and* $\{\sigma_{t2}\}_{t=1}^{\infty}$*:*

   σ_{CRA} *is optimal at every node the CRA moves on.*

ii. *Given* μ_t, $\alpha_t(H_t^p)$ *and* H_t^p,

   a. σ_{ti} *is optimal given* σ_{tj} *with* $i, j \in \{1, 2\}$ *and* $j \neq i$.

   b. $\sigma_{Firm\ t}$ *is optimal given* $\{\sigma_{CRA\ t}, \sigma_{t1}, \sigma_{t2}\}$.

iii. *Given* $\alpha_t(H_t^p)$ *and* H_t^p, μ_t *and* $\alpha_{t+1}(H_{t+1}^p)$ *are derived from the perturbed time* t *strategies of the players as Bayes rule limits of (totally mixed) perturbed strategies converging to* $(\sigma_{Firm\ t}, \sigma_{CRA\ t})$

   where $\sigma_{CRA\ t}$ *is the time* t *component of the CRA's strategy:*

   $\sigma_{CRA\ t} : \zeta_0 \times H_t^p \times A \times \{p_H, p_L\} \rightarrow \triangle\{H, L\}$ *consistent with* σ_{CRA}.

iv. *If* $\alpha_T(H_t^p) = 0$ *or* $\alpha_T(H_t^p) = 1$ *for some* T, *then* $\alpha_t = \alpha_T\ \forall t \geq T$.

I note here that the Lemmas 1, 2 and 3 listed here are from Elamin (2010a). These Lemmas still hold in this environment, and they detail the investors' optimal strategies given their beliefs. The only difference between this environment and the environment of Elamin (2010a) is that the beliefs themselves may be different. But optimal strategies given the beliefs are still the same.

Lemma 1. *Under Condition 1 and Condition 2, and given* μ*,*

1. *An optimal response of the investors given beliefs in the contingency following access and announcement is a vector* $((R_1, b_1), (R_2, b_2))$ *for the two investors s.t. for* $i \in \{1, 2\}$*,* (R_i, b_i) *solves the following problem (Problem P1):*

$$(1-\epsilon)Max_{R_i,b_i}(\bar{R} - R_i)b_i \ s.t.$$
$$0 \le b_i \le 1$$
$$qLog(1 - b_i) + (1 - q)Log(1 + (R_i(1 - \epsilon) - 1)b_i) = 0$$

   where:

   $q = \mu(A, L)p_L + (1 - \mu(A, L))p_H$ *if there was access and L was announced.*

   $q = \mu(A, H)p_L + (1 - \mu(A, H))p_H$ *if there was access and H was announced.*

2. *An optimal response of the investors given beliefs in the contingency following no access is a vector* $((R_1, b_1), (R_2, b_2))$ *for the two investors s.t. for* $i \in \{1, 2\}$*,* (R_i, b_i) *solves the following problem (Problem P2):*

$$Max_{R_i,b_i}(\bar{R} - R_i)b_i \ s.t.$$
$$0 \le b_i \le 1$$
$$qLog(1 - b_i) + (1 - q)Log(1 + (R_i - 1)b_i) = 0$$

   where:

   $q = \mu(NA)p_L + (1 - \mu(NA))p_H$

Lemma 2. *Under Condition 1 and Condition 2, there exists a solution* (R, b) *that solves P1. Moreover the solution is unique and is characterized by the following two equations:*

$$b = 1 - \left[\frac{q}{(1-q)((1-\epsilon)\bar{R}-1)}\right]^{1-q}$$
$$R = \frac{1}{q}\left[(1-q)\bar{R} - \frac{1}{(1-\epsilon)}\right]\frac{1-b}{b}$$

Lemma 3. *Under Condition 1 and Condition 2, there exist a solution* (R, b) *that solves P2. Moreover, the solution is unique and is characterized by the following two equations:*

$$b = 1 - \left[\frac{q}{(1-q)(\bar{R}-1)}\right]^{1-q}$$
$$R = \frac{1}{q}\left[(1-q)\bar{R} - 1\right]\frac{1-b}{b}$$

2.4 Informationally-Efficient Equilibrium

In considering the concept of efficiency to use, I note that what is most important in this game is that the investors be informed of the type of project they are investing in. A CRA is a signaling device that is supposed to reveal its information about the projects it rates mitigating the information asymmetry between firms and investors. The notion of efficiency I will consider will reflect that role of the CRA. Intuitively speaking in an informationally-efficient equilibrium the investors are always informed of the type of project they face, and there is no information asymmetry. Hence in an informationally-efficient equilibrium the CRA fully fulfills its role revealing all the information it has about the projects it rates when accessed. A more formal definition follows now.

