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## Labor market and Schooling Choice: Italy vs US

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### ABSTRACT

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The aim of this paper is to study the effect of labor market institutions on schooling choice. I introduce college decision in a search on-the-job model, where wage offers distribution depends on workers' innate ability and schooling level. The interaction between workers' heterogeneous abilities and labor market structure endogenously determine both college wage premium and college decisions. As in the classical search on-the-job model, higher labor mobility and higher wage dispersion increase workers' probability to reach the right tail of the wage distribution. This provides agents with a larger incentive to shift wage distribution to the right by attending school. The model fits US and Italian data, two countries that present wide differences in educational achievements. Specifically, the US presents a larger college premium, higher enrollment and graduation rates in college. I find that the labor markets in US and Italy differ significantly in two dimensions. The US is characterized by i) higher job mobility and ii) higher return of innate ability. The former accounts for around 30% of the different enrollment and graduation rates. The latter rises the observed college premium in US above Italy, despite the effect of schooling on wage being higher in Italy, and can account for almost 50% of differences in educational attainment.

JEL: I20, J01, J02, J24, J60. Keywords: Labor Market, Schooling Choice, Job Mobility, Wage Dispersion.

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## Introduction

When looking at the private return to education, previous literature has mainly focused on the wage differentials, as a measure of productivity differentials. An interesting issue, to which little attention has been paid, is the relation between labor market institutions and return to schooling. Do different institutional arrangements significantly affect the market value of education? In this paper, by adding schooling decision to a search model with agents heterogenous in their innate abilities, I build a theoretical framework in which both college wage premium and college decisions are endogenously determined by the interaction of labor market features and agents' private ability. In the model agents i) attend college in order to accumulate human capital <sup>1</sup> and ii) make employment decisions as in a classical search on-the-job model, with wage offers increasing in both innate ability and human capital accumulated. Agents with higher ability can accumulate faster human capital (hence at a lower cost), and therefore will decide to accumulate more human capital, by enrolling in college.

From a theoretical point of view I find that a more flexible market generates an higher incentive for college attendance. In the simple search model, the value of employment depends strongly not only on the average offered wage, but also on other moments of the wage distribution and on wage offer dynamics (namely, the probabilistic process of wage offer and/or matching). Likewise in my model, the return of education can be significantly altered by all parameters characterizing the labor market. Specifically, high worker mobility and low wage compression enhance worker's probability to reach the right tail of wage distribution, and therefore the incentive to shift to the right the wage offers' distribution by accumulating capital.

I fit the model to US and Italian data. These two countries were chosen because they significantly differ in both the structure of the labor market, and general educational

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<sup>1</sup>The accumulation process is similar to the classical Ben-Porath(1967) fashion framework, but beside a monetary cost, student bear the opportunity cost of foregone wages.

achievements. On the one hand, economic literature has always stressed the presence of more frictions in Italian labor market with respect to the American one, namely low worker mobility and a rigid highly centralized wage setting system. On the other hand the US is characterized by a higher college wage premium, higher cost of college education, higher college enrollment and completion rates. The model fits data from both countries and is able to explain the differences in the market for college education as an outcome of the structure of the labor market, without assuming any technological differences. I find that the US and Italy differ significantly in two dimensions. First of all, US labor market is characterized by higher mobility. The higher probability of job-to-job transitions allows American worker to climb the wage ladder faster, providing them with an higher return to schooling. I find that differences in labor mobility can account for almost 40% of differences in college choice between the US and Italy. Second, innate abilities have a remarkably lower effect on wages in Italy. This affects both the observed college premium and enrollment decision. I find that the college degree is expected to increase wage by less than 20% in both countries with the remaining premium being explained by differences in pre-existent abilities between those who obtained a college degree and those who didn't. Since individual abilities have a small effect on Italian wages, the observed college premium is significantly lower in Italy than in the US (27% versus 60%). Running a counterfactual simulation, I find that differences in return to ability can explain almost 50% of the difference in graduation and enrollment rates between the US and Italy.

This paper relates to two important branches of economic literature, one studying the determinants of return to education, and the other investigating labor market institutions. Theoretical literature has discussed several factors that might affect the market value of education. We can generally identify two lines of research: one focusing on productivity enhancement by schooling (introduced by Schultz, 1961 and Becker, 1964) and a second stressing the role of school as a device for ability signaling (this literature traces back to Spencer,

1973). According to the first hypothesis, more educated workers can expect a higher wage since attending school increase their productivity. In the second scenario, the differences in productivity are ex-ante, and school works as a signaling device, selecting the more skilled workers.<sup>2</sup>

Following the first approach, different returns to education (both across countries and time) has been explained by differences in technology. For example, most of the growth models aimed at explaining the dramatic increase in the college premium experienced by the US in the last 25 years, have basically looked exclusively at the production side. The most common explanation for such a rise is skill biased technological change. Acemoglu(2002) endogenizes the technological change, showing that international trade with less developed countries might constitute an incentive for firms to adopt skill-biased technologies. Another common explanation relies on the assumption of high complementarity between Capital and Skilled labor. In Krusell, Ohanian, Rios-Rull and Violante (2000) the drop in the relative price of capital, boosts the demand for skilled workers, hence the wage premium<sup>3</sup>. This paper introduces market institutions as another factor affecting return to education. I show how, even in absence of relevant differences in technology, return to education might vary because of a different structure of the labor market. Eckstein and Wolpin (1995) study the relationship between unemployment duration and return to schooling in a search-matching model. They show how neglecting search frictions can significantly bias estimates of return to schooling. Their paper focuses on the observed return to schooling, without analysis of the schooling choice and how this is related to return to schooling. Embedding endogenous

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<sup>2</sup>The two explanations are clearly not mutually exclusive. Fang,2004 estimates the relative weight of the two factors in determining the college premium.

<sup>3</sup>These theories identify workers with high education as workers employed in high skilled tasks, and low education workers as workers employed in low-skilled tasks. Eckstein (2004) claims that the two are not identical, and that while the college premium has increased the skill wage premium (measured as differences between the remuneration for high-skills requiring jobs and low-skills requiring jobs) is basically constant. The college premium has increased only because over time access to high skills jobs has been restricted to college students. From this point of view the changes in the college premium have to be attributed mostly to a different functioning of the labor market that tends to separate more sharply college graduates from high school graduates.

schooling choice in the model I am able to empirically quantify the effect that frictions have on educational attainment. More over, in Eckstein and Wolpin there are two main frictions that significantly bias the observed return to schooling: unemployment duration and the distribution of accepted wages, as opposed to the distribution of offered wages. In my model the main force is not the probability to exit unemployment, but the probability of job-to-job transition. The latter allows worker to climb the wage ladder faster, and hence provide incentive to shift wage distribution to the right by attending college.

My paper also relates to that part of the economic theory studying the effect of labor market institutions (mainly labor mobility and wage setting procedures) In particular, wage rigidities and high hiring/firing cost has been considered at the root of the so-called Eurosclerosis, claimed to be a major factor behind the poor economic performance of most European countries in the last 20 years. The interaction between mobility and investment in education hasn't receive the same attention. In recent years availability of a new data set has prompted research on labor mobility. Flinn (2002) uses a similar model to estimate the effect of scarce labor mobility on life time welfare inequality in Italy and US. Jolivet et al (2006) test mobility differences between Europe and US, using a similar dataset.<sup>4</sup> It's also a well known fact that wage dispersion in Italy is significantly lower than in the US.<sup>5</sup> Different reasons have been provided, (Italian high inflation, distribution of skills, technological change, educational achievement) but most of the literature has been focusing on the role played by highly centralized unions on the wage setting system.<sup>6</sup> Checchi and Pagani(2004) provide an interesting analysis of the relationship between the structure of the wage setting, the role

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<sup>4</sup>Most of the literature has suggested that the low labor mobility that characterize the Italian labor market is explained by : 1) the strictness of Employment Protection Legislation (EPL hereafter)adopted in Italy (with a strong emphasis on the high firing costs and severance payments); 2) the size of the public sector. For instance Garibaldi (1996), Garibaldi and Violante (2000) Garibaldi and Pacelli study the effect of EPL on unemployment, employment duration and labor mobility. See also Contini(1999) for an extensive survey on work mobility in Italy

<sup>5</sup>In particular during the 1970s Italy experienced a dramatic compression of wage differentials, and in the 1980s the increase in wage inequality was significantly smaller than the one observed in the US.

<sup>6</sup>According to OECD(1999) The percentage of workers' contracts covered by collective bargaining is almost 90% in Italy and less than 20% in the US

of Unions and Italian wage inequality. With this paper I show that low mobility and wage compression, can have a strong effect also on schooling decisions.

The paper is organized as follows. In Section 1) I build a theoretical model able that incorporates schooling decision in the classical framework of a search model. In section 2) I solve the model and discuss the main features generated in equilibrium. In section 3) I present the data used to fit the model to US and Italian markets and present the estimation method. In section 4) the results of the estimation are discussed. Obviously section 5) is reserved for final conclusions.

## 1. The model

In order to study the impact of labor market institutions on return to schooling, and consequently on private investment in education, I use a continuous time model of search on-the-job with an endogenous schooling decision. Infinitely lived agents differ in their innate ability, denoted by  $\gamma$ . Its distribution in the population is denoted by  $G : \Gamma \rightarrow [0, 1]$ , where  $\Gamma \in [1, +\infty)$ . Moreover, at time 0, each agent is endowed with human capital  $h = h_0$ . While ability can't change through time, each agent can increase her human capital by attending college. While in college, she pays a constant fee  $f$  and accumulates human capital through an effort function  $\dot{h} = \rho\gamma^\eta x^{1-\eta}$  where  $x$  denotes the effort exerted,  $\rho > 0$  and  $\eta \in (0, 1)$ . The higher  $\eta$  is the higher is the effect of innate ability in accumulating human capital in college. The parameter  $\rho$  is a general index of how efficiently college allows the student to accumulate human capital. In each moment the student can decide to drop out of college and join the job market or continue studying. The maximum level of human capital a worker can attain is  $\bar{h}$ , corresponding to a college degree.

