



## *Romer's Model of Expanding Varieties*

Econ 4960: Economic Growth



## *The AK model and Policy Debates*

- ✦ The fact that savings rate can affect the growth rate (and in a big way) made the AK model very popular in policy discussions.
- ✦ It makes government policy very important for growth.
- ✦ In a famous paper, Lucas (1990) called tax cuts on savings as the “*largest genuinely free lunch I have seen in 25 years in this business.*”
- ✦ Even today when candidates fiercely debate about taxes, an important part of discussion revolves around growth

Econ 4960: Economic Growth



## *The AK model and Policy Debates*

- ✦ King and Rebelo (1990, JPE): The “welfare effect” of a 10 percent increase in income tax is 40 times larger in an (AK) endogenous growth model (65% of consumption) than it is in a neoclassical growth model (1.6% of consumption)
- ✦ Stokey and Rebelo (1995) and Lucas (1990) argue that if endogenous growth models are calibrated to plausible values the effect on welfare is not likely to be large.
- ✦ Note that this “gift” of the AK model is also its “curse.”
- ✦ Because if tax differences are so important for growth, how come countries like Sweden with extremely high tax rates grow as fast as the US?

Econ 4960: Economic Growth

## *Shortcomings of the AK model*

- ✦ **Growth is the outcome of accidents**---actions that are completely unintentional.
- ✦ **Externalities must be substantial**: For example, the capital bought by an investor contributes twice as much to others’ production than to his/her own. Same for human capital: Your education benefits others more than it benefits you.
- ✦ Alternatively stated, the Social return on many types of investments **far exceed** their private return.
- ✦ If externalities are really that big, individuals will typically find a way to capitalize on them (A doctor will not distribute advise on the street, etc.)
- ✦ Exponent on **externality must be exactly  $1-\alpha$** . Otherwise, there will be no BGP

Econ 4960: Economic Growth



## Back to Romer's Model

- ✦ A key concept in Romer's model will be the production of ideas.
- ✦ But ideas are inherently difficult to even define, let alone measure.
- ✦ One imperfect measure is the number of patents, although it tells us little about the value of patents.
- ✦ Another measure is the resources devoted to R&D

Econ 4960: Economic Growth

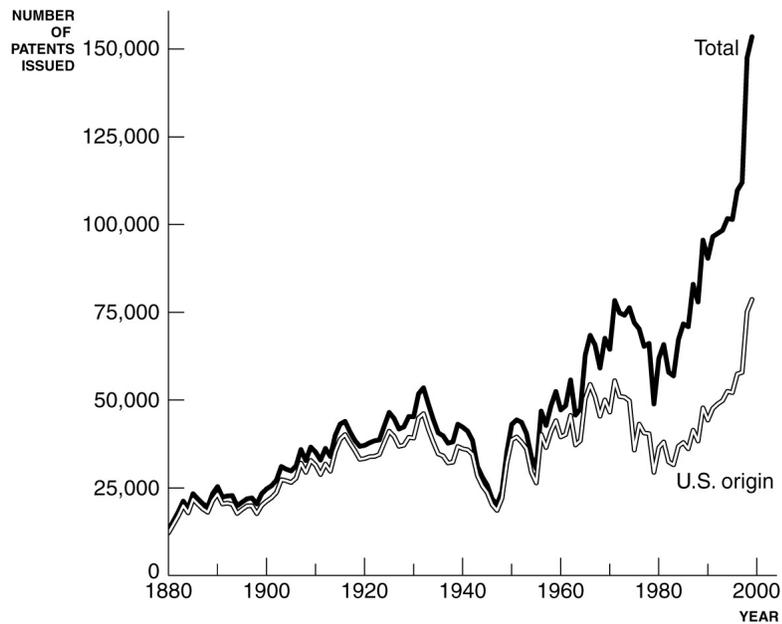
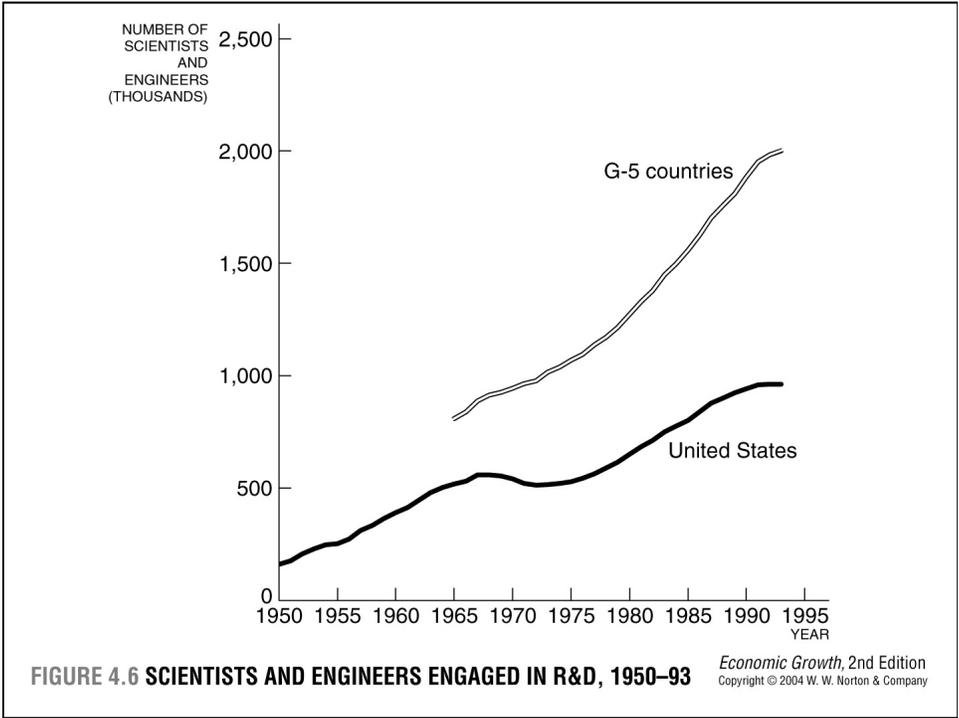