Definition 2. *A sequential equilibrium is informationally-efficient if on the equilibrium path the investors are correct in their beliefs about the type of project they face.*

I clarify two possible confusions that might occur to the reader. The first confusion concerns what is meant by beliefs being correct. The second confusion concerns what is meant by the equilibrium path. Beliefs are correct in equilibrium if they are updated according to the definition of the equilibrium used. Here beliefs are always correct in the sense that they adhere to the sequential equilibrium definition used. They are computed by Bayes rule on the equilibrium path, and are Bayes rule limits off the equilibrium path. What is meant by correct in Definition 2 is that the investors believe they face the bad project in the periods where nature selects a bad project. And they believe they face the good project when nature selects a good project. The best way to clarify the other misconception concerning the equilibrium path is by defining a slightly different concept which I call on the equilibrium path in period t. Play is on the equilibrium path in period t if firm t plays according to its equilibrium strategy. Play is on the equilibrium path in the infinitely repeated game if it is on the equilibrium path every period. In an informationally-efficient equilibrium, in any period t the beliefs of the time t investors should be correct when play is on the equilibrium path in period t. Let us see what an informationally-efficient equilibrium requires of the beliefs of investors after a deviation. Assume that at period 1 the bad firm was not supposed to access the CRA but it did. In an informationally-efficient equilibrium, the CRA does not have to mitigate the information asymmetry in that particular period and the investors might not have correct beliefs in that particular period. But in any period following that first period, if the firms play according to their equilibrium strategies then the CRA should mitigate the information asymmetry and the investors should have correct beliefs. What is important to understand here is that following a deviation in period t, the CRA expects all firms playing afterwards to stick to their equilibrium strategy, and hence in any informationally-efficient equilibrium the CRA's continuation payoff will be fixed after every possible contingency. The relevance of this subtle point should be clear after reading the proofs of Propositions 1, 2, and 3.

I have specified the equilibrium of interest. Now let us look if we can find that as an equilibrium in the game developed. To prepare for the coming proposition, I now define a cuttoff reputation value α^*. Let α^* be the $0 < \alpha < 1$ that solves the following equation:

$$\alpha\big[(1-\epsilon)\bar{R}-\frac{((1-\epsilon)\bar{R}-1)^{p_L}}{(1-p_L)^{1-p_L}{p_L}^{p_L}}\big]+(1-\alpha)\big[(1-\epsilon)\bar{R}-\frac{((1-\epsilon)\bar{R}-1)^{p_H}}{(1-p_H)^{1-p_H}{p_H}^{p_H}}\big] = \bar{R}-\frac{(\bar{R}-1)^{p_L}}{(1-p_L)^{1-p_L}{p_L}^{p_L}} \tag{1}$$

I note that under Condition 1 and Condition 2, a solution α^* to the above equation exists by the intermediate value theorem. Moreover the solution α^* is unique because of monotonicity in α of the LHS of the equation. The equation determines the cuttoff reputation level at which the bad firm is indifferent between access and no access when the strategic CRA says it is the good firm after it accesses and the investors believe the CRA, and the bad firm is known to be the bad firm when it does not access. We note here that if the reputation of the CRA is above this cuttoff, the bad firm would never choose to access even when it is considered to be the bad firm after no access. This remark is key to understanding the proof of the following Proposition.

Proposition 1. *Under Condition 1 and Condition 2, in the infinitely repeated game with reputation, independent of* $0 \leq \delta < 1$*, if* $\alpha_1 \geq \alpha^*$ *then there is an informationally-efficient equilibrium. If* $\alpha_1 < \alpha^*$ *then there is no informationally-efficient equilibrium.*

Proof. .

Let us prove the first part of the proposition first. Assume $\alpha_1 \geq \alpha^*$ and $0 \leq \delta < 1$. The following is an informationally-efficient equilibrium. Good firms access, and bad firms do not. The strategic CRA always says L (good) after access by p_L firm and by p_H firm. Beliefs of investors after they see no access is that it is the bad firm, after they see access and L that it is the good firm, and after they see access and H that it is the bad firm. Reputation level stays constant at α_1 on the equilibrium path, and goes to 1 if the investors see access and H and stays at 1 forever. This proves the first part of the Proposition.