The labor market is structured as in the classical search model. Job offers to unemployed and employed workers accrue at constant Poisson rates  $\lambda^u$  and  $\lambda^e$  respectively. Exogenous layoffs accrue at the constant Poisson rate  $\delta$ . If employed, the worker receives a flow of earnings of  $w$ . The latter is a function of  $h$  and  $\gamma$ :  $w = h^\alpha \gamma^\beta \omega$  with  $\alpha \in (0, 1), \beta > 0$

and  $\omega$  be employee-firm match specific component drawn from a set  $\Omega \in \mathbb{R}^+$  with associated distribution function  $F$ .

An Agent utility function is increasing in consumption  $c_t$  and decreasing in effort  $x_t$ . For simplicity I assume linearity in both arguments even if the assumption is not necessary for any of the qualitative results derived. No saving technology is available, so  $c_t = w_t$  if the agent is working,  $c_t = 0$  if the agent is unemployed and  $c_t = -f$  if the agent is enrolled in school. Individuals seek to maximize the expected discounted (at a positive rate  $r$ ) utility. If  $r > 0$  the agent will find it optimal to concentrate all human capital investment at the beginning. At time 0, an agent decides whether to enroll in college and, conditional on being enrolled, for how long. Once she exits college, she will never enroll again. Hence, I first describe the college decision made by the agent at time 0, and I will later study the employment choice she faces after.

## A. College Decision

Given the non-negativity of the discount rate, the problem of a worker continuously deciding whether to be in college or not, can be reduced to the choice of *a*) the time to exit college  $T$  ( $T = 0$  implies that the worker won't enroll in college at all) and *b*) the effort  $x_t$  exerted for each  $t \in [0, T]$ , *c*) the exit value of human capital,  $h_T$ . Let  $U(h, \gamma)$  denote the expected discounted future income of an unemployed worker, endowed with ability  $\gamma$  and accumulated human capital  $h$ . We can write the agent problem at time 0 as:

$$\begin{aligned} \max_{T, x_t, h_T} \int_0^T e^{-rt} (-f - x_t) dt + e^{-rT} U(h_T, \gamma) \\ \text{s.to} \\ \dot{h}_t = \rho \gamma^\eta x_t^{1-\eta} \\ h_0 = \bar{h}_0 \end{aligned}$$

## B. Employment Choice

The problem of an employed worker follows the usual characterization of a search on-the-job model. With  $V(\omega, h, \gamma)$  representing the value of a worker with ability  $\gamma$  and accumulated skills  $h$  of being employed at a wage rate  $h^\beta \gamma^\alpha \omega$ :

$$(r + \lambda^e + \delta) V(\omega, h, \gamma) = h^\alpha \gamma^\beta \omega + \delta U(h, \gamma) + \lambda^e \int \max[V(\omega', h, \gamma), V(\omega, h, \gamma)] dF(\omega') \quad (1)$$

$$(r + \lambda^u) U(h, \gamma) = \lambda^u \int \max[V(\omega', h, \gamma), U(h, \gamma)] dF(\omega') \quad (2)$$

## C. Solution of the Model

The optimal behavior of the agent is fully described by:

1. A "stopping time" policy  $T(\gamma) \geq 0$ ;
2. An effort policy function  $x_t(\gamma, h)$  for  $t$  between 0 and  $T$ <sup>7</sup>;
3. An acceptance space  $\Omega^u(\gamma, h) \subseteq \Omega$ , such that an unemployed worker with ability  $\gamma$  and skills  $h$  will accept a job offer of  $\omega$  if and only if  $\omega \in \Omega^u(\gamma, h)$ ;
4. An acceptance space  $\Omega^e(\omega, \gamma, h) \subseteq \Omega$ , such that a worker with ability  $\gamma$  and skills  $h$ , currently working at wage  $w = h^\alpha \gamma^\beta \omega$  will accept a job offer of  $\omega'$  if and only if  $\omega' \in \Omega^e(\omega, \gamma, h)$ .

The existence of a unique bounded solution for  $V(\cdot), U(\cdot)$  is guaranteed as in the standard search model. In order to characterize 3) and 4) we can observe that  $U$  is homothetic in both its arguments, hence  $U(h, \gamma) = h^\alpha \gamma^\beta \bar{U}$  where  $\bar{U} = U(1, 1)$ <sup>8</sup>. This implies that  $\Omega^u(\gamma, h) = \Omega^u = \{\omega \in \Omega | \omega > \omega^*\}$ , and  $\Omega^e(\omega, \gamma, h) = \Omega^e(\omega) = \{\omega' \in \Omega | \omega' > \omega\}$ . If moreover  $\lambda^e \leq \lambda^u$  the solution of an unemployed agent is represented by a non trivial reservation wage

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<sup>7</sup>The choice  $T = 0$  represents the decision of not enrolling in college. If the choice of  $T$  and  $x_t$  is such that  $h_T > \bar{h}$  the agent graduates.

<sup>8</sup>See Appendix 1.

policy for unemployed (that is  $\Omega^u \neq \Omega$ ).

In order to characterize the schooling choice decisions we reformulate the problem in terms of an optimal exit strategy. Indeed, we can restrict the college decision of the agent to: *a*) an optimal value of human capital when exiting college  $h^*$  and *b*) the effort strategy  $x_t$  to reach it. Given the deterministic nature of the human capital accumulation process the optimal time to exit college  $T$  is univocally determined by *a*) and *b*). The college decision problem is therefore reformulated as:

$$\begin{aligned} \max_{h^*, x_t} \int_0^T e^{-rt} (-f - x_t) dt + e^{-rT} U(h^*, \gamma) \\ \text{s.to} \\ \dot{h}_t = x_t^{1-\eta} \gamma \rho \\ h_0 = \bar{h}_0 \\ T = \inf\{t | h_t \geq h^*\} \end{aligned}$$

Let's denote with  $W_{h^*}(h, \gamma)$  the value of an enrolled student that will exit college once the optimal level of human capital  $h^*$  is reached. By  $\hat{I}$ 's lemma  $\dot{W}_b(h, \gamma) = \dot{h} \frac{\partial W}{\partial h}$  since  $\gamma$  is constant over time. The evolution of  $W(\cdot)$  is therefore described by the following differential equation:

$$rW_{h^*}(h, \gamma) = \max_x [-f - x + \rho \gamma^\eta x^{1-\eta} DW_{h^*}(h, \gamma)] \quad (3)$$

with boundary condition

$$W_{h^*}(h^*, \gamma) = U(h^*, \gamma). \quad (4)$$

In appendix 2 I prove that the previous differential equation has a unique bounded solution given by:

$$W_{h^*}(h, \gamma) = -\frac{f}{r} + \left[ \left( \frac{1-\eta}{\eta} \right)^\eta \frac{r^\eta}{\gamma^\eta \rho} [h - h^*] + \left( \bar{U} h^{*\alpha} \gamma^\beta + \frac{f}{r} \right)^{1-\eta} \right]^{\frac{1}{1-\eta}}$$

The optimality of  $h^*(\gamma)$  is determined by the “smooth pasting condition”

$$DW_{h^*}(h^*, \gamma) \begin{cases} = DU(h^*, \gamma) & \text{if } h^*(\gamma) \in (h_0, \bar{h}) \\ \geq DU(h^*, \gamma) & \text{if } h^*(\gamma) = h \\ \leq DU(h^*, \gamma) & \text{if } h^*(\gamma) = \bar{h} \end{cases}$$

We can rewrite the previous inequalities substituting the corresponding expression:

$$\frac{1}{\gamma^\eta \rho (1-\eta)^{1-\eta} \eta^\eta} (r \bar{U} h^{*\alpha} \gamma^\beta + f)^\eta \underset{\leq}{\geq} \alpha \bar{U} h^{*\alpha-1} \gamma^\beta \quad (5)$$

The left hand side of (5) represents the instantaneous marginal cost of staying in college. It is composed by the opportunity cost of not looking for a job  $r \bar{U} h^{*\alpha} \gamma^\beta$  and the monetary cost of being enrolled  $f$ . Notice that innate ability has two opposite effects on the marginal cost of being in college. On the one side, higher ability reduces the cost of college by lowering the time required to reach a given level of human capital; on the other side it increases the labor market value of the worker, that is the opportunity cost of not searching for a job. The right hand side is the marginal benefit of joining the labor market. Notice that the latter is decreasing in  $h$  since  $\alpha \in (0, 1)$ , that is the value of looking for a job is increasing but concave in the level of human capital accumulated. Consider the function

$$\begin{aligned} S(b, \gamma) &= DW_b(b, \gamma) - DU(b, \gamma) \\ &= \frac{1}{\gamma^\eta \rho (1-\eta)^{1-\eta} \eta^\eta} (r \bar{U} b^\alpha \gamma^\beta + f)^\eta - \alpha \bar{U} b^{\alpha-1} \gamma^\beta \end{aligned}$$

Since  $\lim_{b \rightarrow 0} S(b, \gamma) = -\infty$ ,  $\lim_{b \rightarrow \infty} S(b, \gamma) = \infty$ , and  $\frac{\partial S}{\partial b} > 0$  there is a unique  $b^*(\gamma)$  such that  $S(b^*(\gamma), \gamma) = 0$ . Hence, considering corner solutions we can solve for the optimal  $h^*$ :

$$h^*(\gamma) = \min\{\bar{h}_0, \max[b^*(\gamma), \bar{h}]\}$$

Notice that  $h^*(\cdot)$  is non-decreasing in  $\gamma$ <sup>9</sup>. This implies that agents endowed with a higher ability will decide to acquire an higher level of skills. This allows me to identify two threshold ability levels that completely describe both enrollment and graduation decisions. Defining

$$\gamma^e = \gamma | h^*(\gamma) = \bar{h}_0$$

$$\gamma^g = \gamma | h^*(\gamma) = \bar{h}$$

1. A worker with skills  $\gamma \leq \gamma^e$  will not enroll in college;
2. A worker with skills  $\gamma \in (\gamma^e, \gamma^g)$  will enroll in college and drop out after reaching a human capital level  $h^*(\gamma) < \bar{h}$ ;
3. A worker with skills  $\gamma \geq \gamma^g$  will enroll in college and graduate.

