N13. The Size Distribution of Cities

- Up to this point have looked at cities in isolation: *the internal structure of cities*
- Now look at cities as part of the larger economy
- Begin by a discussion of the size distribution of cities

Facts about City Size Distributions

- Look at distribution of MSAs(metropolitan statistical area), CMSA (consolidated metropolitan area) from 2000
- Rank Size Rule or Zipf's Law: (empirical regularity).

$$\begin{aligned} rank * size &= \textit{constant} \\ \ln(rank) + \ln(size) &= \ln(\textit{constant}) \\ \ln(rank) &= \ln(\textit{constant}) - \ln(size) \end{aligned}$$

- MSA/CMSA data, run regression with top 100 get

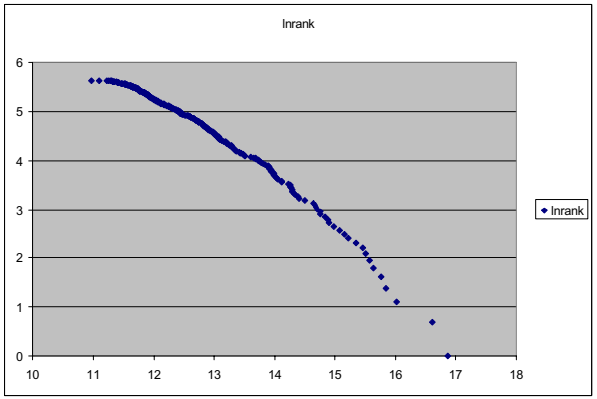
$$\ln(rank) = 18.132 - 1.04 \ln(size)$$

with a standard error on slope of .018 and an R^2 of .98

Metro Populations (MSA and CMSA) from Census 2000

	rank	population	lnrank	lnpop
New York--Northern New Jersey--Long Island, NY--NJ--CT--PA CMSA	1	21,199,865	0.000	16.86951
Los Angeles--Riverside--Orange County, CA CMSA	2	16,373,645	0.693	16.61118
Chicago--Gary--Kenosha, IL--IN--WI CMSA	3	9,157,540	1.099	16.03009
Washington--Baltimore, DC--MD--VA--WV CMSA	4	7,608,070	1.386	15.84472
San Francisco--Oakland--San Jose, CA CMSA	5	7,039,362	1.609	15.76703
Philadelphia--Wilmington--Atlantic City, PA--NJ--DE--MD CMSA	6	6,188,463	1.792	15.6382
Boston--Worcester--Lawrence, MA--NH--ME--CT CMSA	7	5,819,100	1.946	15.57666
Detroit--Ann Arbor--Flint, MI CMSA	8	5,456,428	2.079	15.5123
Dallas--Fort Worth, TX CMSA	9	5,221,801	2.197	15.46835
Houston--Galveston--Brazoria, TX CMSA	10	4,669,571	2.303	15.35658
Atlanta, GA MSA	11	4,112,198	2.398	15.22947
Miami--Fort Lauderdale, FL CMSA	12	3,876,380	2.485	15.17041
Seattle--Tacoma--Bremerton, WA CMSA	13	3,554,760	2.565	15.0838
Phoenix--Mesa, AZ MSA	14	3,251,876	2.639	14.99474
Minneapolis--St. Paul, MN--WI MSA	15	2,968,806	2.708	14.90367
Cleveland--Akron, OH CMSA	16	2,945,831	2.773	14.8959
San Diego, CA MSA	17	2,813,833	2.833	14.85006
St. Louis, MO--IL MSA	18	2,603,607	2.890	14.77241
Denver--Boulder--Greeley, CO CMSA	19	2,581,506	2.944	14.76388

Plot of ln(rank) against ln(pop2000)



- Robust relationship across time and space
- Kind of power law holds in other contexts, firm size distribution, animal populations,...

Challenge to Theory of System of Cities

- Theory discusses various tradeoffs between agglomeration economies and congestion costs
- Industries differ in importance in agglomeration benefits
- So get a size distribution of cities
- But why follow Zipf's law? What is the chance of that?

A Model of Random Growth

- Take N cities. Let $s_{i,t}$ be size of city i at time t
- Let $g_{i,t}$ be growth of city i at time t,

$$s_{i,t+1} = (1 + g_{it})s_{i,t}$$

- Suppose $g_{i,t}$ is i.i.d across firms: Gibrats Law. Distribution of growth independent of initial size
- Suppose have a minimum size $s_{i,t} \geq \underline{s}$
- Then distribution of cities sizes converges approximately to rank-size rule.

Example with Random Growth

Growth = .5 or 1.5 with 50/50 probability. Initial size is 1. Minimum size =.1

city	period 1	period 2	period 3	period 5	period 5	period 6
1	1.000	0.500	0.250	0.125	0.188	0.281
2	1.000	1.500	0.750	1.125	1.688	0.844
3	1.000	0.500	0.250	0.125	0.100	0.150
4	1.000	0.500	0.750	1.125	0.563	0.844
5	1.000	0.500	0.250	0.375	0.188	0.281
6	1.000	1.500	2.250	1.125	1.688	0.844
7	1.000	0.500	0.750	0.375	0.563	0.844
8	1.000	1.500	0.750	0.375	0.188	0.100
9	1.000	1.500	0.750	0.375	0.188	0.281
10	1.000	1.500	2.250	1.125	1.688	2.531

For example, N=100, and 100 time periods. At end of random process get:Ln(rank) = 1.98 – 1.04ln(size), R^2 = .98

Agglomeration Benefits

Sources of gains usually can be classified into one of three kinds of benefits:

1. Scale economies along with transportation cost
2. Reallocation of resources in response to change
3. Gains from competition

Scale Economies leading to Localization/Urbanization

- Concentrated Market (Krugman, others)

—Fixed costs

—transportation costs

—Love of Variety (magnifies this effect)

- Knowledge Spillovers (Lucas, others)

Reallocation of Resources in Response to Change

- Dress factory story

—10 different colors of cloth, each must be produced in own factory

—Fashion goddess picks 5 different style dresses, each made of two colors out of the 10. All colors used for one style dress

—If know blue goes with red for sure, can have a blue factory next to a red factory in the middle of nowhere.

—But with uncertainty, need all dresses near each other.

- General argument for concentrating resources

Related Idea: Better Matching

- Power Couples
- Costa and Kahn paper

TABLE III
PROBABILITY OF LOCATIONAL CHOICE BY HOUSEHOLD TYPE

	1940	1970	1980	1990
Conditional on power couple				
Large metropolitan area	0.321	0.391	0.414	0.495
Midsize metropolitan area	0.254	0.313	0.325	0.295
Small and nonmetropolitan area	0.426	0.296	0.261	0.210
Conditional on part-power couple				
Large metropolitan area	0.319	0.362	0.371	0.421
Midsize metropolitan area	0.268	0.326	0.334	0.308
Small and nonmetropolitan area	0.413	0.312	0.295	0.271
Conditional on low-power couple				
Large metropolitan area	0.266	0.301	0.308	0.339
Midsize metropolitan area	0.240	0.299	0.312	0.292
Small and nonmetropolitan area	0.494	0.399	0.380	0.369