FIGURE 4.5 PATENTS ISSUED IN THE UNITED STATES, 1880–1999

*Economic Growth*, 2nd Edition  
Copyright © 2004 W. W. Norton & Company



*Economic Forces Underlying  
Technical Progress*

Econ 4960: Economic Growth



## Preliminary ideas

- ✦ In contrast to Solow model, Romer's model is better interpreted as a model of the growth of the World economy.
- ✦ Or alternatively stated: how the **world technology frontier** moves forward
- ✦ Notice that this interpretation of Romer's model is rather new.
- ✦ **Implication:** One has to think about different factors than emphasized here to understand how poor countries catch up with the rich, growth miracles, etc.

Econ 4960: Economic Growth



## Elements of the Model

- ✦ We start from a simple version and progressively build towards the more general model.
- ✦ Here assume that production function is Cobb-Douglas:
$$Y = K^\alpha (AL_Y)^{1-\alpha}$$
- ✦ Remember from last lecture that this production function exhibits IRS in all three inputs (because of the non-rivalrous nature of ideas).
- ✦ Capital accumulation:  $\dot{K} = s_K Y - dK$
- ✦ Population growth:  $\dot{L} / L = n$

Econ 4960: Economic Growth



## Novel component: Production of Ideas

- ✦ The previous three equations are the same as in the Solow model. **So what's new?**
- ✦ Romer assumes that the growth rate of new ideas is proportional to the number of people trying to discover new ideas:  $\dot{A} = \bar{\delta} L_A$
- ✦ Where  $\bar{\delta}$  is the rate at which new ideas are discovered.
- ✦ Question: Should  $\bar{\delta}$  be constant, increasing or decreasing in A?
- ✦ Write:  $\bar{\delta} = \delta A^\phi$ . If the exponent is positive (negative) ideas will be easier (harder) to invent as more ideas are around

Econ 4960: Economic Growth



## Novel component: Production of Ideas

- ✦ **Question:** Does it make sense of the growth rate of ideas to be linear in the number of “inventors”?
- ✦ That is, if we increase the number of medical researchers by 10-fold are we going to cut down the time necessary to cure cancer, AIDS, etc., by tenfold?
- ✦ A better specification may be:  $\dot{A} = \delta L_A^\lambda A^\phi$
- ✦ A plausible assumption is that  $\lambda < 1$ , which indicates a negative externality (or congestion effects, such as in traffic, labor markets, etc.)
- ✦ Therefore,  $\lambda < 1$  is called “stepping on toes effect.”
- ✦  $\phi > 0$  is called “standing on shoulders effect”

Econ 4960: Economic Growth



## *Allocation of Labor Across Uses*

- ✦ We have:  $L_Y + L_A = L$
- ✦ How does the society decide on optimal allocation of labor between production and innovation?
- ✦ To answer this fully, we would need some high-tech math (Dynamic programming to be exact) and solve for utility maximization.
- ✦ Here we will make a Solow-like assumption and assume that it is given by a constant:  $L_A / L = s_R$

Econ 4960: Economic Growth



## *Solving the Model*

- ✦ A key question to ask in any model of economic growth: Is there a BGP?
- ✦ Short answer: yes.
- ✦ Now we need to show how.
- ✦ First, we have:  $g_y = g_k = g_A$
- ✦ How can you show this? (Exercise!)
- ✦ So if we can find the rate of growth of A, we can find the growth rate in the economy.

Econ 4960: Economic Growth



## Solving the Model (cont'd)

- What is the rate of technological progress along the BGP?

$$\dot{A} = \delta L_A^\lambda A^\phi \Rightarrow g_A \equiv \frac{\dot{A}}{A} = \delta \frac{L_A^\lambda}{A^{1-\phi}}$$

- By definition of BGP, we must have  $g_A$  is a constant.

$$\text{Which implies: } 0 = \lambda \frac{\dot{L}_A}{L_A} - (1-\phi) \frac{\dot{A}}{A} \quad (*)$$

- Notice that along a BGP we need this:  $\frac{\dot{L}_A}{L_A} = \frac{\dot{L}}{L} = n$

- Substitute this into (\*):  $g_A = \frac{\lambda n}{1-\phi}$

- We've just solved the model!

Econ 4960: Economic Growth



## What does this equation tell us? $g_A = \frac{\lambda n}{1-\phi}$

- So this is the solution to the basic version of Romer's model. What does it mean?
  - There is no growth without population growth.
    - To understand this consider the case when  $\lambda=1$ ,  $\phi=0$ .
    - Is this implication plausible? How do you test it?
  - Romer's model has a special case that implies long run growth even with zero population growth:  $\lambda = 1$  and  $\phi = 1$

$$\dot{A} = \delta L_A A \rightarrow \frac{\dot{A}}{A} = \delta L_A$$

- But this specification would imply that TFP growth in the world should have accelerated dramatically in the past 200 years!

Econ 4960: Economic Growth



## *Policy and other Implications*

- ✦ Notice that saving rate has no effect on long-run growth rate. (as in the Solow model)
- ✦ The fraction of population employed in R&D has no effect on long-run growth.
- ✦ They do however have level effects (as in Solow).