This allows me to easily compute both the enrollment rate (the percentage of the population that will enroll in college, hereafter denoted  $ER$ ) and the graduation rate (the percentage of the population who will obtain a college degree, hereafter denoted  $GR$ ) forecasted by this model:

$$ER = 1 - G(\gamma^e)$$

$$GR = 1 - G(\gamma^g)$$

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<sup>9</sup>It comes from a simple application of the Inverse Function theorem to the function  $S(\cdot)$

Finally we can solve for the optimal length of schooling. In the Appendix we show that:

$$T(\gamma) = A^{-1} \ln \left( \frac{B(\gamma)}{Ah_0 + B(\gamma) - Ah^*(\gamma)} \right)$$

where

$$A = r \frac{1 - \eta}{\eta}$$

and

$$B(\gamma) = \rho \gamma^n \left( \bar{U} h^{*\alpha} \gamma^\beta + \frac{f}{r} \right)^{1-\eta} \left( r \frac{1 - \eta}{\eta} \right)^{1-\eta}$$

First of all notice that since  $h^* \geq h_0$ , the argument in the logarithm function is always greater or equal to 1, that is  $T \geq 0$ . Notice that the optimal time spent by an agent to reach a given level of target human capital  $h^*$  is decreasing in  $F(\gamma)$ . Indeed  $F(\gamma)$  represents the optimal speed of the human capital accumulation. The first factor  $\rho \gamma^n$  is a technological component, while the second reflects the optimal effort exerted and is increasing in the two component of the cost of college, the monetary cost  $f$  and the opportunity cost  $\bar{U} h^{*\alpha} \gamma^\beta$ . Both factors are increasing in  $\gamma$ , since an agent endowed with higher ability i) is a priori able to accumulate human capital faster, ii) exerts an higher level of effort since her opportunity cost of not looking for a job is higher. Notice that this doesn't imply that  $\frac{\partial T}{\partial \gamma} < 0$  since the optimal level of human capital  $h^*(\gamma)$  is increasing in  $\gamma$  as well. The function  $T(\gamma)$  is typically reversed U-shaped. Indeed notice that  $T(\gamma^e) = 0$  since  $h^*(\gamma^e) = \bar{h}_0$ , but also  $\lim_{\gamma \rightarrow \infty} F(\gamma) = \infty$  that implies  $\lim_{\gamma \rightarrow \infty} T(\gamma) = 0$ . On the one hand agents with very low ability spend a small time in college since their target human capital is low. On the other hand agents with very high ability spend a small time in college since they accumulate human capital faster and the optimal human capital is bounded above.

## 2. Schooling Choice and Labor Market: Comparative analysis

In this section we study how different features of the labor market might affect schooling decision. The next four theorems<sup>10</sup> show that labor market institutions can have a strong effect on educational attainment. In particular, in our model both enrollment and graduation rates are positively affected by i) job mobility and ii) wage inequality.

**THEOREM 1.** *Enrollment rates and Graduation rates are increasing in  $\lambda^e$ .*

The intuition is as follows. By changing jobs while employed, workers are able to reach the right tail of the wage distribution faster. This increases the incentive for the worker to shift the wage distribution to the right, and they can do that by accumulating human capital. Notice that even if both  $\lambda^u$  and  $\delta$  are held constant, in equilibrium workers will stay employed more often because an increase in  $\lambda^e$  lowers the reservation wage. Nevertheless the positive effect of job-to-job mobility on the return to education doesn't depend on employment probability. Even if we assume a non-optimal fixed reservation wage policy, that is we hold constant the time that a worker stays employed, a higher  $\lambda^e$  will still imply an higher optimal investment in human capital. The key mechanism is the possibility of the worker to climb the wage ladder and find employment opportunity with higher value.

Since often economies with higher job-to-job mobility are also characterized by higher separation rate between firms and employees, in the next theorem we study the effect on return of education of variations of all mobility parameters.

**THEOREM 2.** *Let's define a family of economies  $\mathcal{E}(\kappa)$  where  $\kappa$  is a measure of mobility such that  $\{\lambda^u, \lambda^e, \delta\}(\kappa) = \{\kappa\bar{\lambda}^u, \kappa\bar{\lambda}^e, \kappa\bar{\delta}\}$ . Enrollment rates and Graduation rates are increasing in  $\kappa$ .*

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<sup>10</sup>All proofs are in the Appendix 3

In this case we study the effect on return to education of a proportional variation of all mobility parameters but the intuition behind Theorem 2 is the same as in Theorem 1. Also in this case the mechanism driving the result doesn't lie in an increased employment probability. Indeed, as proved in the appendix, since the reservation is increasing in  $\kappa$ , the long term employment probability is lower. Again the key incentive to shift right the wage distribution lies in the increased probability to reach in a shorter time the highest wages.

The next theorem studies the relationship between wage dispersion and return to education. To study the impact of dispersion, we use the concept of Rothschild and Stiglitz mean preserving spread.

**THEOREM 3.** *Consider a family of distribution function  $F(\omega, \sigma)$ , where  $\sigma$  is our measure of dispersion, that is i)  $F_\sigma(\omega) \geq 0$  ( $\leq 0$ ) for all  $\omega \leq \theta$  ( $\geq \theta$ ), ii)  $\int_{-\infty}^{\infty} F_\sigma(\omega) d\omega = 0$ . Enrollment rates and Graduation rates are increasing in  $\sigma$ .*

The intuition is as follows. It's already known that in the classical search model wage dispersion increases the reservation wage and the value of unemployment because of the asymmetric effect of thicker tails. Since the reservation wage policy truncates the distribution of wages to the right, the impact of a thicker right tail (higher probability of high wages) overweighs the impact of a thicker left tail (higher probability of low wages). A similar mechanism applies here. Since a thicker upper tail allows workers to reach higher wages with higher probability, agents have a bigger incentive to shift the wage distribution by accumulating human capital.

**THEOREM 4.** *Enrollment rates and Graduation rates are increasing in  $\beta$ .*

In general, return to ability could either boost or depress enrollment and graduation rates. From one point of view a higher return to ability increases the opportunity cost of schooling by raising the expected wage. On the other hand there might be complementarities between return to education and return to ability in the labor market. For example, ability might matter only in jobs that require a certain level of schooling, while being irrelevant for other

tasks. Or schooling can be a device through which an agent can reveal her real ability. In both cases the higher is the effect of ability on wages, the higher is the incentive to attend school. To simply model the effect of complementarities between schooling and ability we assume that both skills and ability enter in the wage function in a multiplicative way. Using this analytical formulation, the effect of ability on the return to education due to the above mentioned complementarities is stronger than the effect of ability on the opportunity cost of schooling, driving the result stated in the previous theorem. I conclude this section by pointing out that in this model all the intuitive mechanisms common in the schooling choice literature, that doesn't need any further explanation.

**THEOREM 5.** Enrollment rates and Graduation rates are increasing in return to schooling  $\alpha$ .

### 3. Data: NLSY and EuroPanel

American data are the 1979 cohort of NLSY. Italian data come from the European Community Household Panel (ECHP). The ECHP is an 8-year (1994-2001) longitudinal survey of individuals developed by the Statistical Office of the European Communities (Eurostat). For purpose of comparability we restrict the NLSY data to the first 8 waves and we drop from the Italian sample individuals that at the first observation are more than 22 years old. Since we study schooling choice we focus on individuals that, at last observation, have at least a high school degree. The main demographic statistics are summarized in Table 1.

TABLE 1: *Sample Basic Demographic*

<b>Characteristics</b>	<b>Italy</b>	<b>US</b>
<b>Sample Size</b>	3682	9896
<b>Gender(%)</b>		
Male	48.96	48.83
Female	51.03	51.17
<b>Average Age at last observation</b>	23.18	22.54

In Table 2 I summarize the statistics relative to schooling choice. I classify individuals according to the educational level achieved at the last observation. A first glance of schooling choice provides us with some preliminary observations. The percentage of Italians who decide

not to enroll in college in Italy is 57.25%, a percentage significantly higher than the 41.05% observed in the US sample<sup>11</sup>.

TABLE 2: *Education Statistics*

Characteristics	Italy	US
<b>Schooling at the last observation (%)</b>		
<i>High School</i>	57.25	41.05
<i>Some College</i>	31.02	33.36
Still in College	10.95	3.53
Drop-out	20.07	29.83
<i>College Graduate</i>	11.73	25.60
<b>Years enrolled in College</b>		
<i>Total</i>		
Mean	1.67	1.68
Standard Deviation	2.38	1.98
<i>Still in College</i>		
Mean	4.47	2.44
Standard Deviation	1.94	1.32
<i>Drop-out</i>		
Mean	2.34	1.90
Standard Deviation	1.40	1.21
<i>Graduate</i>		
Mean	5.27	4.17
Standard Deviation	1.28	1.62

Moreover, some extra information about schooling decision can be inferred. In particular data seem to suggest that, conditional on being enrolled, Italians tend to stay in college a lot longer. Despite the difference in enrollment rate, the average number of years of college attendance is surprisingly very similar in both countries. As mentioned above, this implies that conditional on being positive, the length of college attendance is much higher in Italy. This is true for those who finished college at the last observation time( 5.27 years for Italian, versus the 4.17 of US) and above all for those who are still in college (4.47 for Italy versus 2.44). Considering that more than 10% of Italian are still in College at the end of our observation period (out of 42.75% of individuals who have been enrolled), compared to the 3% (out of 59%) of the American counterpart, we can conclude that the average length of a college degree is significantly higher in Italy than in the US. In my model this might be explained by: i) lower abilities of the average Italian college student (lower  $\eta$ ), ii) higher level

<sup>11</sup>If data from the same observation period were available the difference would probably be more remarkable, since both countries experience an historical increase in the enrollment rates.

of knowledge incorporated in the Italian college degree (higher  $\bar{h}$ ), iii) lower efficiency of the learning process in Italian College (lower  $\rho$ ) iv) lower cost of college (lower  $f$ ). Our model is flexible enough to account for (and disentangle) the four effects. In table 3 we compare the dynamics of Italian and US Labor markets. Both Panel Data allow us to build Employment histories for each individual. Since ECHP reports only month-by-month basis data (compared to the weekly based NSLY reports) our time unit measure will be one month (for both wage and transition data).