Now to prove the second part, assume there is an informationally-efficient equilibrium and $\alpha_1 < \alpha^*$. There are three cases to consider. First, at any time period the bad firm accesses and it is revealed to be the bad firm and the good firm does not access. That would never arise because the bad firm would profitably deviate to no access. Second, at any time period both types of firms access and the CRA truthfully reveals the types. This case would never arise either. The bad firm would prefer to deviate to no access. The worst that could happen when the bad firm does not access is that it is thought to be the bad firm. But even then, it saves the fee. These two cases show that the bad firm would never access when its true type is revealed after access.

To prepare for the third case we need to understand the possible CRA's continuation payoffs in an informationally-efficient equilibrium. It might be useful at this point to review the comments after Definition 2. The two cases discussed before show that the bad firm would never access in an informationally-efficient equilibrium. Hence to separate the types in an informationally-efficient equilibrium, the good firm has to accesses and the CRA reveals the true type of the project. Notice that because of the definition of an informationally-efficient equilibrium, the good firm has to access every period after every possible contingency. This means that in any informationally-efficient equilibrium the continuation payoff of the CRA on the equilibrium path is fixed no matter what happens today. At any period, the continuation payoff of an informationally-efficient equilibrium is determined by the expected payments coming from access by good firms from that period on. Hence let b^*_L be the optimal issuance size investors are willing to buy after access when they know that they face the project with default p_L (the good project). From Lemma 2 we know that $b^*_L = 1 - \left[\frac{p_L}{\left((1-\epsilon)\bar{R}-1\right)(1-p_L)}\right]^{1-p_L}$. The continuation payoff after every possible contingency in an informationally-efficient equilibrium is fixed at $\frac{\eta\epsilon b^*_L}{1-\delta}$.

Now we are ready to present the third case, bad firms do not access and good firm access. The strategic CRA says L (good) after access by good firm, and threatens the bad firm with enough punishment to force it not to access. But no matter what the strategic CRA says it will do after access by p_H (bad) firm, consider the following profitable deviation: the bad firm accesses and the CRA says L (good). The unsuspecting investors think they are on the equilibrium path where only good firms access. After access and an L announcement by the CRA, the investors believe the project is good giving offers accordingly. The reputation level stays the same, and the CRA and the bad firm both profit from the (wrong) belief of the investor that the project is good. The bad firm gets higher profits because it is thought to be the good firm (by Condition 2 and the monotonicity of profits in beliefs). The CRA's continuation payoff is fixed from tomorrow on at $\frac{\eta\epsilon b_L^*}{1-\delta}$ no matter what it says today. With this deviation it gets ϵb_L^*, which is the highest payment it could get in a period, and the continuation is constant. Hence the total payment is definitely higher than saying bad, getting the fee for a bad project today and getting the same continuation payoff from tomorrow on. The proposed equilibrium unravels. This concludes our proof. □

Proposition 1 showed that there is an informationally-efficient equilibrium when there are only two types of projects and when the reputation level is high enough. The intuition is very simple. Assume that the reputation of the CRA is almost 1. Bad firms would not want to access, and good firms would want to access. Access by the bad firms would almost surely reveal they are the bad firms. The worst that could happen if they do not access is that they are considered to be the bad firms. But even then they save the fee. The good firms on the other hand would want to access because that almost surely signals they have the good project. When the good firms access and the bad firms do not, the reputation level stays constant in the game.

But let us try to see the limits of our result. Is this existence result an anomaly that only occurs in the model we consider or is this a particular instance of a more general result? I will ask now two questions. First, is the existence of an informationally-efficient equilibrium robust to changes in the information structure of the game? Second, does the result extend to when the number of projects increase to more than two? Proposition 2 answers the first question in the negative, and Proposition 3 answers the second question in the negative.

3 Firms Informed of CRA's Type

This section motivates the first robustness question and checks if the existence result obtained in Proposition 1 extends to the case where the firms are informed of the type of the CRA.