TABLE 3: *Labor Dynamics*

Characteristics	Italy	US
<b>Unemployment rate at the last observation</b>		
<i>Total</i>	23.42	4.87
<i>High School</i>	20.67	7.11
<i>Some College</i>	21.07	6.41
<i>College Graduate</i>	30.46	3.73
<b>Exit rate from Unemployment (%)</b>		
<i>Total</i>	4.51	11.66
<i>High School</i>	4.04	11.21
<i>Some College</i>	4.17	11.85
<i>College Graduate</i>	5.17	14.62
<b>Transition Rate from Employment to Unemployment (%)</b>		
<i>Total</i>	.052	1.97
<i>High School</i>	0.66	2.70
<i>Some College</i>	0.57	1.86
<i>College Graduate</i>	0.35	0.88
<b>Transition Rate from Job to Job(%)</b>		
<i>Total</i>	1.08	3.77
<i>High School</i>	0.80	3.40
<i>Some College</i>	1.07	3.78
<i>College Graduate</i>	1.15	4.04

Our data confirm the reputation of Italy as a country characterized by low mobility. Obviously the main effect of the slow exit from unemployment is on the youth unemployment. The combination of low exit rate from unemployment and long duration of college explains why the unemployment rate at the last observation is higher for college graduate than High School graduate. Finally Table 6 reports the statistics relative to the wage distribution. I report the wage statistics for wage at the last observation period and for the wage of the first job. Wages are expressed in thousands of 1979 dollars, for comparability reasons.

TABLE 4: *Wage Statistics*

	Last job wage		First job Wage	
	Italy	US	Italy	US
<i>High School Graduate</i>				
Mean	1683	1691	1258	1397
Standard Deviation	489	936	493	688
Coefficient of Variation	.2906	.5535	.3919	.4925
<i>Some College</i>				
Mean	1688	1799	1314	1673
Standard Deviation	567	926	621	802
Coefficient of Variation	.3359	.5147	.4726	.4794
<i>College Graduate</i>				
Mean	1865	2413	1686	2030
Standard Deviation	714	1263	683	988
Coefficient of Variation	.3828	.5234	.4051	.4867

The college premium is, in both cases, higher in US than in Italy. But while the college premium is significantly lower in Italy when considering job at the last observation (.48% versus 10%) the difference is much smaller when looking only at the first job (60% versus 27%). This is strictly related to the different duration of college. Considering the length of our panel and that Italians spend on average 5 years in college to graduate, high school can benefit wage growth for a significantly longer period of time. Indeed the average wage for a high school graduate is 1258 at the first job and 1683 in 2001 (last observation), with an average increase of 425 dollars. (33% growth rate). For College graduate the increase is only 179 dollars (10% growth rate). Another striking difference between US and Italy is the dispersion of wages. In particular, as expected, Italian wages are way more compressed than US wage.

## A. Parametrization

The parameter  $\gamma$  enters in the optimization problem twice. On the one hand it enters the human capital accumulation function as  $\gamma^\eta$ , on the other hand  $\gamma^\beta$  is a component of wage offers. For this reason I can't identify the distribution of  $\gamma$  separately from the exponents  $\eta$  and  $\beta$ . Therefore I fix a given distribution for  $\gamma$ , allowing for  $\beta$  (effect of ability on wages) and  $\eta$  (effect of ability on human capital accumulation) to be different across countries. In

particular I assume that:

$$\gamma = 1 + x; \text{ where } x \sim (0, 1)$$

Also I normalize  $h_0=1$  in both countries. Again  $h_0$  is essentially a scale parameter, and can't be separately identified from the other parameters. For each individual, firm-employee match specific productivity is assumed to be drawn from a log-normal distribution, that is

$$\omega = e^\epsilon \text{ where } \epsilon \sim N(\mu_\epsilon, \sigma_\epsilon)$$

The college cost  $f$  is taken from the average private expenditure per student, as in OECD 1999<sup>12</sup>. The average private expenditure per student, enrolled in tertiary education (expressed in 1979 dollars) is \$3121 in Italy and \$6689 in US. Since all data are monthly based, I assume a monthly cost for college of \$260 for Italy and \$557 for US.

## B. Estimation method

The 10 parameters of the model  $(\lambda^u, \lambda^e, \delta, \mu_\epsilon, \sigma_\epsilon, \bar{h}, \alpha, \beta, \rho, \eta)$  are estimated with a GMM method to estimate . I use a N-steps procedure, to update the weighting matrix. In order to compute the standard errors I estimate the asymptotic variance of the estimator, using consistent estimators for the Jacobian function. Details are discussed in the appendix. I chose 12 moments to match to balance all the information in the data related to the main feature of the model. First of all I consider moments relative to wage distribution for different education level. In particular I use mean and variance of first job wage distribution college graduate, high school graduate and college dropout. Second, I include three transition moments, relatively the observed transition probabilities from unemployment to employment, employment to unemployment, and from job to job. The last three moments

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<sup>12</sup>It doesn't include only fees and tuitions but also all related expenses such as housing, supplies, etc.

describe schooling decisions : Enrollment rate, graduation rate after 5 years, and drop out rate after 5 years.

## 4. Results

In the next table I report the results of my estimation for both US and Italy.

TABLE 5: *Estimation Results*

	Italy	US		Italy	US	Italy	US
	Wage distribution			Job Mobility		Human Capital	
$\mu_\epsilon$	6.7684	6.6245	$\lambda^u$	.0462	0.1307	$\bar{h}$	1.6979
	(0.0124)	(0.0231)		(0.0001)	(0.0001)		(.1121)
$\sigma_\epsilon$	.6650	.7180	$\lambda^e$	.0202	0.0835	$\rho$	153
	(0.0223)	(0.0962)		(0.0001)	(0.0001)		(12.47)
$\beta$	0.0270	0.2918	$\delta$	.0050	0.0202	$\eta$	0.88
	0.0431	0.8745		(0.0000)	(0.0001)		(.0429)
$\alpha$	0.3450	0.3334					0.95
	(0.1121)	(0.0987)					(0.0231)

We can observe that the distribution of  $\epsilon$ , the idiosyncratic component of wage, is very similar in the two countries, with the US distribution being slightly more disperse. Also, the impact of human capital on wage ( $\alpha$ ) is virtually the same in both countries. Nevertheless the implied wage distributions for the two countries differ significantly in another dimension. Innate abilities seem to have a much stronger effect in US than Italy. The reference parameter  $\beta$  is indeed almost 10 times higher in the US than in Italy (0.2918 versus 0.0270). Interesting enough, such a difference is not found when estimating  $\eta$ , the effect of abilities on human capital accumulation. The difference in  $\beta$  can be economically interpreted in different ways. A first explanation can be related to the wage setting system, and how this is able to reward individual characteristics. In a highly centralized wage setting system, with small or no room for individual bargaining, like the Italian one, it is not surprising to register a relatively small impact of personal ability on wages. On the other hand,  $\beta$  and  $\eta$  can be seen as a measure of the correlation between ability relevant in school ( $\gamma^\eta$ ) and ability relevant on the labor market ( $\gamma^\beta$ ). Lower values for  $\beta$  and  $\eta$  can therefore be interpreted as characteristics of a schooling system that requires abilities other than those relevant in a productive job. In any case, a low  $\beta$  has two effects on my model. First, it reduces the dispersion of observed

wage, since ability is one of the sources of wage variability. Second, as discussed previously,  $\beta$  measures the complementarity between wage and schooling, hence a lower  $\beta$  reduces the return to college and consequently the optimal choice of schooling.

Remarkable differences can also be found amongst the parameters governing job mobility. The estimates, all characterized by small standard errors, reflect the labor dynamics observed before.

Finally, estimations of all parameters characterizing the human capital accumulation process, reveal some interesting features. The amount of human capital incorporated in a college degree ( $\bar{h}$ ) is higher in Italy than in US. Moreover, given that the impact of human capital on wages ( $\alpha$ ) is virtually the same in both countries, we can conclude that, the wage rise that a worker can expect by getting a college degree is not higher in US than Italy. Therefore other factors do account for the relevant difference in the observed wage premium.

In the next table I compare the fit of the model to some relevant moments.

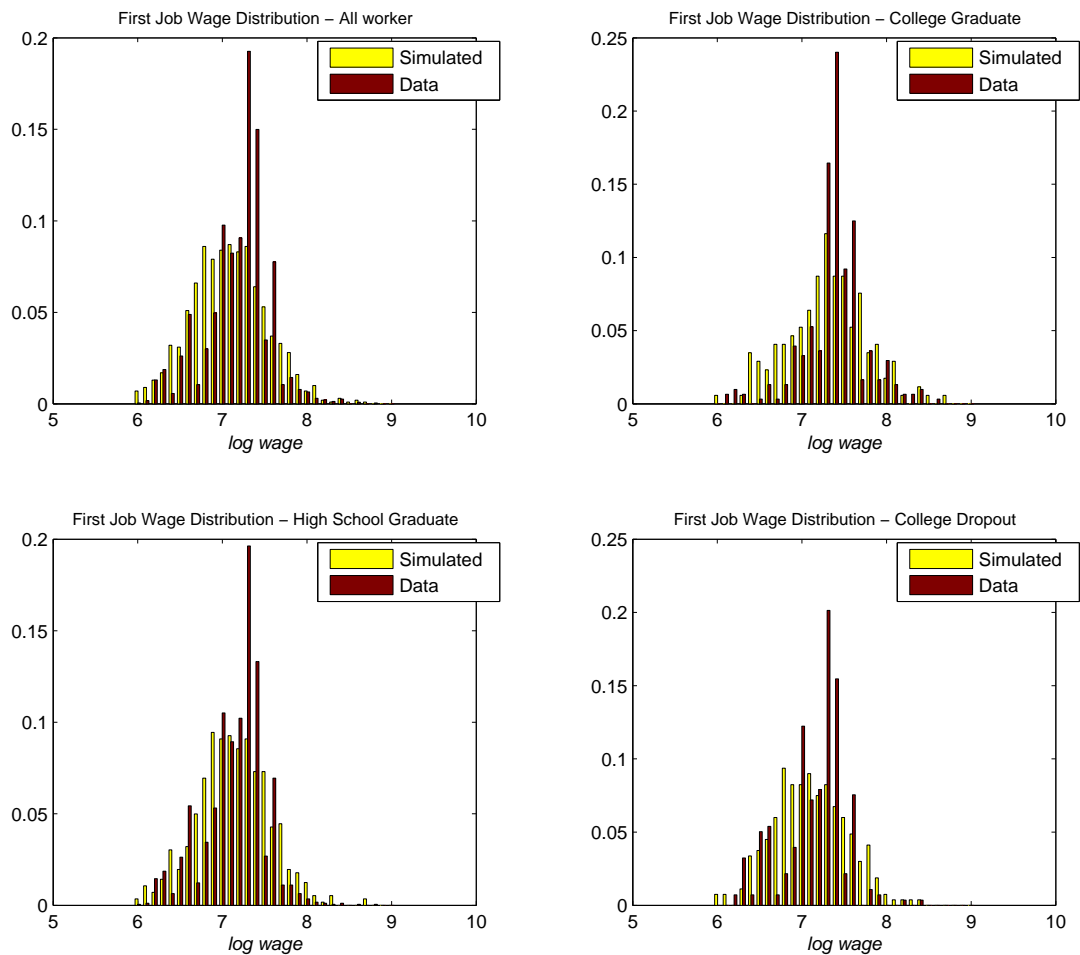
TABLE 6: *Moments fit*

	US		Italy	
	Data	Model	Data	Model
College Wage Premium	1.55	1.601	1.24	1.27
Some College Wage Premium	1.20	1.22	1.04	1.08
Enrollment rate	58.95	57.12	40.75	43.34
Dropout rate in 5 ys	.45	.53	.41	.33
Graduation Rate in 5 ys	0.25	0.26	0.08	0.04
Unemployment-Employment	11.66	12.01	4.51	4.55
Employment-Unemployment	1.97	2.01	0.052	0.049
Job-to-Job	3.77	3.89	1.08	0.91

In Figure 1 and 2, I compare the observed first wage distribution with the one generated by the model. The model is able to fit fairly well the observed premium for both college graduates and college drop out in both countries. A closer look at figure 1 and 2 reveals that the model is not able to capture some other dimensions of wage distribution. In particular Italian wage seems to be more compressed than our model predicts. The inability of my model to capture the high concentration around the mode seems to be particularly relevant for college dropouts . Indeed, in the model the human capital level is constant for both high

school graduate ( $\bar{h}_0$ ) and college graduate ( $\bar{h}$ ), while is heterogenous across college dropout, adding therefore another source of variability. A different problem arises when comparing data and model prediction for US data. In this case model predictions seems to fail not on the dispersion of the distribution, but on the symmetry. It is a well known fact that US wage distribution is strongly right skewed because of existing minimum wage policies. Indeed, the asymmetry is stronger for high school graduates, where the minimum wage constraint is more likely to bind, but disappears for college graduates. My model fails in capturing this feature.

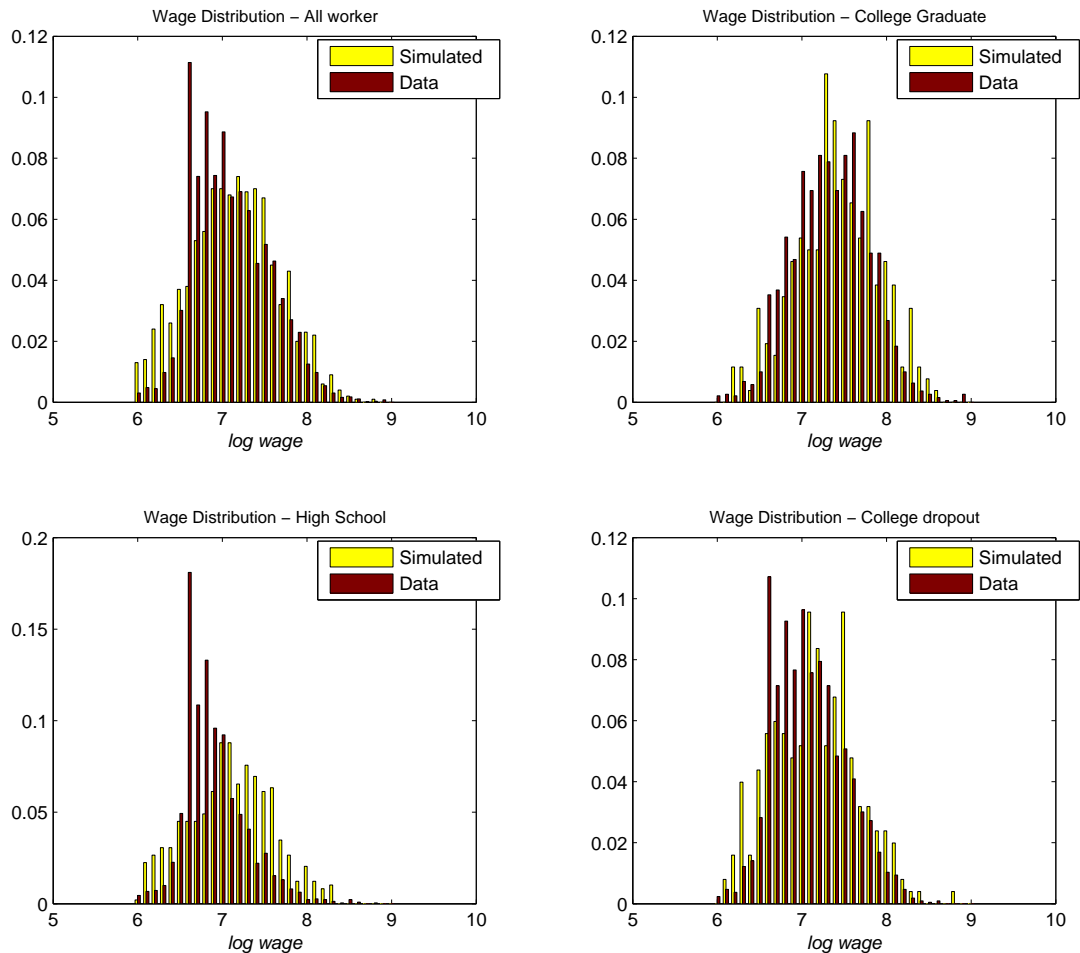
FIGURE 1 Italian Wage distribution



The model also fits observed behavior of college students pretty well. I underestimate the speed of the human capital accumulation process of Italian students, that is the time Italian

students need to reach their target level of human capital. The model predicts that in 5 years only 37% of workers reached their optimal target capital (either by graduating or by dropping out) while in the data only 50% of enrolled students are still in college after 5 years.

FIGURE 2 US Wage distribution



### A. Return to College

In Table 7 I display estimates for different measures of return to college. First I report the percentage increase of wage exclusively due to college attendance (rows 1 and 2 for college graduate and college drop out respectively). Then I compute the implied average wage ratios (College graduate versus high school graduate and college dropout versus high school

graduate ). The two indices are potentially very different since the second takes into account the endogenously determined average ability of each educational group. Indeed my model predicts an endogenous positive correlation between schooling and ability, that is agents with higher ability will attend college. Therefore observed wage is increasing in schooling level for two reasons: i) direct effect of human capital on wage, ii) different abilities. College graduates will receive higher wages not only because they attend college but also because their ability was higher. For convenience I use the following notation: cg=collge graduate, cd=college dropout; hs=highschool graduate.

TABLE 7: *Return to Education*

		Italy	US
<b>Wage gain</b>			
<i>College graduate</i>	$E(h^\alpha cg)/E(h^\alpha hs)$	1.2061	1.1361
<i>College dropout</i>	$E(h^\alpha cd)/E(h^\alpha hs)$	1.0681	1.0661
<b>Wage premium (unconditional)</b>			
<i>College graduate</i>	$E(h^\alpha\gamma^\beta cg)/E(h^\alpha\gamma^\beta hs)$	1.2644	1.6049
<i>College dropout</i>	$E(h^\alpha\gamma^\beta cd)/E(h^\alpha\gamma^\beta hs)$	1.0845	1.2212

A first, surprising, result is that the effect on wage of a college degree, per se, is lower in US than Italy (13% versus 20%). On the other hand the equilibrium wage premia is higher in the US than Italy (60% versus 26%)<sup>13</sup>. This implies that the difference between the observed college premia in Italy and the US is due to the differences in the pre-existent abilities between those who decide to enroll in college and those who don't. By decomposing the the logarithm wage premium we can assess the relative weight of the two components (ability versus schooling) in the overall wage premium. For a given educational group  $i = cg(\text{college graduate}), cd(\text{college dropouts})$

$$E(\log(w)|i) - E(\log(w)|hs) = \beta(E(\log(\gamma)|i) - E(\log(\gamma)|hs)) + \alpha(E(\log(h)|i)\log(\bar{h}))$$

The first term  $\beta(E(\log(\gamma)|i) - E(\log(\gamma)|hs))$  represents the endogenously determined difference in ability between the two groups. It's obviously increasing in  $\beta$ , therefore the observed wage premium is higher if the return of ability on wages is higher. The second term

<sup>13</sup>The same pattern is observed for college dropouts.

$\alpha(E(\log(h)|i)\log(\bar{h}))$  represent the “real” return to college, that is the expected increase in wage determined by college attendance. In the following table I report the result of the decomposition for both college graduates and college dropouts (in parenthesis, the relative weight).