The CRAs operate under what is known as the issuer-pays business model. Issuers of the securities themselves pay the CRA raising some doubts about their clout with the CRA. Moreover the process of rating a security is a give and take process between the issuer of the security and the CRA. There is a high level of interaction between the firms who have the projects and the CRA. This interaction endows the firm with a deeper knowledge of the CRA's inner workings. I model these close interactions between the firms and the CRA as a change in the information structure I started this paper with. Assume that the firms know the true type of the CRA. They know that the CRA is the strategic type when it is. They know the CRA is the truthful type when it is. It is only the investors who have the incomplete information about the type of the CRA because they are far removed from direct dealing with the CRA. Investors see the CRA as a black box which issues ratings. Will the existence result of Proposition 1 hold in this environment? I will skip making the adjustments to the strategies of the firms and to the equilibrium concept because these adjustments are straightforward.

Proposition 2. *Under Condition 1 and Condition 2, in the infinitely repeated game with reputation if the firms are informed of the true type of the CRA, then for every* $\delta : 0 \leq \delta < 1$ *and for every* $\alpha : 0 \leq \alpha_1 < 1$ *there is no informationally-efficient equilibrium.*

Proof. .

Assume that there is an informationally-efficient equilibrium. The arguments in the proof of Proposition 1 show that the case to consider is where the good firm accesses and is revealed to be the good firm, and the bad firm does not access. The same profitable deviation as before still holds. Assume that Nature selected the strategic type, and the CRA has threatened the bad firm by revealing its type when it is the strategic type with high enough probability to convince it not to access. No matter what the strategic CRA says it will do, it will always say the project is good when it is presented with the contingency of access by the bad firm. Hence the profitable deviation is that the bad firm accesses the CRA, the CRA says the project is good, and the investors think the project is good. The equilibrium unravels and our Proposition is proved. □

I refer the reader to the conclusion of this paper for a discussion of the intuition for the existence of an informationally-efficient equilibrium in Proposition 1, and the nonexistence results of Proposition 2 and Proposition 3.

4 More than Two Project Types

This section checks if the existence of an informationally-efficient equilibrium when reputation is high enough in Proposition 1 extends to the case of more than two projects. Obviously in reality there are more than two project types, and I refrain from motivating the obvious. Here just like before changing the information structure for our first robustness exercise, only the CRA knows its type. The investors and firms have to use the public history to update their probability that the CRA is the truthful type.

This paragraph tries to make the necessary changes to Condition 2 and quickly sets up the new game. Assume there are N projects with N different probabilities of default $p_1 < p_2 < \ldots < p_N$. Let $N \geq 3$. Also assume the prior on each project is $\eta_n > 0$ where $\sum_{n=1}^{N} \eta_n = 1$. A time t public history becomes an element $h_t^p \in \left[\{NA, (A,1), (A,2), \ldots, (A,N)\} \times \Re^2 \times [0,1]^2 \times \{1,2\} \times \{D,P\}\right]^{t-1}$. Firm t's strategy becomes $\sigma_{Firm\ t} : H_t^p \times \{p_1, \ldots, p_N\} \to \triangle\{A, NA\}$ and the CRA's strategy becomes $\sigma_{CRA} : \zeta_0 \times H^p \times A \times \{p_1, \ldots, p_N\} \to \triangle\{1, \ldots, N\}$. Investor ti, sees the public history from the past and picks his offer given the $N+1$ contingencies he might find himself in today. Time t investor i's strategy is $\sigma_{ti} : H_t^p \times \{NA, (A,1), \ldots, (A,N)\} \to \Re \times [0,1]$.

The condition we will have now in this environment will allow the before-worst firm to get higher profits when it accesses the CRA and its true type is revealed than not access and be considered the worst firm.

Condition 3. $(1-\epsilon)\bar{R} - \frac{((1-\epsilon)\bar{R}-1)^{p_{N-1}}}{(1-p_{N-1})^{1-p_{N-1}}{p_{N-1}}^{p_{N-1}}} \geq \bar{R} - \frac{(\bar{R}-1)^{p_N}}{(1-p_N)^{1-p_N}{p_N}^{p_N}}$

Condition 3 is in line with our informationally-efficient equilibrium definition. If this condition does not hold there is no hope of getting an informationally-efficient equilibrium since the before-worst firm would never access and pay the fee to reveal its type. The worst firm never accesses to reveal its type since it prefers to always not access and save the fee. But then the worst and the before-worst firm pool together and no informationally-efficient equilibrium exists. I also note here that if this condition holds for the before-worst firm, then it holds for every firm that has a better project. Hence when Condition 3 holds, every firm other than the worst firm prefers to access and pay the fee when their true type is revealed than not access and save the fee but be considered the worst firm.