TABLE 8: *Wage premium decomposition*

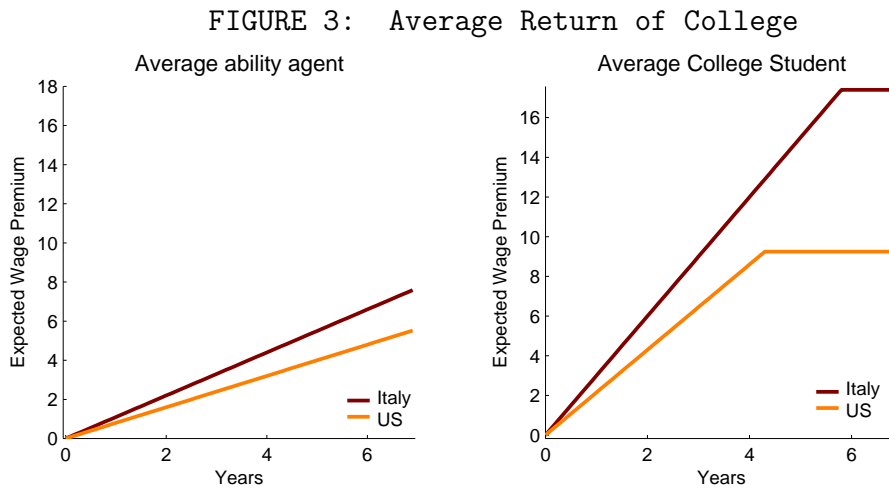
	$\Delta E(\log(w))$	$\beta$	$\Delta E(\log(\gamma))$	$\beta \Delta E(\log(\gamma))$	$\alpha \Delta E(\log(h))$
<i>College graduate</i>					
<b>Italy</b>	0.234	0.016	1.748	0.047(20.13%)	0.188(79.87%)
<b>US</b>	0.468	0.358	1.1347	0.331(70.78%)	0.137(29.22%)
<i>College dropout</i>					
<b>Italy</b>	0.079	0.016	0.5444	0.015(18.31%)	0.065(81.69)
<b>US</b>	0.198	0.358	0.4612	0.135 (68.01%)	0.063(31.99%)

The decomposition confirms the previous intuition. Italian degrees incorporate more “knowledge” but the wage premium is higher in the US because of differences between ex-ante individual differences (that accounts for 70% of wage premium in US versus 20% in Italy%). As argued before this might depend on either a labor market that “rewards” more individual ability<sup>14</sup>, and/or a higher correlation between schooling decision and the return of ability<sup>15</sup>. Both mechanisms are described by the variable  $\beta$ . Finally, I present the implied classical return to schooling parameters. In the following graphs I show the return of schooling, as the expected wage premium (over a high school graduate) per year of schooling, for the average individual, and an average college student respectively. We can observe that in both cases, the expected return of schooling is higher in Italy. In Italy an average individual can expect that each year of college increases the wage by 1.15%, while an average American can expect only an increase of 0.75% per year of college. Only considering the college worker the expected premia are of 4.3% and 2.3% respectively. Obviously my coefficients look lower than those estimated by using the standard Mincer Regression. If personal ability is not

<sup>14</sup>For instance,  $\beta=0$  means that wage are set independently of personal ability.

<sup>15</sup>A higher correlation between schooling decision and the return of ability on wage implies either that the abilities necessary in school are those that have a higher value in the labor market and/or that the schooling selection is more efficient.

completely observable, the coefficients of a Mincer regression will tend to "overestimate" the real one.



The difference will be higher the higher is the return to individual ability on wage, the noisier is the observation of personal ability, and the higher is the correlation between ability and schooling choice. For the arguments explained above, we would expect the difference between our coefficient and the results of previous estimation be wider in the American case. The most recent literature estimates that expected wage growth for year of schooling is between 2.5% and 5% for Italy and between 7% and 10% for the US. Finally I simulate a set of wage/schooling data, and I perform standard Mincer regressions on year of schooling. We consider two benchmark cases, with full or zero observability of ability.

TABLE 9: *Simulated regression*

Country	Full observability	No observability
<b>Italy</b>	4.3%	4.7%
<b>US</b>	2.3%	6%

Again we see how the Italian coefficients do not improve significantly when we don't condition on personal ability, and they are very close to the typical results of empirical literature. Conversely, the estimated return of schooling for the US increases dramatically when we don't condition on ability and we are still significantly beyond the "typical" coefficients.

## B. Labor Mobility

My estimation for mobility parameters are comparable to previous literature results. The main difference is an higher on-the-job arrival rate for both countries. I focus on young individuals, and it's reasonable to expect an higher job-to-job mobility for workers at the beginning of their carriers. The differences are striking in all dimensions. For example, given my estimation of job arrival rates when unemployed, the average unemployment duration for a young Italian worker is almost 22 months ( $1/\lambda^u(1-F(\omega^*))=21.7$ ), almost three times the average unemployment duration for a young American worker (7.8 months). Also an employed Italian worker will wait more than 4 years (50 months) before receiving a new job offer, against the 11 months of an American. At the same time the American separation rate is 4-fold the Italian separation rate. As discussed earlier, lower mobility implies lower wage growth rate due to search on the job. My parametrization implies an expected annual wage growth rate in the first 5 years of above 7% in the US, and less than 4% in Italy. The effect of this on the value of schooling is significant. In the next table I try to asses the effect of labor mobility on four different moments relative to schooling choice:

1. Average expected value of college degree, computing the difference between the expected lifetime welfare of a college graduate and the expected lifetime income of an high school graduate;
2. Enrollment rate, percentage of high school graduates who enroll for at least one year;
3. Graduation rate, percentage of high school graduates who complete college;
4. Average length of time to Graduation college.

In the first two columns I will list the value of this statistic according to my parametrization, while in the third I report the moments for an hypothetical economy with all Italian parameters but with US labor market mobility.

TABLE 10

Statistics	US	Italy	Italy/US Mobility
<i>Value of College ('000s)</i>	295	101	258
<i>Enrollment Rate</i>	58%	43%	48%
<i>Graduation Rate</i>	27%	16%	19%
<i>Average length of Time to Graduation (years)</i>	4.8	7.2	7.6

Differences on mobility parameters can account for respectively 33% and 30% of the differences in enrollment and graduation rate between Italy and the US.

### C. Wage Dispersion

As shown in the descriptive statistics tables, wages are more disperse in the US, for all possible levels of education. Based on the estimation I perform a decomposition of the variance of the wage, in order to have a better understanding of the relationship between wage compression and schooling choice. In particular since:

$$\log(w) = \alpha h + \beta \gamma + \epsilon$$

we can decompose the Variance of (log of) wages in four components:

$$\begin{aligned} \text{Var}(\log(w)) &= \alpha^2 \text{Var}(\log(h)) + \beta^2 \text{Var}(\log(\gamma)) + 2\alpha\beta \text{Cov}(\log(\gamma), \log(h)) + \text{Var}(\epsilon) \\ \sigma_{\log(w)}^2 &= \alpha^2 \sigma_h^2 + \beta^2 \sigma_\gamma^2 + 2\alpha\beta \sigma_{\gamma h} + \sigma_\epsilon^2 \end{aligned}$$

The first term  $\alpha^2 \text{Var}(\log(h))$  represents the variability of wages due to different levels of human capital. Obviously, in my model, only dropout students displays heterogeneity in that dimension. The second component  $\beta^2 \sigma_\gamma^2$  represents the variability of wages due to innate ability, with  $\beta$  playing a pivotal role. The third term,  $2\alpha\beta \text{Cov}(\log(\gamma), \log(h))$  represents the correlation between schooling and innate ability. Finally the last term is common to all groups, and represents the variance of the idiosyncratic component of the firm-worker match.

TABLE 11: *Wage Dispersion decomposition*

	$\sigma_{\log(w)}^2$	$\beta^2 \sigma_\gamma^2$	$\alpha^2 \sigma_h^2$	$\beta \alpha \sigma_{\gamma h}$	$\sigma_\epsilon^2$
<i>Total</i>					
<b>Italy</b>	0.4275	0.0002	0.0054	0.0019	0.4200
<b>US</b>	0.4996	0.0232	0.0032	0.0149	0.4583
<i>College graduate</i>					
<b>Italy</b>	0.4201	0.0001	0	0	0.4200
<b>US</b>	0.4727	0.0144	0	0	0.4583
<i>College dropout</i>					
<b>Italy</b>	0.4233	0.0001	0.0028	0.0004	0.4200
<b>US</b>	0.4631	0.0010	0.0013	0.0012	0.4583
<i>High School</i>					
<b>Italy</b>	0.4200	0.000	0	0	0.4200
<b>US</b>	0.4601	0.0018	0	0	0.4583

We can observe that both exogenous components of variability (the idiosyncratic shock  $\epsilon$  and the innate ability  $\gamma^\beta$ ) are more disperse. And if the difference of the variability of  $\epsilon$  is less relevant, the variability of  $\gamma^\beta$  in US is 100 times than in Italy. Once again it appears obvious that the way in which individual ability is rewarded differs significantly between Italy and the US, and can play a significant role in explaining the differences in schooling decision. This appears even more obvious when we compute the implied correlation coefficients between ability and wage, and schooling and wage, that is:

TABLE 12

	$\rho_{\gamma w}$	$\rho_{hw}$
<i>Total</i>		
<b>Italy</b>	0.2271	0.1519
<b>US</b>	0.3539	0.3709
<i>College graduate</i>		
<b>Italy</b>	0.0154	
<b>US</b>	0.1745	
<i>College dropout</i>		
<b>Italy</b>	0.0769	0.0929
<b>US</b>	0.1022	0.1019
<i>High School</i>		
<b>Italy</b>	0.0109	
<b>US</b>	0.0626	

As before, I perform a simple exercise, to asses the relevance of wage compression on schooling choices. I compute the predicted schooling choice for an Italian economy with US wage dispersion by changing either or both exogenous sources of wage compression in my model ( $\sigma_\epsilon$  and  $\beta$ ).

TABLE 13

Statistics	US	Italy	Italy	Italy	Italy
			$\sigma_\epsilon = 0.7180$	$\beta = .2918$	$\sigma_\epsilon = 0.7180$ $\beta = .2918$
<i>Value of College ('000s)</i>	295	101	133	413	450
<i>Enrollment Rate</i>	58%	43%	46%	51%	51%
<i>Graduation Rate</i>	27%	16%	17%	19%	20%
<i>Time to Graduation (ys)</i>	4.8	7.2	7.4	7.5	7.5

## 5. Conclusions

This paper represents a first attempt to build a model able to analyze at both a quantitative and qualitative level the interaction between schooling choice and labor market institutions. The classical search model represents the natural candidate to study the effect of market frictions on worker's behavior. By adding endogenous schooling choice I am able to build a simple framework where college decision and wage premium are endogenously determined by the interaction of market frictions and workers heterogeneous abilities. The model fits well US and Italian data, and reveals how differences in labor market institutions between the two countries can significantly account for differences in both educational attainment and observed college premium.

Two features are particularly relevant. First, low Italian labor mobility decreases the return to education and therefore optimal investment in human capital. Second, estimates suggest wages are dramatically less elastic to innate ability in Italy than in the US. This accounts for 1) low observed Italian college premium; 2) low Italian wage dispersion. Moreover, if schooling and ability are complementary, a low return to ability reduces return to schooling and therefore educational attainments. In the model, wage elasticity with both schooling and innate ability are exogenous, but the results suggests that policies affecting wage settings can play a pivotal role in shaping the return to education. The relevance of a more detailed analysis of wage setting system is confirmed by the empirical results of the estimation. Indeed while the model fits really well the first moments of Italian and US data, it fails to capture other features of wage distributions. In particular, it underestimates Italian

wage concentration and doesn't capture the skewness of the US. Both features can be easily theoretically related to the role of collective bargaining and minimum wage respectively. I believe that a model able to endogenize the features of the wage setting system (such as collective versus individual bargaining, minimum wage policies, tax schedules) could shed an important light on the relationship between labor market and human capital investment.

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## Appendix 1

Let  $\bar{U}$  and  $\bar{V}(\omega)$  solve

$$(r + \lambda^e + \delta) \bar{V}(\omega) = \omega + \lambda^e \int \text{Max}[\bar{V}(\omega'), \bar{V}(\omega)] dF(\omega') + \delta \bar{U} \quad (6)$$

$$(r + \lambda^u) \bar{U} = \lambda^u \int \text{Max}[\bar{V}(\omega'), \bar{U}] dF(\omega') \quad (7)$$

It's easy to verify that each solution is the problem must satisfy, for each pair  $(\gamma, h)$ ,  $V(\omega, \gamma, h) = \bar{V}(\omega)h^\alpha\gamma^\beta$  and  $U(\gamma, h) = \bar{U}h^\alpha\gamma^\beta$ . The properties of the optimal policy functions for unemployed and employed workers are standard.

## Appendix 2

We need to solve the differential equation

$$rW_b(h, \gamma) = \max_x -f - x + \rho\gamma^\eta x^{1-\eta} DW_b(h, \gamma) \quad (8)$$

with boundary condition

$$W_b(b, \gamma) = U(b, \gamma) \quad (9)$$

Solving the first-order condition yields

$$x = \gamma(\rho(1 - \eta)DW_b)^{\frac{1}{\eta}} \quad (10)$$

Substituting this back into the Bellman equation gives:

$$rW_b(h, \gamma) = -f + \gamma(1 - \eta)^{\frac{1-\eta}{\eta}} \eta (\rho DW_b)^{\frac{1}{\eta}} \quad (11)$$

This has a general solution

$$W(h, \gamma) = -\frac{f}{r} + \left( \frac{1-\eta}{\eta} \frac{r}{\gamma \rho^{\frac{1}{\eta}}} \right)^{\frac{\eta}{1-\eta}} [h - D]^{\frac{1}{1-\eta}} \quad (12)$$

where the constant  $D$  follows from the boundary condition

$$W(b, \gamma) = U(b, \gamma) = \bar{U} h^\alpha \gamma^\beta \quad (13)$$

$$D = b - \left( \frac{\eta}{1-\eta} \frac{\gamma \rho^{\frac{1}{\eta}}}{r} \right)^\eta \left( \bar{U} h^\alpha \gamma^\beta + \frac{f}{r} \right)^{1-\eta} \quad (14)$$

Hence

$$W(b, \gamma) = -\frac{f}{r} + \left[ \left( \frac{1-\eta}{\eta} \frac{r}{\gamma \rho^{\frac{1}{\eta}}} \right)^\eta [h - b] + \left( \bar{U} h^\alpha \gamma^\beta + \frac{f}{r} \right)^{1-\eta} \right]^{\frac{1}{1-\eta}} \quad (15)$$

Consider an agent whose optimal target education is  $b$ .

$$T = \inf\{t | h_t = b\} \quad (16)$$

The dynamic of  $h_t$  is described by the following differential equation:

$$\begin{aligned} \dot{h}_t &= (\rho \gamma^\eta D W (1-\eta))^{\frac{1-\eta}{\eta}} \rho \gamma^\eta \\ &= \rho \gamma^\eta \left[ \left[ \left( \frac{1-\eta}{\eta} \right)^\eta \frac{r^\eta}{\rho \gamma^\eta} [h_t - b] + \left( \bar{U} b^\alpha \gamma^\beta + \frac{f}{r} \right)^{1-\eta} \right]^{\frac{\eta}{1-\eta}} \left( \frac{1-\eta}{\eta} \right)^\eta r^\eta \right]^{\frac{1-\eta}{\eta}} \\ &= \rho \gamma^\eta \left[ \left( \frac{1-\eta}{\eta} \right)^\eta \frac{r^\eta}{\rho \gamma^\eta} [h_t - b] + \left( \bar{U} b^\alpha \gamma^\beta + \frac{f}{r} \right)^{1-\eta} \right] \left( \frac{1-\eta}{\eta} \right)^{1-\eta} r^{1-\eta} \\ &= (h_t - b) r \frac{1-\eta}{\eta} + \left( \bar{U} b^\alpha \gamma^\beta + \frac{f}{r} \right)^{1-\eta} \left( r \frac{1-\eta}{\eta} \right)^{1-\eta} \rho \gamma^\eta \end{aligned}$$

The previous differential equation has a general solution:

$$h_t = [B_0]e^{At} + b - \frac{F}{a} \quad (17)$$

where

$$A = r \frac{1-\eta}{\eta}; \quad (18)$$

$$F = \left( \bar{U}b^\alpha \gamma^\beta + \frac{f}{r} \right)^{1-\eta} \left( r \frac{1-\eta}{\eta} \right)^{1-\eta} \rho \gamma^\eta \quad (19)$$

We can solve for the coefficient  $B_0$  by using the boundary condition at time 0,  $h_0 = \bar{h}_0$ . There fore

$$h_t = \left( h_0 - b + \frac{F}{A} \right) e^{At} - \frac{F}{A} + b$$

We can now compute time in college  $T$  with the boundary condition:

$$\begin{aligned} h_T &= b \\ \left( h_0 - \frac{F}{A} + b \right) e^{AT} - \frac{F}{A} + b &= b \\ F &= e^{AT}(Ah_0 + F - Ab) \\ e^{AT} &= \frac{F}{Ah_0 + F - Ab} \\ T &= A^{-1} \ln \left( \frac{F}{Ah_0 + F - Ab} \right) \end{aligned}$$

### Appendix 3

I first prove a proposition that relates  $\bar{U}$  to the optimal schooling choice.

PROPOSITION 1. For each  $\gamma$ ,  $h^*(\gamma)$  is non-decreasing in  $\bar{U}$

**Proof.** Let's denote with  $K$  the coefficient  $\frac{1}{\gamma^\eta \rho(1-\eta)^{1-\eta} \eta^\eta}$  and recall the definition of  $h^*$

$$h^*(\gamma) = \min\{\bar{h}_0, \max[b^*(\gamma), \bar{h}]\} \quad (20)$$

where  $b^*$  is the unique value solving

$$k (r\bar{U}b^{*\alpha}\gamma^\beta + f)^\eta - \alpha\bar{U}b^{*\alpha-1}\gamma^\beta = 0 \quad (21)$$

Total differentiating equation (3) with respect to  $\bar{U}$  yields

$$\begin{aligned} & r\eta b^{*\alpha}\gamma^\beta K (r\bar{U}b^{*\alpha}\gamma^\beta + f)^{\eta-1} - \alpha b^{\alpha-1}\gamma^\beta + \\ & + \frac{\partial b^*}{\partial \bar{U}} \left[ K\alpha r\bar{U}b^{*\alpha-1}\gamma^\beta \eta (r\bar{U}b^{*\alpha}\gamma^\beta + f)^{\eta-1} - \alpha(\alpha-1)\bar{U}b^{*\alpha-1}\gamma^\beta \right] \end{aligned} \quad (22)$$

Rearranging the terms

$$\frac{\partial b^*}{\partial \bar{U}} = \frac{\alpha\bar{U}b^{*\alpha-1}\gamma^\beta - r\eta b^{*\alpha}\gamma^\beta K (r\bar{U}b^{*\alpha}\gamma^\beta + f)^{\eta-1}}{K\alpha r\bar{U}b^{*\alpha-1}\gamma^\beta \eta (r\bar{U}b^{*\alpha}\gamma^\beta + f)^{\eta-1} - \alpha(\alpha-1)\bar{U}b^{*\alpha-1}\gamma^\beta} \quad (23)$$

The denominator is positive, since  $\alpha \in (0, 1)$ . Using equation 3 we can simplify the numerator:

$$\begin{aligned} & \alpha b^{*\alpha-1}\gamma^\beta - r\eta b^{*\alpha}\gamma^\beta K (r\bar{U}b^{*\alpha}\gamma^\beta + f)^{\eta-1} = \\ & = \alpha b^{*\alpha-1}\gamma^\beta - r\eta b^{*\alpha}\gamma^\beta \frac{\alpha b^{*\alpha-1}\gamma^\beta}{r\bar{U}b^{*\alpha}\gamma^\beta + f} = \alpha b^{*\alpha-1}\gamma^\beta \left[ 1 - \eta \frac{r\bar{U}b^{*\alpha}\gamma^\beta}{r\bar{U}b^{*\alpha}\gamma^\beta + f} \right] > 0 \end{aligned} \quad (24)$$