To prepare for the proof of Proposition 3, for $n \neq N$ let b_n^* be the optimal issuance size investors are willing to buy after access when they know that they face the project with default p_n. From Lemma 2 we know that $b_n^* = 1 - \left[\frac{p_n}{\left((1-\epsilon)\bar{R}-1\right)(1-p_n)}\right]^{1-p_n}$. I remind the reader that $b_1^* > b_2^* > \ldots > b_{N-1}^*$. The optimal issuance size increases when the investors know that they face a less risky project.

Proposition 3. *Under Condition 1 and Condition 3, in the infinitely repeated game with reputation if the number of projects is more than two then independent of the discount factor* $\delta : 0 \leq \delta < 1$ *and the initial reputation level* $\alpha_1 : 0 \leq \alpha_1 < 1$ *there is no informationally-efficient equilibrium.*

Proof. .

We first note that a bad firm will never access if its true type is revealed. No access is always a profitable deviation. When it does not access, the worst the investors could believe about the firm is that it is the bad firm. But even then the bad firm saves the fee. So in an informationally-efficient equilibrium the bad firm will not access the CRA. Now note that an informationally-efficient equilibrium separates between the types on the equilibrium path. Hence in an informationally-efficient equilibrium all the firms with types better than the worst will access and the CRA will reveal their type truthfully.

We note now that everything in the paragraph above has got to happen in every time period along the equilibrium path. Now fix the first period of an informationally-efficient equilibrium and let us look at the continuation payoffs from period 2 on. In every contingency along the equilibrium path every firm better than the worst will access and its type will be revealed to the investors. Hence the continuation payment of the CRA after every contingency that happens on the equilibrium path is constant and in particular is equal to $\frac{\sum_{n=1}^{N-1} \eta_n \epsilon b_n^*}{1-\delta}$.

We now know that the continuation from period 2 onwards is fixed at $\frac{\sum_{n=1}^{N-1} \eta_n \epsilon b_n^*}{1-\delta}$ and that all the firms except the worst firm will access and their true type will be revealed. Consider the following profitable deviation for the CRA, after access by a p_2 firm, it announces it is a p_1 firm. The investor would assume we are still on the equilibrium path and would act accordingly increasing the CRA's current payment ($\epsilon b_1^* > \epsilon b_2^*$), and the future payment is still fixed by the equilibrium's continuation payment ($\frac{\sum_{n=1}^{N-1} \eta_n \epsilon b_n^*}{1-\delta}$). This profitable deviation concludes our simple proof. □

5 Conclusion

In this paper I have shown that reputation is not potent enough for the CRA to mitigate information asymmetry in the structured finance market. In the model with reputation, we expected that the fear to lose this reputation will discipline the CRA and help it fulfill its role fully.

What we found is that in the model with non-verifiable ratings, if there are only two types of projects and only the CRA is informed of its type, then there is a cutoff reputation level such that if the initial reputation is above the cutoff then an informationally-efficient equilibrium exists. If the initial reputation is below the cutoff then there is no informationally-efficient equilibrium. Although at a first glance the result is encouraging, but we fail to extend the result in two possible directions. The first direction is a change of the information structure of the game. If the firms are informed of the type of the CRA, then there is no informationally-efficient equilibrium. The second direction is that even when only the CRA is informed of its type, if there are more than two project types there is no informationally-efficient equilibrium. It does seem that the existence result we got in the two project types hinges on the fact that when reputation is high, the bad firms do not want to access. The fact that there is the access and no access choice then separates the types. The bad firms do not access, and the good firms access. The separation here is not really happening because of what the CRA itself does.

When the firms know the true type of the CRA, the intuition above fails. The bad firms are not afraid to access the CRA any more when it is the strategic type. Hence there is no informationally-efficient equilibrium. And when there are more than two types of projects, the access and no access decision is not enough any more to separate the types. The actions of the CRA itself are needed to help separate between the types. The CRA can not reveal truthfully when it is the strategic type, and there is no informationally-efficient equilibrium. If investors believe the CRA will always reveal truthfully, then the CRA prefers to deviate today to make the highest gains in the current payment. This increases its current payment without having any effect on the continuation payoff which is fixed in any informationally-efficient equilibrium.

Therefore because of the lack of robustness of our existence result and because the world is composed of more than two types of projects, I conclude that reputation is not strong enough to ensure efficiency in the structured finance market.

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