■

## Proof of Theorem 1

Integrating by parts (1) and (2):

$$r\bar{U} = \lambda^u \int_{\omega^*} \frac{1 - F(\omega)}{\delta + r + \lambda^e(1 - F(\omega))} d\omega \quad (25)$$

where

$$\omega^* = (\lambda^u - \lambda^e) \int_{\omega^*} \frac{1 - F(\omega)}{\delta + r + \lambda^e(1 - F(\omega))} d\omega \quad (26)$$

Total differentiating the previous equation with respect to  $\lambda^e$

$$\begin{aligned} \frac{\partial \omega^*}{\partial \lambda^e} &= -\frac{\partial \omega^*}{\partial \lambda^e} (\lambda^u - \lambda^e) \frac{1 - F(\omega^*)}{\delta + r + \lambda^e(1 - F(\omega^*))} - \\ &\quad - \int_{\omega^*} \frac{1 - F(\omega)}{\delta + r + \lambda^e(1 - F(\omega))} d\omega - \\ &\quad - (\lambda^u - \lambda^e) \int_{\omega^*} \frac{(1 - F(\omega))^2}{(\delta + r + \lambda^e(1 - F(\omega)))^2} d\omega \end{aligned} \quad (27)$$

Rearranging the terms:

$$\frac{\partial \omega^*}{\partial \lambda^e} = \frac{-\frac{\omega^*}{\lambda^u - \lambda^e} - (\lambda^u - \lambda^e) \int_{\omega^*} \frac{(1 - F(\omega))^2}{(\delta + r + \lambda^e(1 - F(\omega)))^2} d\omega}{1 + (\lambda^u - \lambda^e) \frac{1 - F(\omega^*)}{\delta + r + \lambda^e(1 - F(\omega^*))}} \quad (28)$$

From equation (25) and (26)

$$U = \frac{\lambda^u \omega^*}{(\lambda^u - \lambda^e) r} \quad (29)$$

that implies

$$\begin{aligned} \frac{\partial \bar{U}}{\partial \lambda^e} &= \frac{1}{r} \frac{\lambda^u}{(\lambda^u - \lambda^e)^2} \omega^* + \frac{\lambda^u}{\lambda^u - \lambda^e} \frac{\partial \omega^*}{\partial \lambda^e} \\ &= \frac{1}{r} \frac{\lambda^u}{\lambda^u - \lambda^e} \left[ \frac{\frac{\omega^*}{\lambda^u - \lambda^e} + \omega^* \frac{1 - F(\omega^*)}{\delta + r + \lambda^e(1 - F(\omega^*))}}{1 + (\lambda^u - \lambda^e) \frac{1 - F(\omega^*)}{\delta + r + \lambda^e(1 - F(\omega^*))}} \right. \\ &\quad \left. - \frac{\frac{\omega^*}{\lambda^u - \lambda^e} + (\lambda^u - \lambda^e) \int_{\omega^*} \frac{(1 - F(\omega))^2}{(\delta + r + \lambda^e(1 - F(\omega)))^2} d\omega}{1 + (\lambda^u - \lambda^e) \frac{1 - F(\omega^*)}{\delta + r + \lambda^e(1 - F(\omega^*))}} \right] \\ &= \frac{\lambda^u}{r} \frac{\int_{\omega^*} \frac{(1 - F(\omega))(1 - F(\omega^*))}{(\delta + r + \lambda^e(1 - F(\omega))(\delta + r + \lambda^e(1 - F(\omega^*)))} d\omega - \int_{\omega^*} \frac{(1 - F(\omega))^2}{(\delta + r + \lambda^e(1 - F(\omega)))^2} d\omega}{1 + (\lambda^u - \lambda^e) \frac{1 - F(\omega^*)}{\delta + r + \lambda^e(1 - F(\omega^*))}} > 0 \end{aligned} \quad (30)$$

where the last inequality comes from the fact that  $\frac{1-F(\omega)}{\delta+r+\lambda^e(1-F(\omega))}$  is decreasing in  $\omega$ .

■

## Proof of Theorem 2

Let's define a family of economies  $\mathcal{E}(\kappa)$  where  $\kappa$  is a measure of mobility such that  $\{\lambda^u, \lambda^e, \delta\}(\kappa) = \{k\bar{\lambda}^u, \kappa\bar{\lambda}^e, \kappa\bar{\delta}\}$ .

$$\begin{aligned} \frac{\partial \omega^*}{\partial \kappa} &= \frac{r}{\kappa^2} (\lambda^u - \lambda^e) \int_{\omega^*} \frac{(1-F(\omega))}{(\delta+r+\lambda^e(1-F(\omega)))^2} d\omega \\ &\quad - \frac{\partial \omega^*}{\partial \kappa} \frac{(1-F(\omega^*))}{(\delta+r+\lambda^e(1-F(\omega^*)))^2} \\ \frac{\partial \omega^*}{\partial \kappa} &= \frac{\frac{r}{\kappa^2} (\lambda^u - \lambda^e) \int_{\omega^*} \frac{(1-F(\omega))}{(\delta+r+\lambda^e(1-F(\omega)))^2} d\omega}{1 + \frac{(1-F(\omega^*))}{(\delta+r+\lambda^e(1-F(\omega^*)))^2}} \geq 0 \end{aligned}$$

and follows that

$$\frac{\partial U}{\partial \kappa} = \frac{1}{r} \frac{\partial \omega^*}{\partial \kappa} > 0 \quad (31)$$

■

## Evolution of the employment probability

Let's define  $p_t$  the probability of being employed at time  $t$ . The evolution of  $p - t$  is described by the following differential equation:

$$\dot{p}_t = -p_t \delta + \lambda_u (1 - F(\omega^*)) (1 - p_t) = h - p_t (\delta + \lambda_u (1 - F(\omega^*))) \quad (32)$$

that has a general solution of that type:

$$p_t = A_0 e^{\alpha t} + A_1 \quad (33)$$

After solving for  $A_1$  and  $\alpha$  we can find  $A_0$  by using the boundary condition at time 0.

$$p_t = A_0 e^{(\delta + \lambda_u(1 - F(\omega^*)))t} + \frac{\lambda_u(1 - F(\omega^*))}{\lambda_u(1 - F(\omega^*)) + \delta} \quad (34)$$

$$p_0 = 0 \rightarrow A_0 + \frac{\lambda_u(1 - F(\omega^*))}{\lambda_u(1 - F(\omega^*)) + \delta} = 0 \quad (35)$$

$$p_t = \frac{\lambda_u(1 - F(\omega^*))}{\lambda_u(1 - F(\omega^*)) + \delta} (1 - e^{(\delta + \lambda_u(1 - F(\omega^*)))t}) \quad (36)$$

The long term employment probability is

$$\lim_{t \rightarrow \infty} p_t = \frac{\lambda_u(1 - F(\omega^*))}{\lambda_u(1 - F(\omega^*)) + \delta} \quad (37)$$

### Proof of Theorem 3

Integrating by parts we can rewrite

$$\begin{aligned} \frac{\partial \omega^*}{\partial \sigma} &= (\lambda^u - \lambda^e) \int_{\omega^*} \frac{-(\delta + r)F_\sigma(\omega)}{(\delta + r + \lambda^e(1 - F(\omega)))^2} d\omega = \\ &= (\lambda^u - \lambda^e) \left[ \int_{\omega^*}^{\theta} \frac{-(\delta + r)F_\sigma(\omega)}{(\delta + r + \lambda^e(1 - F(\omega)))^2} d\omega + \int_{\theta} \frac{-(\delta + r)F_\sigma(\omega)}{(\delta + r + \lambda^e(1 - F(\omega)))^2} d\omega \right] \geq \\ &\geq \frac{-(\lambda^u - \lambda^e)(\delta + r)}{(\delta + r + \lambda^e(1 - F(\theta)))^2} \int_{\omega^*} F_\sigma(\omega) \geq 0. \end{aligned}$$

hence

$$\frac{\partial \bar{U}}{\partial \sigma} = \frac{\lambda^u r}{(\lambda^u - \lambda^e)} \frac{\partial \omega^*}{\partial \sigma} > 0. \quad (38)$$

■

### Proof of Theorems 4,5

The result comes from a simple application of inverse function theorem to equation (21) ■

## Appendix 4: Estimation method

The estimation procedure used is a N-step GMM, where the weighting matrix is approximated until convergence of the estimator. Let's  $\theta$  be the set of parameters to estimate, that is  $\theta = \{\mu_\epsilon, \sigma_\epsilon, \lambda^e, \lambda^u, \delta, \bar{h}, \beta, \alpha, \rho, \eta\}$  and  $X = \{x_i\}_1^I$  the set of observations. Let's denote with  $M(X)$ , a vector of some given population moments, and with  $\hat{M}(\theta)$  the vector of the corresponding moment generated by the model under a given parametrization. The estimator  $\hat{\theta}_n$  solves:

$$\hat{\theta}_n = \min_{\theta} G(X; \theta)' W_n G(X; \theta)$$

where  $G(X; \theta) = M(X) - \hat{M}(\theta)$  and  $W_n$  is a symmetric weighting matrix update according to the operator  $W_{n+1} = [G(X; \theta_n)G(X; \theta_n)']^{-1}$  with  $W_1 = I$ . The procedure is reiterated until convergence of the parameters, that is until  $\max[\hat{\theta}_{n+1} - \hat{\theta}_n] \leq e^{-10}$ . Finally, I estimate of the asymptotic variance of the parameters, assuming asymptotic normality of the estimator.

$$V(\hat{\theta}) = (\hat{J}' \hat{W} \hat{J})^{-1} \hat{J}' \hat{W} \hat{J} (\hat{J}' \hat{W} \hat{J})^{-1}$$

where  $\hat{W} = [G(X; \hat{\theta})G(X; \hat{\theta})']^{-1}$  and is the estimation of the Jacobian matrix  $\hat{J} = \left. \frac{\partial M(\theta)}{\partial \theta} \right|_{\theta=\hat{\theta}}$